NUMERICAL MODELLING AND SIMULATION FOR ONE-DIMENSIONAL FLUID STRUCTURE INTERACTION IN BLOOD FLOW

TANG AIK YING

UNIVERSITI TEKNOLOGI MALAYSIA
NUMERICAL MODELLING AND SIMULATION FOR ONE-DIMENSIONAL FLUID STRUCTURE INTERACTION IN BLOOD FLOW

TANG AIK YING

A thesis submitted in fulfilment of the requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

NOVEMBER 2017
To my beloved father, mother and family
ACKNOWLEDGEMENT

First of all, I would like to express my gratitude to my supervisors, Prof. Dr. Norsarahaida Saidina Amin and Dr. Airil Yasreen Mohd Yassin for their encouragement and support during this work. They have been spending their valuable time to guide the work by providing the necessary suggestions and advices that strengthen this thesis.

Besides, I wish to thank to my friends, Mohd Al-Akhbar Mohd Noor, Mohd Zhafri Jamil and Mokhtazul Haizad Mokhtaram for their willingness to share their knowledge on research and programming works. Next, I would like to thank to Noor Azlina Subani and Tan Yu Ting for their ideas and supports during my studies in UTM.

Last but not least, special thanks to my parents and siblings for their continuous support and encouragement during my studies. Thus, I am able to continue the studies with their motivation and understanding along the years of my studies.
Fluid structure interaction (FSI) needs to be considered in modeling biofluids because the interaction between blood flow and vessel wall is of great clinical interest. However, the interaction between blood flow and vessel wall make FSI problems complex and challenging. Spurious oscillations were observed from numerical solutions and in the case of Bubnov-Galerkin finite element method, the oscillations occurred at relatively high pressure differences. In this thesis, Streamline-Upwind Petrov Galerkin (SUPG) stabilization scheme was formulated to solve one-dimensional FSI problems in blood flow to eliminate the spurious oscillations and to obtain stable numerical solutions for stenotic vessel. A pressure-area constitutive relation to complement the continuity equation and momentum equation was formulated by adopting the collapsible model. The geometry of stenotic vessel consists of single smooth and single irregular stenosis, multi-smooth and multi-irregular stenosis in this thesis. Numerical results show that there are no vessel collapse phenomena in single smooth stenosis and multi-smooth stenosis cases. Vessel collapse phenomena are observed for single-irregular stenosis with 85% cross sectional area amplitude at distal pressure of 47 mmHg while for multi-irregular stenosis with 60% and 85% cross sectional amplitudes at proximal stenosis and distal stenosis respectively, at distal pressure of 36 mmHg. In addition, paradoxical collapse motion along the time phase cycle is obtained in unsteady cases for single irregular stenosis and multi-irregular stenosis with the distal resistance of 2.73 mmHg/(ml/s) and 2.44 mmHg/(ml/s) respectively when sinusoidal pressure variation is applied at the inlet boundary. In conclusion, numerical results show that vessel collapse phenomena occurs when there is supercritical flow at the minimum cross sectional area of the stenotic vessel which is lower than the minimum cross sectional area at static condition and hence lead to the negative transmural pressure at that position.
ABSTRAK

