Characterization of the koch fractal embedded hexagonal loop frequency selective surface structure for X-band application

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Article Info
ABSTRACT
This paper presented the bandstop Koch fractal embedded hexagonal loop frequency selective surface (FSS) for the X-band application. The simulated transmission coefficient response ($S_{21}$) had been obtained by using CST software. The surface current distribution and the electric field density are illustrated to explain in detail the $S_{21}$ of the fractal based FSS structure. The proposed structure is highly insensitive towards angular stability and also polarization independent up to 70°. The parametric analysis on the effect of the periodicity, width, and height of the fractal FSS structure on the $S_{21}$ has been illustrated and discussed thoroughly.

Keywords:
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1. INTRODUCTION
The frequency selective surface (FSS) which also known as a spatial filter, is made up of 1-D or 2-D infinite periodic array of similar elements that allows the transmission and reflection of the plane wave that incident onto the structure [1]. An American physicist, David Rittenhouse, had discovered the potential of FSS to be used as spatial filter when he found out that the light spectrum’s colors being filtered through a silk scarf while observing a lamp, and became the pioneer for the study of the FSS structure. The filter response of a FSS is highly dependent on the two different array geometries, which are the patches and the slots. The patch array and the slot array represent the low pass filter characteristic and high pass filter characteristic, respectively, which can be explained by the Babinet’s principle. The combination of both arrays can yield the bandpass and bandstop filter characteristics. For the traditional FSS, a longer electrical length is required especially for the low frequency response. Furthermore, some applications require a smaller-sized FSS screen to achieve better filter performance. The parameters such as multiband and wide BW behavior, angular stability, and polarization independent properties are unachievable by utilizing the traditional FSS geometry, which can deteriorate the filter’s performance. To address these limitations, the utilization of the fractal structure is suggested to miniaturize the dimension of the FSS.
In 1991, the idea of utilizing fractal geometry such as Minkowski loop and the Hilbert curve had been introduced by Parker and El sheikh [2], to miniaturize the FSS structure. Few factors had led to the utilization of fractal geometry onto the FSS structure. The first factor is that a large number of traditional FSS unit cell required for obtaining the specified frequency response based on the mutual interactions of the unit cells [3]. The small size of the FSS screen is required for some applications to achieve high angular stability response. Furthermore, the traditional FSS geometry unable to provide the best filter’s performance in terms of multiband and broad bandwidth (BW) behaviors, angular stability and polarization independence [4]. Thus, fractal geometry is introduced to overcome these limitations. The uniqueness of the self-similarity of the fractal frequency selective surface structure had been reported widely in numerous studies for different applications [3, 5-11], which includes multiband and wide BW behavior, angular independent operations and low grating lobes appearance in a compact design.

Previously, the utilization of hexagonal geometry for FSS structure is proven to have higher angular stability than the traditional square geometry of FSS [12, 13]. The hexagonal geometry also able to employ the multiband and wide BW behavior that have high insensitivity towards angular stability and polarization independence. The hexagonal slot geometry had been applied together with Koch fractal structure as in [12] to achieve multiband and wide BW passband filter. For this paper, the utilization of Koch fractal to the hexagonal loop FSS structure is analyzed and discussed in detail in terms of frequency response behavior, angular stability and polarization independence. When the Koch fractal structure embedded to the hexagonal loop FSS structure, with the increased of the iteration level to 2nd level, the transmission coefficient ($S_{21}$) exceeding -10 dB over the range of 2.23GHz (from 8.24 GHz to 10.47 GHz), which is resonated at frequency of 9.5 GHz and thus applicable for X-band application. The proposed fractal FSS structure has higher stability toward angular stability at higher iteration level. However, for the TE mode polarization, the BW increased, while for the TM mode polarization, the BW decreased continuously as the oblique angles increased. The BW enhancement for the entire X band (8 GHz to 12 GHz) can be further study in the future. The parametric study which includes the periodicity, width and also the height of the proposed Koch fractal hexagonal loop FSS structure had been simulated by the CST software and discussed in detail in the results and discussions section.

