

STEADY AND UNSTEADY MHD MIXED CONVECTION FLOW OF
CASSON AND CASSON NANOFLUID OVER A NONLINEAR
STRETCHING SHEET AND MOVING WEDGE

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To My Beloved Mother and Father, Brothers, Wife and Children

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ABSTRACT

Casson fluid is a shear thinning fluid which is one of the non-Newtonian fluids that exhibit yield stress. In this fluid, if a shear stress less than the yield stress is applied, it behaves like a solid, whereas if vice-versa the fluid starts to move. The advantage of Casson fluid is that it can be reduced to Newtonian fluid at very high wall shear stress. Due to these reasons, the steady and unsteady two-dimensional, electrically conducting mixed convection flow of Casson fluid was studied in this thesis. Flow that was generated due to nonlinear stretching sheet and moving wedge filled with and without nanoparticles were given attention. Specific problems were studied with various effects include, porous medium, thermal radiation, chemical reaction, slip and convective boundary conditions. Similarity transformations were used to convert nonlinear governing equations into nonlinear ordinary differential equations. The obtained equations were then solved numerically via the implicit finite difference scheme, known as Keller-box method. Moreover, an algorithm was developed in MATLAB software in order to obtain the numerical solutions. The accuracy of the numerical results was validated through comparison with the results available in the published journal. The effects of pertinent parameters on velocity, temperature and concentration profiles as well as wall shear stress, heat and mass transfer rates were displayed graphically and also presented in tabular form. Findings reveals that, when Casson fluid parameter increases the momentum boundary layer thickness reduces in both cases, nonlinear stretching sheet and moving wedge. It is noticed that in the case of moving wedge, the strength of magnetic parameter reduces the wall shear stress. Whereas, opposite trend is observed in the case of nonlinear stretching sheet. In both geometries, the influence of Brownian motion and thermophoresis parameters on the nanoparticles concentration is notably more pronounced.

ABSTRAK

Bendalir Casson adalah bendalir penipisan ricih yang merupakan satu daripada bendalir bukan Newtonan yang mempamerkan tegasan alah. Dalam bendalir ini, jika tegasan ricih digunakan kurang daripada tegasan alah, bendalir bersifat seperti pepejal, manakala jika sebaliknya bendalir akan mula bergerak. Kelebihan bendalir Casson adalah ia boleh berubah menjadi bendalir Newtonan ketika tegasan ricih dinding yang sangat tinggi. Disebabkan oleh alasan ini, aliran olakan campuran pengaliran elektrik, berkeadaan mantap dan tidak mantap dua dimensi bagi bendalir Casson dikaji dalam tesis ini. Aliran yang terjana oleh lembaran regangan tak linear dan baji bergerak yang diisi dengan dan tanpa partikel nano telah diberi perhatian. Masalah khusus telah dikaji dengan pelbagai kesan, termasuk bahantara berliang, sinaran terma, tindak balas kimia, keadaan sempadan gelincir dan berolakan. Penjelmaan serupa telah digunakan untuk mengubah persamaan menakluk tak linear kepada persamaan pembezaan biasa tak linear. Persamaan yang diperolehi kemudiannya diselesaikan secara berangka melalui skim beza terhingga tersirat, yang dikenali sebagai kaedah kotak-Keller. Selain itu, algoritma telah dibangunkan dalam perisian MATLAB bagi mendapatkan penyelesaian berangka. Kejituan keputusan berangka telah disahkan melalui perbandingan dengan keputusan yang boleh didapati dalam jurnal yang telah diterbitkan. Kesan parameter penting terhadap profil halaju, suhu dan kepekatan serta tekanan ricih dinding, kadar pemindahan haba dan jisim telah dipaparkan secara grafik dan juga dalam bentuk jadual. Dapatan mendedahkan bahawa, apabila parameter bendalir Casson meningkat ketebalan lapisan sempadan momentum mengurang bagi kedua-dua kes, lembaran regangan tak linear dan baji bergerak. Diperhatikan bahawa dalam kes baji bergerak, kekuatan parameter magnetik mengurangkan tegasan ricih dinding. Manakala, trend berlawanan diperhatikan dalam kes lembaran regangan tak linear. Bagi kedua-dua geometri, pengaruh parameter gerakan Brownian dan parameter termoforesis ke atas kepekatan partikel nano didapati lebih ketara.