Interaksi struktur bendalir (FSI) perlu diambil kira dalam pemodelan biofluids kerana interaksi antara aliran darah dengan dinding salur pembuluh mempunyai kepentingan klinikal yang besar. Walau bagaimanapun, interaksi antara aliran darah dengan dinding salur pembuluh menjadikan masalah FSI rumit dan mencabar. Ayunan palsu telah dapat diperhatikan dari penyelesaian berangka dan dalam kes kaedah unsur terhingga Bubnov-Galerkin, ayunan berlaku ketika terdapat perbezaan tekanan yang agak tinggi. Dalam tesis ini, skim penstabilan Streamline-Upwind Petrov Galerkin (SUPG) digubal untuk menyelesaikan masalah FSI satu dimensi dalam aliran darah untuk menghapuskana ayunan palsu dan mendapatkan penyelesaiaan berangka yang stabil bagi salur pembuluh stenosis. Hubungan konstitutif tekanan-kawasan untuk melengkapi persamaan keselanjaran dan persamaan momentum telah diformulasi dengan menggabungkan model boleh runtuh. Geometri salur pembuluh stenosis yang terdiri daripada stenosis tunggal yang seragam dan stenosis tunggal yang tidak seragam, stenosis pelbagai yang seragam dan stenosis pelbagai yang tidak seragam digunakan dalam tesis ini. Keputusan berangka menunjukkan bahawa tiada fenomena keruntuhan salur pembuluh dalam kes stenosis tunggal yang seragam dan stenosis pelbagai yang seragam. Fenomena keruntuhan salur pembuluh telah dikesan dalam kes stenosis tunggal yang tidak seragam dengan 85% amplitud kawasan keratan rentas stenosis pada tekanan distal 47 mmHg sementara dalam kes stenosis pelbagai yang tidak seragam masing-masing dengan 60% dan 85% amplitud keratan rentas pada stenosis proksimal dan stenosis distal, pada tekanan distal di 36 mmHg. Tambahan pula, keruntuhan salur pembuluh paradox di sepanjang masa kitaran fasa telah didapati untuk kes stenosis tunggal yang tidak seragam dan stenosis pelbagai yang tidak seragam dengan rintangan distal masing-masing 2.73 mmHg/(ml/s) dan 2.44 mmHg/(ml/s) apabila variasi tekanan sinusoidal digunakan pada sempadan masuk. Kesimpulannya, keputusan berangka telah membuktikan fenomena keruntuhan salur pembuluh terjadi apabila terdapat aliran superkritikal di kawasan keratan rentas minimum salur pembuluh stenosis adalah kurang daripada kawasan keratan rentas minimum pada keadaan static dan seterusnya membawa kepada tekanan transmural negatif pada kedudukan tersebut.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DECLARATION</td>
<td></td>
<td>ii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td></td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td></td>
<td>iv</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td></td>
<td>v</td>
</tr>
<tr>
<td>ABSTRAK</td>
<td></td>
<td>vi</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td></td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td></td>
<td>xiii</td>
</tr>
<tr>
<td>LIST OF ABBREVIATIONS</td>
<td></td>
<td>xvi</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td></td>
<td>xvii</td>
</tr>
<tr>
<td>LIST OF APPENDIX</td>
<td></td>
<td>xviii</td>
</tr>
</tbody>
</table>

1 INTRODUCTION 1

1.1 Research Background 1
1.2 Problem Statement 2
1.3 Objectives of Research 3
1.4 Scope of Research 4
1.5 Significance of Research 5
1.6 Outline of Thesis 5

2 LITERATURE REVIEW 8

2.1 Introduction 8
2.2 Physiological Background 9
2.3 FSI in Blood Flow 12
2.4 One-dimensional FSI in Blood Flow 16
  2.4.1 Governing Equations 16
  2.4.2 Numerical Works 25
2.5 Stabilization Technique 30
2.6 Concluding Remarks 33

3 MATHEMATICAL MODELS 35
  3.1 Introduction 35
  3.2 Derivation of Governing Equations 36
    3.2.1 Continuity Equation 36
    3.2.2 Momentum Equation 40
    3.2.3 Pressure-area Constitutive Relation 44
  3.3 Mathematical Model 1 48
  3.4 Mathematical Model 2 51
  3.5 Mathematical Model 3 53

4 NUMERICAL TECHNIQUE AND FORMULATION 63
  4.1 Introduction 63
  4.2 Characteristic System 64
  4.3 SUPG Formulation 68
  4.4 Boundary Conditions and Compatibility Conditions 73
  4.5 Numerical Discretization 77
  4.6 Nonlinear Solvers 87
  4.7 Stability Criterion 89
  4.8 Algorithm Code 92

5 STEADY FLOW IN STRAIGHT AND STENOTIC VESSEL 94
  5.1 Introduction 94
  5.2 Oscillations Phenomenon 95
  5.3 Verification of SUPG Numerical Solutions 99
    5.3.1 Straight Vessel: Verification with Analytical Solutions 99
5.3.1.1  p-A Model 1 99
5.3.1.2  p-A Model 2 103
5.3.2  Stenotic Vessel: Comparison with Numerical Solutions of Downing and Ku (1997) 108
5.4  Parametric Variations Study 114
  5.4.1  Single Stenosis 116
  5.4.2  Multi-Stenosis 127
5.5  Concluding Remarks 143

6  UNSTEADY FLOW IN STENOTIC VESSEL 145
  6.1  Introduction 145
  6.2  Verification of SUPG Numerical Solutions 146
  6.3  Parametric Variations Study 150
    6.3.1  Single Stenosis 151
    6.3.2  Multi-Stenosis 160
  6.4  Concluding Remarks 172