2. DESIGN OF FRACTAL-BASED FREQUENCY SELECTIVE SURFACE

For the hexagonal loop structure, (1) shows that the resonant frequency ($f_r$) can be deduced when the electrical length of the geometry structure is half wavelength. As the FSS structure does not have the ground plane, the $f_r$ is highly dependent on the electromagnetic properties FSS structural element.

$$f_r = \frac{c}{2 \times \lambda_{\text{eff}}}$$  \hspace{1cm} (1)

$$\lambda_{\text{eff}} = \frac{\varepsilon_{\text{eff}} + \varepsilon_r}{2}$$  \hspace{1cm} (2)

where $c$ is the speed of light, $\varepsilon_{\text{eff}}$ is the effective permittivity and $\varepsilon_r$ is the permittivity of the dielectric material.

For the Koch fractal structure, the Iterated Function System (IFS), which was invented by John Hutchinson, was applied to the fractal structure by using the affine transformations, as reported in several previous studies [14-17]. Based on (3), a two-dimensional affine transformation, $w$, was introduced on the Euclidean plane, in which the parameters are represented by six variables (i.e., $a, b, c, d, e,$ and $f$).

$$w\left(\begin{array}{c} x \\ y \end{array}\right) = \left(\begin{array}{cc} a & b \\ c & d \end{array}\right)\left(\begin{array}{c} x \\ y \end{array}\right) + \left(\begin{array}{c} e \\ f \end{array}\right)$$  \hspace{1cm} (3)

where the matrix of $a, b, c$ and $d$, is defined as a linear transformation that includes a combination of rotation, scaling, and a sheer, and then translated by the matrix of $e$ and $f$.

For the acquired low frequency, a large FSS dimension is required to accommodate the longer electrical lengths, and this could result in the appearance of grating lobes [1, 18]. The grating lobe phenomenon arises due to large spacing between the unit cell elements (D) as well as the angle of the incident wave depicted in (4).

$$D < \frac{\lambda_o}{1 + \sin \theta}$$  \hspace{1cm} (4)

where $\lambda_o$ is the resonant wavelength of the FSS and $\theta$ is the angle of the incident wave.
Based on the (4), the periodicity of the unit cell should be less than the $\lambda_0$ at initial incident angle. With a higher value of incident angle, the unit cell periodicity should be half the free space wavelength to prevent the occurrence of the grating lobe phenomena—a limitation that can be prevented by implementing a fractal structure to miniaturize the FSS. For example, extensive studies over the past decade have integrated patch or slot elements with the fractal structures such as the Koch fractal structure [3, 19], the Minkowski [20] and the Minkowski island [21], the Interdigital fractal structure [22], the Sierpinski fractal structure [23], the Swastika fractal structure [24] etc., to reduce the size of the FSS element.

All the hexagonal loop and the proposed Koch fractal hexagonal loop FSS structures had been simulated by using the CST software. The FR4 substrate with $\varepsilon_r$ of 4.3 and the 1.6-mm-thick lossy substrate, with 0.035-mm-thick copper layer with a conductivity of $5.8 \times 10^7$ S/m, had been used as the dielectric substrate. Figure 1 shows the hexagonal loop and the proposed Koch fractal hexagonal loop FSS structures with optimized parameters such as the unit cell periodicity ($p = 7.22$ mm), the width of the hexagonal loop ($w = 0.20$ mm) and the side length of the hexagonal loop ($l = 3.33$ mm), which resonated at 11.03 GHz. The proposed Koch fractal hexagonal loop FSS structure which employed the first and the second iteration levels of the Koch fractal structure are shown in Figure 1(b) and 1(c). Figure 1d shows the simulation of a unit cell of the proposed FSS structure with unit cell boundary condition to simulate the frequency response of the proposed FSS.