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LIST OF SYMBOLS

Roman Letters

A	-	Unsteadiness parameter
a	-	Reference length along the flow
\mathbf{B}^*	-	Total magnetic field
\mathbf{B}_1	-	Transverse magnetic field
\mathbf{b}	-	Induced magnetic field
B_0	-	Magnitude of applied magnetic field
Bi_1, Bi_2	-	Biot numbers
c_f	-	Specific heat of fluid
c_p	-	Specific heat capacity of nanoparticles
C	-	Concentration
C_s	-	Wall concentration
C_∞	-	Concentration at free stream
C_0	-	Reference concentration
Cf_x	-	Skin friction coefficient
$\frac{D}{Dt}$	-	Material time derivative
D_m	-	Coefficient of mass diffusivity
D_B	-	Brownian diffusion coefficient
D_T	-	Thermophoresis coefficient
d_p	-	Nanoparticle diameter
\mathbf{E}	-	Total electric field
e_{ij}	-	$(i, j)^{th}$ component of the deformation rate
\mathbf{F}	-	Body force

Gr, Gr_n	-	Thermal Grashof number
Gm, Gm_n	-	Mass Grashof number
g	-	Gravitational acceleration
h_p	-	Specific enthalpy of nanoparticles
h_f	-	Convective heat flux
h_s	-	Convective mass flux
I	-	Identity tensor
J	-	Current density
j_A	-	Molar flux
j_p	-	Diffusion mass flux for nanoparticles
j_{p.B}	-	Nanoparticles mass flux due to Brownian diffusion
j_{p.T}	-	Nanoparticles mass flux due to thermophoretic effects
K	-	Porosity parameter
k_1	-	Permeability of porous medium
k_0	-	Permeability constant
k_f	-	Thermal conductivity of fluid
k_B	-	Boltzmann constant
k_c	-	Rate of chemical reaction
k_2	-	Constant of reaction rate
k^*	-	Mean absorption coefficient
Le	-	Lewis number
M	-	Magnetic parameter
m	-	Wedge angle parameter
N_1	-	Velocity slip
N_b	-	Brownian motion parameter
N_t	-	Thermophoresis parameter
Nu_x	-	Nusselt number
n	-	Nonlinear stretching sheet parameter
Pr	-	Prandtl number

p	-	Pressure
p_h	-	Hydrostatic pressure
p_d	-	Dynamic pressure
p_y	-	Yield stress
\mathbf{q}	-	Energy flux relative to nanofluid velocity
\mathbf{q}_r	-	Rosseland approximation
q_w	-	Wall heat flux
q_s	-	Wall mass flux
\mathbf{R}^*	-	Darcy Resistance
R_d	-	Radiation parameter
k_c	-	Rate of chemical reaction
R	-	Chemical reaction parameter
Re_x	-	Reynold's number
Sh_x	-	Sherwood number
Sc	-	Schmidt number
\mathbf{T}	-	Cauchy stress tensor
T_f	-	Temperature at wall
T	-	Temperature
T_∞	-	Temperature at free stream
T_0	-	Reference temperature
t	-	Time
u	-	Velocity in x – direction
u_w	-	Velocity at wall
u_e	-	Free stream velocity
\mathbf{V}	-	Velocity vector field
v	-	Velocity in y -direction
\mathbf{V}_T	-	Thermophoretic velocity
x	-	Dimensionless coordinate axis along the flow
y	-	Dimensionless coordinate axis normal to the flow