7  CONCLUSION 173
  7.1  Summary of Research 173
  7.2  Suggestion for Future Work 178

REFERENCES 179
Appendix A 191-205
## LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE NO.</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Various momentum equations for one-dimensional FSI in blood flow</td>
<td>19</td>
</tr>
<tr>
<td>2.2</td>
<td>Various pressure-area constitutive relations for one-dimensional FSI in blood flow</td>
<td>23</td>
</tr>
<tr>
<td>4.1</td>
<td>Algorithm code for time-dependent one-dimensional FSI blood flow model</td>
<td>93</td>
</tr>
<tr>
<td>5.1</td>
<td>Biological parameters for $p$-$A$ Model 1 (Sochi, 2015)</td>
<td>95</td>
</tr>
<tr>
<td>5.2</td>
<td>Comparison Bubnov-Galerkin numerical results with Sochi (2015) and analytical solution</td>
<td>96</td>
</tr>
<tr>
<td>5.3</td>
<td>Biological parameters for $p$-$A$ Model 2 (Downing and Ku, 1997)</td>
<td>98</td>
</tr>
<tr>
<td>5.4</td>
<td>Numerical data of pressure distributions taken at $x = 0.45$ m for $p$-$A$ Model 1</td>
<td>102</td>
</tr>
<tr>
<td>5.5</td>
<td>Numerical data of pressure distributions taken at $x = 0.045$ m for $p$-$A$ Model 2</td>
<td>107</td>
</tr>
<tr>
<td>5.6</td>
<td>Biological parameters for stenotic vessel (Downing and Ku, 1997)</td>
<td>109</td>
</tr>
<tr>
<td>5.7</td>
<td>Comparison numerical data of cross sectional area, pressure and speed index distributions for steady stenosis model for $P_2 = 60$ mmHg with Downing and Ku (1997)</td>
<td>112</td>
</tr>
<tr>
<td>5.8</td>
<td>Numerical data of minimum area, minimum pressure and maximum speed index at $x = 1.52$</td>
<td>114</td>
</tr>
<tr>
<td>5.9</td>
<td>Baseline conditions for parametric study for single stenosis and multi-stenosis cases</td>
<td>115</td>
</tr>
</tbody>
</table>
5.10 Numerical results with variations of distal pressure for single smooth stenosis when $\lambda_{Ao} = 85\%$ at $x = 2$

5.11 Numerical results with variations of distal pressure for single irregular stenosis when $\lambda_{Ao} = 85\%$ at $x = 2.14$

5.12 Numerical results with variations of $\lambda_{Ao}$ for single irregular stenosis when $P_2 = 47$ mmHg at $x = 2.14$

5.13 Numerical results with variations of $P_2$ for multi-smooth stenosis when $\lambda_{Ao1} = 60\%$ and $\lambda_{Ao2} = 85\%$

5.14 Numerical results with variations of $P_2$ for multi-irregular stenosis when $\lambda_{Ao1} = 60\%$ and $\lambda_{Ao2} = 85\%$

5.15 Summary of numerical results with variations of $\lambda_{Ao1}$ and $\lambda_{Ao2}$ for multi-smooth stenosis at $P_2 = 36$ mmHg

5.16 Summary of numerical results with variations of $\lambda_{Ao1}$ and $\lambda_{Ao2}$ for multi-irregular stenosis at $P_2 = 36$ mmHg

5.17 Numerical results with different length between stenosis for multi-smooth stenosis at baseline settings

5.18 Numerical results with different length between stenosis for multi-irregular stenosis at baseline settings

6.1 Numerical data of minimum cross sectional area for unsteady case with $R_{dis} = 6.1$ mmHg/(ml/s) compared with Downing and Ku (1997)

6.2 Numerical data of maximum speed index for unsteady case with $R_{dis} = 6.1$ mmHg/(ml/s) compared with Downing and Ku (1997)

6.3 Numerical data of average flow rate for unsteady case with $R_{dis} = 6.1$ mmHg/(ml/s) compared with Downing and Ku (1997)

6.4 Inlet pressure and cross sectional area for unsteady parametric variations study

6.5 Summary of unsteady numerical results for single smooth stenosis with $R_{dis} = 3.07$ mmHg/(ml/s)

6.6 Summary of unsteady numerical results for single smooth stenosis with $R_{dis} = 3.90$ mmHg/(ml/s)
6.7 Summary of unsteady numerical results for single smooth stenosis with $R_{dis} = 4.67$ mmHg/(ml/s) 154