![Figure 1](image)

The self-similarity feature of the Koch fractal structure based on (5) has allows the miniaturization of the FSS structure for obtaining the specified X-band frequency.

$$D = \frac{\log N}{\log (\frac{4}{3})}$$

(5)

where $sf$ = scaling fraction
$N$ = copies of the original geometry to be made
$D$ = self-similarity dimension
3. RESULTS AND DISCUSSION

For bandstop fractal based FSS structure, the parameters such as $f_r$, $S_{21}$, BW, angular stability and polarization independence were studied for determining the best performance of the bandstop fractal-based FSS structure for X-band application. Figure 2 shows the simulated $S_{21}$ results of the hexagonal loop and the proposed Koch fractal hexagonal loop FSS structures. Generally, the increasing of the iteration level would shift the $f_r$ to the low frequency region, which is agreeable with the simulated $S_{21}$ results. For the hexagonal loop FSS, the initial values of $f_r$ and BW are 11.03 GHz and 2.67 GHz, respectively. When the Koch fractal structure embedded to the hexagonal loop FSS structure, with the increased of the iteration level to 2nd level, the $f_r$ and BW had decreased to 9.5 GHz and 2.23 GHz (from 8.24 GHz to 10.47 GHz), respectively.

![Figure 2. The simulated $S_{21}$ for the hexagonal loop fractal based FSS structures. (a) hexagonal loop FSS, (b) 1st iteration level and (c) 2nd iteration level of Koch fractal hexagonal loop FSS](image)

Previously, it is known that the ability of the hexagonal geometry in providing better performance of FSS structure in terms of angular stability and polarization [12, 13]. Figure 3 depicted the simulated $S_{21}$ of the basic and the proposed Koch fractal hexagonal loop FSS structure for TE and TM polarized wave incidence at the variation of oblique angles ($0^\circ$, $10^\circ$, $20^\circ$, $30^\circ$, $40^\circ$, $50^\circ$, $60^\circ$, $70^\circ$). By comparing the first and the second iteration level of Koch fractal, the shifting of the $f_r$ as the increasing of oblique angles is less pronounced at higher iteration level of the fractal structure. However, for the TE mode polarization, the BW increased, while for the TM mode polarization, the BW decreased continuously as the oblique angles increased.

![Figure 3. Simulated $S_{21}$ for TE and TM mode polarizations](image)
Figure 3. The simulated $S_{21}$ at variation of oblique angles ($0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ, 70^\circ$)

Figure 4 depicted the surface current distribution and the electric energy density (the distribution of the electric field) for the hexagonal loop and also the proposed Koch fractal hexagonal loop FSS structure, for better understanding of the fractal FSS's physical mechanism. It can be observed that in Figures 4(a), 4(b) and 4(c), the strongest surface current density exists on the left and the right sides of the hexagonal loop structures. As the iteration level increased, the stronger surface current density appeared at the Koch fractal geometry of the hexagonal loop FSS structure. Figure 4(d), 4(e) and 4(f) show the E-field distribution of the hexagonal loop and also the proposed Koch fractal hexagonal loop FSS structures. It can be seen that the edge center of up and down sides of the hexagonal loop and the proposed Koch fractal hexagonal loop FSS structure had the strongest distribution of E-field. Based on this information, it is agreeable that the length of the current path is increased proportionally with the iteration level of the Koch fractal as can be seen in Figure 4, which lead to the reduction of $f_r$ as in (1) and allows the specify $f_r$ to be obtained.

Figure 4. The surface current distribution of (a) hexagonal loop FSS, (b) 1st iteration level of Koch fractal hexagonal loop FSS, and (c) 2nd iteration level of Koch fractal hexagonal loop FSS; The E-field distribution of (d) hexagonal loop FSS, (e) 1st iteration level of Koch fractal hexagonal loop FSS, and (f) 2nd iteration level of Koch fractal hexagonal loop FSS
3.1. Parametric analysis