Greek Letters

α_f	-	Thermal diffusivity of fluid
β	-	Casson fluid parameter
β_T	-	Coefficient of thermal expansion
β_C	-	Coefficient of concentration expansion
$\tilde{\beta}$	-	Proportionality factor
β_1	-	Hartree pressure gradient
∇	-	Vector operator Del
η	-	Similarity variable
γ	-	Moving wedge parameter
μ	-	Dynamic viscosity
μ_B	-	Plastic dynamic viscosity
ν_f	-	Kinematic viscosity
Ω	-	Total angle
π_1	-	Product of deformation rate with itself
π_c	-	Critical value of the product
ϕ	-	Dimensionless concentration
φ	-	Porosity of porous medium
ρ_f	-	Density of fluid
ρ_∞	-	Ambient density
ρ_p	-	Nanoparticles mass density
ψ	-	Stream function
σ	-	Electrical conductivity
σ^*	-	Stefan-Boltzmann constant
θ	-	Dimensionless temperature
τ_{ij}	-	Shear force tensor for Casson fluid
τ	-	Ratio of heat capacities
τ_w	-	Wall skin friction

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CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter deliberates the important area of research which emphasis on non-Newtonian fluids, especially Casson and Casson nanofluid together with some basic and significant physical effects in the study of boundary layer flows. It comprises of an introduction of the research background, problem statement, research objectives, scope of research and the significance of study.

1.2 Research Background

Non-Newtonian fluid is a fluid whose flow properties are different from those of Newtonian fluids. In other words, the flow behavior of non-Newtonian fluids changes under stress. If someone hits, shakes or jumps on such fluids, they get thicker and act like solid after the stress is applied, and return back to its original position when the stress is removed. Many polymer solutions and molten polymer are non-Newtonian fluids, most commonly found substances such as ketchup, jelly, toothpaste, paint, blood and shampoo.

The study of boundary layers flows of non-Newtonian fluids have gained considerable attention of researchers due to its widespread applications in engineering and industry, such as, design of solid matrix heat, nuclear waste water disposal, chemical catalytic reactors, geothermal energy production and ground water

hydrology. It is common belief that for such fluids, no single constitutive equation exists in literature which exhibits all characteristics of these fluids. Due to the complex nature of these fluids, several models have been proposed by researchers to understand the flow behavior of these fluids. Most common non-Newtonian models include power law model (Bég *et al.*, 2012), viscoelastic fluids including Maxwell upper convected models (Bég and Makinde, 2011), Walters-B short memory models (Mohiddin *et al.*, 2010; Prasad *et al.*, 2011), Oldroyd-B model (Tripathi *et al.*, 2012), differential Reiner-Rivlin models (Bég *et al.*, 2008; Rashidi *et al.*, 2012), Bingham plastic model (Vatankhah, 2011) and second grade or third grade models (Sajid *et al.*, 2009; Sahoo and Poncet, 2011).

In all of these models, most commonly used models in modelling of non-Newtonian fluids are power law and second grade or third grade, but those fluids are unable to predict the effect of elasticity and stress relaxation, respectively. Among the non-Newtonian fluids, there is another fluid known as Casson fluid. The Casson fluid is defined as shear thinning fluid which exhibits yield stress and high shear viscosity. The Casson model behaves like Newtonian fluid as wall shear stress becomes large (Subba Rao *et al.*, 2015). This model was originally developed by Casson (Casson, 1959) for the suspensions of pigments which used in preparation of printing inks and silicon suspension. The Casson model is also preferred rheological model for blood and chocolate (Singh, 2011). The significant applications of Casson fluid can also be found in polymer processing industries and biomechanics (Das and Batra, 1993). Several researchers (Porwal and Badshah, 2012; Prasad, 2013; Pramanik, 2014) have included Casson fluid model in their investigations for different geometries.