6.8 Summary of unsteady numerical results for single irregular stenosis with $R_{dis} = 2.73$ mmHg/(ml/s) 157

6.9 Summary of unsteady numerical results for single irregular stenosis with $R_{dis} = 3.49$ mmHg/(ml/s) 157

6.10 Summary of unsteady numerical results for single irregular stenosis with $R_{dis} = 4.20$ mmHg/(ml/s) 158

6.11 Summary of unsteady numerical results for multi-smooth stenosis with $R_{dis} = 2.60$ mmHg/(ml/s) 163

6.12 Summary of unsteady numerical results for multi-smooth stenosis with $R_{dis} = 2.96$ mmHg/(ml/s) 164

6.13 Summary of unsteady numerical results for multi-smooth stenosis with $R_{dis} = 4.73$ mmHg/(ml/s) 164

6.14 Summary of unsteady numerical results for multi-irregular stenosis with $R_{dis} = 2.44$ mmHg/(ml/s) 167

6.15 Summary of unsteady numerical results for multi-irregular stenosis with $R_{dis} = 3.77$ mmHg/(ml/s) 168

6.16 Summary of unsteady numerical results for multi-irregular stenosis with $R_{dis} = 4.55$ mmHg/(ml/s) 168

6.17 Computational time for single smooth stenosis, single irregular stenosis, multi-smooth stenosis and multi-irregular stenosis 171
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE NO.</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Stages of atherosclerosis (Mayoclinic.org)</td>
<td>10</td>
</tr>
<tr>
<td>2.2</td>
<td>Types of stenosis (wikidoc.org)</td>
<td>10</td>
</tr>
<tr>
<td>2.3</td>
<td>Arteriography with critical stenosis in right common iliac artery (Ferrari <em>et al</em>, 2004)</td>
<td>11</td>
</tr>
<tr>
<td>2.4</td>
<td>25 months later, iliac occlusion with clinical deterioration (Ferrari <em>et al</em>, 2004)</td>
<td>11</td>
</tr>
<tr>
<td>2.5</td>
<td>Coupling scheme for incompressible FSI (Andersson and Ahl, 2011)</td>
<td>13</td>
</tr>
<tr>
<td>2.6</td>
<td>Behavior of collapsible tube (Shapiro, 1977)</td>
<td>21</td>
</tr>
<tr>
<td>2.7</td>
<td>Tube law capturing bovine and canine artery (Downing, 1995)</td>
<td>22</td>
</tr>
<tr>
<td>2.8</td>
<td>Advection skew to mesh with homogenous natural outflow boundary condition (Brooks and Hughes, 1982)</td>
<td>32</td>
</tr>
<tr>
<td>2.9</td>
<td>Transient pure advection: comparison results at steps 20, 40, 60, 80, 100 (Brooks and Hughes, 1982)</td>
<td>33</td>
</tr>
<tr>
<td>3.1</td>
<td>One-dimensional orientation (Sherwin <em>et al</em>, 2003b)</td>
<td>36</td>
</tr>
<tr>
<td>3.2</td>
<td>Finite control volume fixed in space (Anderson, 1995)</td>
<td>37</td>
</tr>
<tr>
<td>3.3</td>
<td>Control volume for derivation of unsteady one-dimensional continuity and momentum equation in blood flow model (Anderson, 1995)</td>
<td>38</td>
</tr>
<tr>
<td>3.4</td>
<td>Pressure distributions in $x$ direction acting on the control volume</td>
<td>42</td>
</tr>
<tr>
<td>3.5</td>
<td>Physical interpretation for a circular cylinder as a model of blood vessel (Westerhof <em>et al</em>, 2004)</td>
<td>44</td>
</tr>
</tbody>
</table>
3.6 Geometry of one-dimensional compliant wall (Westerhof et al, 2004) 45
3.7 Change in vessel radius as a result of increasing pressure, $p$ (Waite and Fine, 2007) 46
4.1 Incoming and outgoing waves propagation 73
5.1 Oscillation due to the employment of Bubnov-Galerkin formulation for $p$-$A$ Model 1 97
5.2 Oscillation due to the employment of Bubnov-Galerkin formulation for $p$-$A$ Model 2 98
5.3 Stabilization of numerical solutions with the employment of SUPG formulation for $p$-$A$ Model 1 100
5.4 Stabilization of numerical solutions with the employment of SUPG formulation for $p$-$A$ Model 2 106