The parametric analysis on the physical mechanism of the proposed Koch fractal hexagonal loop FSS structure had been done for investigating the effect of the periodicity (p), width (w) and height of the Koch fractal on the $S_{21}$ response. Figure 5 shows the simulated $S_{21}$ response for various ‘p’ value. It can be observed that for both of the iteration level of Koch fractal hexagonal loop FSS structure, the $f_c$ would be shifted to the high frequency region and at the same time, would decrease the value of BW and $S_{21}$ as the periodicity increased. The values of BW and $S_{21}$ with the periodicity of 7.0 mm are 2.33 GHz (8.33 GHz ~ 10.66 GHz) and -30.35 dB respectively, for the first iteration level of the Koch fractal. As the periodicity increased up to 8.0 mm, the decreased values of BW and $S_{21}$ are 1.85 GHz (9.03 GHz ~ 10.88 GHz) and -27.71 dB, respectively. For the second iteration, the values of BW and $S_{21}$ with the periodicity of 7.0 mm are 2.36 GHz (8.0834 GHz ~ 10.438 GHz) and -30.20 dB respectively. As the periodicity increased up to 8.0 mm, it affected the values of BW and $S_{21}$ which are 1.78 GHz (8.84 GHz ~ 10.62 GHz) and -27.54 dB, respectively. This is because the metal area in unit cell decrease, so the reflected power is lower, hence attenuation level in transmission is lower as well [25].

![Figure 5](image1.png)

Figure 5. The simulated $S_{21}$ results of the variation of the periodicity of hexagonal loops fractal based FSS structures. (a) 1st iteration level and (b) 2nd iteration level of Koch fractal hexagonal loop FSS

The simulated $S_{21}$ response for variation of ‘w’ is depicted in Figure 6. For both of the iteration level, the initial width of the proposed Koch fractal hexagonal loop FSS structure provides the BW of 2.14 GHz (8.62 GHz ~ 10.76 GHz) for the first iteration and 2.23 GHz (8.24 GHz ~ 10.47 GHz) for the second iteration level of fractal FSS structure. It can be seen that the increase of ‘w’ up to 0.5 mm had resulted in an increased value of BW and $f_c$, which is more significant for the second iteration level of the Koch fractal hexagonal loop FSS structure. However, the increased of ‘w’ would disturb the compactness of the fractal FSS structure.

![Figure 6](image2.png)

Figure 6. The simulated $S_{21}$ results of the variation of ‘w’ of hexagonal loops fractal based FSS structures. (a) 1st iteration level and (b) 2nd iteration level of Koch fractal hexagonal loop FSS
Figure 7 shows the effect of the height of the Koch fractal geometry on the $S_{21}$ response. The height of the Koch fractal is highly dependent on the indentation angle of the Koch fractal geometry structure, as the changes of the indentation angle are proportional to the changes of the height of the Koch fractal. The height of the Koch fractal is varied from 0.34 mm up to 0.98 mm for variation of indentation angle (20°, 30°, 40°, 50°, 60°, 70°, 80°). Based on Figure 7, at both iteration level, the value of $f_r$ had shifted to the left and the value of BW had decreased further. As the height increased, the electrical length of the Koch fractal is increased which lead to the decreased of the $f_r$, that is agreeable with the (1).

Figure 7. The simulated $S_{21}$ results of the variation of the height of the Koch fractal hexagonal loops FSS structures. (a) 1st iteration level and (b) 2nd iteration level of Koch fractal hexagonal loop FSS

4. CONCLUSION

In this paper, the design, the analysis of simulated $S_{21}$, and also the parametric analysis (periodicity, width, height) for the proposed Koch fractal hexagonal loop FSS structure is presented. The employment of Koch fractal to the hexagonal loop FSS structure is analyzed and discussed in detail in terms of frequency response behavior, angular stability and polarization independence. The fractal FSS structure is composed of Koch fractal hexagonal loop FSS structure. The proposed fractal FSS design provides $–10$ dB BW of 2.23 GHz (from 8.24 GHz to 10.47 GHz) at $f_r$ of 9.5 GHz, as well as higher angular stability and polarization independence, at second iteration level of Koch fractal geometry. However, for the TE mode polarization, the BW increased, while for the TM mode polarization, the BW decreased continuously as the oblique angles increased. For future work, the BW enhancement for the entire X band (8 GHz to 12 GHz) for the Koch fractal hexagonal loop FSS structure can be further study.

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