Another important type of fluid is nanofluid. Nanofluids are heat transfer liquid with suspended particles (size 1-100nm). Nanofluids are engineered colloids made of base fluid and nanoparticles. There are several general applications of nanofluids such as industrial cooling, vehicle cooling, generating new types of fuel, reducing fuel in electric power generation plant, cancer therapy, imaging and sensing. The thermal conductivity of convectional heat transfer fluids such as oil, water and ethylene glycol mixture, plays a vital role on the heat transfer coefficient between the heat transfer of medium and surface (Chamkha *et al.*, 2011). This new class of fluid was first described by (Choi and Eastman, 1995). In their investigations, they pointed

out that these types of fluids by suspending nano-sized particles can be used to enhance the thermal conductivity of base fluid. The nano-sized particles can be made of metals; Copper (Cu), Aluminum (Al), Titanium (Ti), Iron (Fe), Silver (Ag), Gold (Au), metal oxides; Copper oxide (CuO), Aluminum oxide (Al₂O₃), Titanium Oxide (TiO₂), carbides; Silicon carbide (SiC) and nitrides; Aluminum nitride (AlN), Silicon nitride (SiN). Wong and De Leon (2010) pointed out that in cancer patients, iron based nanoparticles can be used as delivery vehicles of drugs and radiation.

The thermal conductivity of nanofluids is greater than that of normal fluid because the suspended ultrafine particles significantly improve its capability of energy exchange (Xuan and Roetzel, 2000). Several researchers (Masuda *et al.*, 1993; Lee *et al.*, 1999; Xuan and Li, 2000) agreed upon that small amount of nanoparticles volume fraction (5% or less) can enhance thermal conductivity of base fluid by more than 20%. Such enhancement depends upon shape of particles, dimensions of particles, thermal properties of particles, and volume fractions of suspended particles. Nanofluids coolants are also considered as new technology due to its better thermal performance to secure nuclear safety and economics (Kandasamy *et al.*, 2011).

Later on, Buongiorno (2006) carried out analysis of nanofluids and concluded that out of seven slip mechanisms, only Brownian motion and thermophoresis are significant mechanisms in nanofluids. Based on his predictions, he proposed a model which is known as Buongiorno's model. In this model, Buongiorno wrote the conservation equations in light of these two facts. It is noteworthy to mention that several researchers (Kuznetsov and Nield, 2010; Khan and Pop, 2013; Mahdy and Chamkha, 2015) adopted Buongiorno's model in their study.

Besides the characteristics of non-Newtonian fluids, another important mechanism that influences the behavior of fluid is the application of magnetic field to the fluid. Magnetohydrodynamic (MHD) is the study of the magnetic properties of electrically conducting fluids. Examples of such fluids include plasma, liquid metals, and salt water or electrolytes. The term MHD was first introduced by Hannes Alfvén in 1942. However, interest in the MHD flow began in 1918, when Hartmann invented electromagnetic pump. MHD flows have several important applications in engineering and industry such as petroleum industries, the boundary layer control in

aerodynamics, MHD power generators, accelerators, MHD pumps, and cooling of nuclear reactors. MHD was originally applied to astrophysical and geophysical problems, but very recently to fusion power problems, since the material walls would be destroyed, therefore electromagnetic forces are used to isolate hot plasmas from the wall. MHD flows are also important in metallurgical and metal work process, where magnetic field is used to control the rate of cooling of strips (Nandy and Mahapatra, 2013).

On the other hand, magnetic nanofluids are unique material that possesses characteristics of both liquid and magnet. Magnetic nanofluids are used for numerous studies because they are easily manipulated with an external magnetic field (Mohamad *et al.*, 2013). Magnetic nanoparticles are also important in several potential biomedical applications such as magnetic cell separation, hyperthermia and contrast enhancement in magnetic resonance imaging (MRI). Further applications of magnetic nanofluids can also be found in cancer therapy, blood analysis and construction of loud speakers.

In addition to this, the application of MHD flow together with porous media has been the growing interest in the study of non-Newtonian fluids. It is due to the fact that magnetic field influence the heat generation/absorption process in electrically conducting flows. It is well known that the complicated structure of pores is the unique property of porous medium that distinguishes it from solid bodies. The use of MHD on boundary layer flow through porous medium can be found in geothermal energy recovery, plasma studies, oil extraction, nuclear reactors and thermal energy storage.

Another important mechanism in the analysis of fluid flow is the transport of heat transfer. Heat can be transferred from one place to another by three methods conduction, convection and radiation. If the system comprise of temperature differences then heat will always transferred from higher temperature to lower temperature. Some common examples of energy transfer in our daily life include boiling of water, heating and cooling of room and heat from sun or light bulb.