5.5 Comparison numerical results between Bubnov-Galerkin, SUPG and Downing and Ku (1997) for $P_2 = 60$ mmHg 109
5.6 SUPG numerical solutions for steady stenosis model when $P_2 = 60$ mmHg, 55 mmHg and 50 mmHg 113
5.7 Cross sectional area for single stenosis with $\lambda_{A_0} = 85\%$ 116
5.8 Effect of distal pressure with $\lambda_{A_0} = 85\%$ for single smooth stenosis 119
5.9 Effect of distal pressure with $\lambda_{A_0} = 85\%$ for single irregular stenosis 121
5.10 Effect of $\lambda_{A_0}$ on vessel collapse for single irregular stenosis 124
5.11 Relationship between distal pressure and distal resistance for single smooth stenosis and single irregular stenosis 126
5.12 Cross sectional area for multi-stenosis with $\lambda_{A_{01}} = 65\%$ and $\lambda_{A_{02}} = 85\%$ 127
5.13 Effect of distal pressure with $\lambda_{A_{01}} = 60\%$ and $\lambda_{A_{02}} = 85\%$ for multi-smooth stenosis 130
5.14 Effect of distal pressure with $\lambda_{A_{01}} = 60\%$ and $\lambda_{A_{02}} = 85\%$ for multi-irregular stenosis 132
5.15 Effect of cross sectional area reduction amplitude for multi-smooth stenosis at $P_2 = 36$ mmHg and $\lambda_{Ao2} = 85\%$  
136
5.16 Effect of cross sectional area reduction amplitude for multi-irregular stenosis at $P_2 = 36$ mmHg and $\lambda_{Ao2} = 85\%$  
137
5.17 Effect of length between stenosis for multi-smooth stenosis at baseline setting  
139
5.18 Effect of length between stenosis for multi-irregular stenosis at baseline settings  
141
5.19 Relationship between distal pressure and distal resistance for multi-smooth stenosis and multi-irregular stenosis  
143
6.1 Comparison SUPG numerical results with Downing and Ku (1997) for unsteady stenosis case with $R_{dis} = 6.1$ mmHg/(ml/s) and $f = 1$ Hz, 5 Hz and 10 Hz  
147
6.2 Effect of distal resistance in unsteady flow cases for single smooth stenosis  
152
6.3 Effect of distal resistance in unsteady flow cases for single irregular stenosis  
156
6.4 Pressure distributions along single irregular vessel at $R_{dis} = 2.73$ mmHg/(ml/s)  
159
6.5 Effect of distal resistance in unsteady flow cases for multi-smooth stenosis  
161
6.6 Effect of distal resistance in unsteady flow cases for multi-irregular stenosis  
166
6.7 Pressure distributions along multi-irregular stenotic vessel at $R_{dis} = 2.44$ mmHg/(ml/s)  
170
LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>FSI</td>
<td>Fluid Structure Interaction</td>
</tr>
<tr>
<td>SUPG</td>
<td>Streamline-Upwind Petrov-Galerkin</td>
</tr>
<tr>
<td>DG</td>
<td>Discontinuous Galerkin</td>
</tr>
<tr>
<td>MUSCL</td>
<td>Monotonic Upwind Scheme for Conservation Law</td>
</tr>
<tr>
<td>p-A</td>
<td>Pressure-area</td>
</tr>
</tbody>
</table>
**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$x$</td>
<td>Axial coordinate</td>
</tr>
<tr>
<td>$A$</td>
<td>Cross sectional area</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity</td>
</tr>
<tr>
<td>$c$</td>
<td>Local wave speed</td>
</tr>
<tr>
<td>$Q$</td>
<td>Volumetric flow rate</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure</td>
</tr>
<tr>
<td>$P_e$</td>
<td>External Pressure</td>
</tr>
<tr>
<td>$P - P_e$</td>
<td>Transmural pressure</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Momentum correction factor</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Viscosity friction coefficient</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Fluid dynamic viscosity</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Vessel stiffness for p-A Model 1</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Cross sectional area of the flow at reference pressure $P_o$</td>
</tr>
<tr>
<td>$h_o$</td>
<td>Vessel’s wall thickness at reference pressure $P_o$</td>
</tr>
<tr>
<td>$v$</td>
<td>Poisson ratio</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s elastic modulus</td>
</tr>
<tr>
<td>$L$</td>
<td>Vessel length</td>
</tr>
<tr>
<td>$A_{in}, A_{ou}$</td>
<td>Flow cross sectional area at the inlet and outlet</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Vessel stiffness for p-A Model 2</td>
</tr>
<tr>
<td>$R$</td>
<td>Mean flow radius</td>
</tr>
<tr>
<td>$n_1, n_2$</td>
<td>Tube law exponents</td>
</tr>
<tr>
<td>$F_{frict}, F_f$</td>
<td>Lumped loss in the flow</td>
</tr>
</tbody>
</table>
\( f_L \) - Major laminar loss
\( D_e \) - Hydraulic diameter
\( K_{sep} \) - Separation loss coefficient
\( A_{th} \) - Throat area of stenosis
\( L_s \) - Length of separation region
\( x_{sep} \) - Separation point
\( D_o \) - Nominal vessel diameter
\( A_o(x) \) - Vessel area variation
\( A_{oo} \) - Nominal vessel area
\( \lambda_A(x) \) - Stenosis shape function
\( \lambda_{Ao} \) - Cross sectional area reduction amplitude
\( x_s \) - Starting point of stenosis
\( x_E \) - Stopping point of the stenosis
\( K_p(x) \) - Vessel stiffness variation
\( K_{po} \) - Nominal vessel stiffness
\( \lambda_K(x) \) - Stiffness variation amplitude
\( f \) - Frequency
\( p_1 \) - Perfusion / inlet pressure
\( p_2 \) - Distal / outlet pressure
\( \lambda_{Ao1}, \lambda_{Ao2} \) - Cross sectional area reduction amplitude for multi-stenosis
\( \lambda_{Ko1}, \lambda_{Ko2} \) - Stiffness reduction amplitude for multi-stenosis
\( x_{sep1}, x_{sep2} \) - Separation point for multi-stenosis
\( A_{th1}, A_{th2} \) - Throat area for multi-stenosis
\( x_{s1}, x_{s2} \) - Starting point for multi-stenosis
\( x_{E1}, x_{E2} \) - Stopping point for multi-stenosis
\( L_{BS} \) - Length between stenosis
\( U \) - Matrix of dependent variables
\( F \) - Flux matrix
\( B \) - Force matrix (conservation form)
\( H \) - Jacobian of flux vectors
\( w \) - Weighting function
\( N_l \) - Linear shape function
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(w) )</td>
<td>Operator applied to the test function</td>
</tr>
<tr>
<td>( R(U) )</td>
<td>Residual of one-dimensional FSI equations</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Stabilization parameters</td>
</tr>
<tr>
<td>( S(U) )</td>
<td>Force matrix (quasi-linear form)</td>
</tr>
<tr>
<td>( \lambda, \Lambda )</td>
<td>Eigenvalues of ( H )</td>
</tr>
<tr>
<td>( L, L_{1,2} )</td>
<td>Left-eigenvectors of ( H )</td>
</tr>
<tr>
<td>( C_{TI} )</td>
<td>Time-independent compatibility conditions</td>
</tr>
<tr>
<td>( C_{TD} )</td>
<td>Time-dependent compatibility conditions</td>
</tr>
<tr>
<td>( S )</td>
<td>Speed index</td>
</tr>
<tr>
<td>( R_{dis} )</td>
<td>Distal resistance</td>
</tr>
<tr>
<td>( A_{min} )</td>
<td>Minimum cross sectional area</td>
</tr>
<tr>
<td>( P_{min} )</td>
<td>Minimum pressure</td>
</tr>
<tr>
<td>( S_{max} )</td>
<td>Maximum speed index</td>
</tr>
<tr>
<td>( \bar{Q} )</td>
<td>Average Flow Rate</td>
</tr>
<tr>
<td>( S_1, S_2 )</td>
<td>Vessel position for multi-stenosis</td>
</tr>
<tr>
<td>( A_{S1}, A_{S2} )</td>
<td>Cross sectional area at vessel position ( S_1, S_2 )</td>
</tr>
<tr>
<td>( P_{S1}, P_{S2} )</td>
<td>Pressure at vessel position ( S_1, S_2 )</td>
</tr>
<tr>
<td>( S_{S1}, S_{S2} )</td>
<td>Speed index at vessel position ( S_1, S_2 )</td>
</tr>
</tbody>
</table>
LIST OF APPENDIX

<table>
<thead>
<tr>
<th>APPENDIX</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>SUPG Source Code</td>
<td>191</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Research Background

Fluid structure interaction (FSI) is encountered and applicable in many different branches of engineering and science. For example, FSI is a crucial consideration in the design of many engineering systems such as aircraft and bridges. In general, FSI is defined as the interaction between the deformable structures with an internal or surrounding fluid flow. Such deformation can be either stable or oscillatory. The deformation of the structure contributes to the changes in boundary conditions of the fluid flow. Fluid flows encountered in our daily life include amongst others meteorological phenomena, environmental hazards, processes in human body such as blood flow and breathing.

FSI is more often considered in modelling biofluids because the interaction between the blood flow and vessel wall is of great clinical interest, for example, in studying cardiovascular diseases which are a major cause of death in developed countries (Mortazavinia et al., 2012). The consideration of the interaction between blood flow and vessel wall had seldom being consideration in the previous studies.
due to the difficulty in solving the coupled fluid and solid equations (Zhang et al., 2003). Although the assumptions of the rigid wall surfaces yield results that are reasonable accurate, there are still have some considerations to be taken into such that the elastic nature of the arterial wall, stresses on the arterial wall that play crucial role in arterial disease as well as the material property alterations with the development of the atherosclerotic lesion. (Kanyanta et al., 2009; Friedman et al., 2010; Siogkas et al., 2011).

Recent studies about the effect of rigid wall and FSI on flow distributions in arterial modelling had been carried out. The axial velocities of rigid wall are higher compared to the ones in compliant model. Such situation is explained through mass conservation theory where the internal fluid pressure exerted on the vessel wall pushes the vessel wall outward consistently and slows fluid flow due to the flow area expansion. These findings showed that incorporating FSI has significant effects on blood flow characteristics, yet FSI models are computationally expensive when the arterial geometry is highly complicated. (Lee and Xu, 2002; Siogkas et al., 2011; Mortazavinia et al., 2012; He et al., 2016).

1.2 Problem Statement

FSI describes the wave propagation in arteries driven by the pulsatile blood flow. From theoretical point of view, such problems are complex and challenging due to high nonlinearity of the problem. Two-dimensional and three-dimensional mathematical models are solved with the aid of the commercial software or black-box solvers, yet there are some considerations such as added mass effect, coupling conditions between fluid and structure and suitable boundary conditions to avoid the wave reflection influence the numerical stability.
Despite of the numerical stability obtained from the physical problems, there is wiggling phenomena observed in computational fluid dynamics (CFD) problems especially flow with high Peclet number or high Reynolds number. Same phenomenon is expected for one-dimensional FSI blood flow problems for relatively high pressure differences. Besides, numerical formulation and simulation become complicated to include the geometrical variation of the vessel such as the spatial variation of area and corresponding stiffness resulting from the attempt to model the stenosis. Moreover, flow in stenotic vessel is further complicated when there is choked flow or flow transition where vessel collapse is observed.

Thus, sets of governing equations together with suitable boundary conditions are important to study the flow behavior in straight and stenotic vessel. Numerical technique and formulation with oscillations free is significant in ensuring the attainment of reliable information and numerical results.

1.3 Objectives of Research

The objectives of this research are specified as follows:
1. To develop numerical method based on finite element method with Streamline-Upwind Petrov-Galerkin (SUPG) stabilization scheme to solve one-dimensional blood flow in a stenosed artery.
2. To determine the effect of geometry to the flow behavior by including the irregular shape and multi-stenosis geometry.
3. To determine the effect of area reduction amplitude to the flow behavior in smooth and irregular stenosis.
4. To determine the effect of distal pressure to the flow behavior in smooth and irregular stenosis.
5. To identify the physiological conditions for vessel collapse phenomena in stenotic vessel.
1.4 Scope of Research

The scope of this study is on the numerical modelling and simulation in one-dimensional FSI blood flow cases. One-dimensional, incompressible, Newtonian flow is considered in this study. Continuity equation, momentum equation and pressure-area constitutive relation are coupled and solved numerically with the employment of compatibility conditions at the boundary nodes. Besides, finite element method with SUPG stabilization formulation is employed as space discretization and first-order forward difference is employed as time discretization. For straight vessel, two types of pressure-area constitutive relations are coupled together with continuity equation and momentum equation, that are, nonlinear elastic model and collapsible model, which are termed as \( p-A \) Model 1 and \( p-A \) Model 2 respectively. Pressure differences for \( p-A \) Model 1 range from 400 Pa to 2500 Pa while pressure differences for \( p-A \) Model 2 range from 10 mmHg to 45 mmHg.

For stenotic vessel, one-dimensional, incompressible Newtonian flow with frictional losses is considered. Collapsible model is applied as pressure-area constitutive relation to describe the flow in stenotic vessel and capture the vessel collapse phenomena. Four different geometry of stenosis are discussed, which are single smooth stenosis, single irregular stenosis, multi-smooth stenosis and multi-irregular stenosis. For single stenosis and multi-stenosis cases, cross sectional area reduction amplitude vary from 60% to 85% and stiffness reduction amplitude is set 10. Perfusion pressure is set at 100 mmHg. Distal pressure is varying from 47 mmHg to 70 mmHg for single stenosis cases and 36 mmHg to 60 mmHg for multi-stenosis cases.
1.5  **Significance of Research**

First of all, SUPG stabilization scheme is formulated to solve one-dimensional FSI in straight and stenotic vessel. This study is significant as this would be the first application of SUPG in the study of one-dimensional FSI blood flow problems. Besides, one-dimensional FSI governing equations are solved numerically with the employment of compatibility conditions to minimize the wave reflection at the boundary. Compatibility conditions and SUPG stabilization term are derived from the characteristic system, which emphasizing the physical nature of the problem. With the approach that proposed in this study, the understanding on the characteristic flow behavior, physiological conditions to induce vessel collapse, relationship between cross sectional area, volumetric flow rate and pressure of the flow are observed.

1.6  **Outline of Thesis**

This thesis consists of seven chapters, including this introduction chapter. Chapter 1 introduces the general information about the thesis, including the research background, problem statement, objectives, scopes and significance of this study. Chapter 2 presents the literature review about FSI in blood flow. The chapter begins with the difficulty and challenges of considering FSI in two-dimensional and three-dimensional mathematical models which then contributes to the research on one-dimensional FSI model. Then, the governing equations and numerical works on one-dimensional FSI in blood flow are detailed. Chapter 3 discusses about the mathematical models which are used throughout the study, including the governing equations, initial conditions and boundary conditions.
Chapter 4 deliberates about the numerical technique and formulation in the study. The chapter begins with finite element method, followed by the SUPG formulation. The formulation of SUPG involves the stabilization term, which is correspond in adding the diffusion along the characteristic direction with the appropriate stabilization parameter. Nonreflecting boundary conditions which known as compatibility conditions are derived from the characteristic system of governing equations to minimize the outgoing characteristic waves at the boundary. Eigenvalues and left-eigenvectors of the Jacobian flux vectors are solved from the method of characteristics system in order to get the time-independent and time-dependent compatibility conditions. The coupled governing equations are discretized into matrix form and solved with Newton-Raphson nonlinear iterative solver. The stability criterion is discussed and an algorithm code is presented at the end of the chapter.

Chapter 5 discusses about the steady flow in the straight vessel, followed by stenotic vessel. Bubnov-Galerkin finite element is formulated for p-A Model 1, as in the work reported in Sochi (2015). However, when Bubnov-Galerkin finite element is applied to the higher pressure difference, that is, in the range higher than reported in Sochi (2015), spurious oscillations occur. In confirming the occurrence of the oscillations, p-A Model 2 is studied. Similar phenomenon is observed for both p-A models. Hence, SUPG stabilization scheme is formulated and the numerical results are validated with the analytical solutions for both p-A models. The analytical solution for p-A Model 1 is taken from Sochi (2015) while the analytical solution for p-A Model 2 is derived. Then SUPG formulation is extended to stenotic vessel and the numerical results are compared with the numerical works in Downing and Ku (1997). SUPG numerical results are shown to eliminate spurious oscillations obtained from Bubnov-Galerkin finite element formulation and provide reliable information of the flow. Afterward, SUPG formulation is extended to parametric variations study with the irregular geometry of stenosis. Four cases are studied and vessel collapse conditions are identified for single irregular stenosis and multi-irregular stenosis. The relationship between vessel collapse, pressure and speed index is discussed.
Chapter 6 discusses about the unsteady flow in stenotic vessel. Sinusoidal time variations are applied at the inlet boundary to mimic the pressure variation of 120/80 mmHg. Initially, SUPG unsteady numerical results are compared with Downing and Ku (1997). Parametric variations study is concerned as in previous chapter. Steady flow numerical results in previous chapter are applied as the initial values of unsteady flow cases in this chapter. Effect of distal resistance for all the cases are plotted and discussed. Furthermore, vessel conditions for each phase cycle are demonstrated by the plotting of the pressure distributions along the stenotic vessel throughout the phase cycle. Finally, the thesis ends with Chapter 7. Summary of research and some suggestions for future works are stated.
REFERENCES