In the above mentioned three modes of heat transfer, radiation and convection have gained considerable attention because of its practical applications in science and engineering. Radiation is the form of energy transport consisting of electromagnetic waves. Since all matters emit and absorb electromagnetic radiation therefore radiation heat transfer is everywhere. The interaction of heat transfer with small parts is called radiation. The radiation heat transfer cannot be ignored when high temperature is required for the preparation of final product. Radiative heat transfer also plays a vital role in several engineering areas such as nuclear power plant, gas turbine and various propulsion devices from aircraft, missiles, satellites and space vehicles (Kumara *et al.*, 2015).

In moving fluids, the effect of radiation is significant in applications including heat removal from nuclear fuel debris, underground disposal of radiative waste material, storage of food stuffs, dislocating of fluids in packed bed reactors (Akbar *et al.*, 2015). The phenomenon of thermal radiation usually occurs due to emission by hot walls and moving fluid (Lavanya and Chenna, 2014). In polymer industry, thermal radiation is used to control heat transfer process. Generally, absorption/emission, absorption coefficient and inclusion of radiation term are three main difficulties arise in the computation of highly non-linear partial differential equations of radiative flow (Pal and Mondal, 2009).

Convection is defined as, the transfer of heat energy between surface and moving fluid at different temperature. In other words, convection is the combination of diffusion and bulk motion of molecules. The fluid velocity is low near the surface and diffusion dominates whereas the influence of bulk motion away from the surface enhanced and dominates. Convection heat transfer further divided into forced convection and natural or free convection. Forced convection occurs if fluid motion is induced by external resources such as pump, fan, and fluid machinery. While the fluid motion induced by buoyancy forces or natural forces due to density variation caused by temperature differences is referred as natural or free convection. In the boundary layer, the density of the fluid rises with heating and as a result raises the fluid, and is replaced by cooler fluid that will also heat and rise and the process continues. This continuous process is also called natural or free convection. The mechanism of mixed convection occurs when forced convection and natural or free convection act

simultaneously to transfer heat. The interaction of both pressure and buoyant forces is also called mixed convection.

In general, the presence of acceleration field and fluid density gradient are two conditions that are required for fluid to be set in motion in free convection. The most common acceleration field is gravity. Since temperature changes density of all fluids, it follows that a temperature gradient will set up a density gradient. Free convection in flow loops results in the circulation of the working fluid, referred to as natural circulation, which plays a major role in nuclear plants during shutdown. Free convection caused mixing in ocean and lakes and also is the source of all weather pattern (Lavanya and Chenna, 2014). Generally, velocities of free convection flow are much smaller than forced convection therefore corresponding rates of convection transfer are also smaller. In several systems, multimode heat transfer effects are involved, mixed convection plays a vital role in the design or performance of these systems as large resistance is provided to heat transfer. Moreover, free and mixed convection are preferred to forced convection when the minimum rate of heat transfer or minimum operating cost is required.

In the study of heat transfer, one of the significant variables is temperature, and it is important to write the net buoyancy force in terms of temperature difference, that represent the variation of fluid density with temperature at constant pressure. Mathematically, it can be written as (Jaluria 1980)

$$\beta_T = -\frac{1}{\rho_f} \frac{\rho_\infty - \rho_f}{(T_\infty - T)}, \quad (1.1)$$

$$\rho_\infty - \rho_f = \rho_f \beta_T (T - T_\infty), \quad (1.2)$$

where β_T is the volume expansion coefficient. In equation (1.2), $\rho_\infty - \rho_f$ is density difference and is known as Boussinesq approximation. Since, it gives rise to buoyancy force and sustains flow.

Another important phenomenon in the convective flows is the simultaneous effect of heat and mass transfer on fluids because sometimes, in convective flows, concentration difference also occurs together with temperature differences. In nature, the presence of pure water or air is not possible, where there is always a chance of chemical reaction between foreign mass and water or air (Kandasamy *et al.*, 2009). The classification of chemical reaction as homogeneous or heterogeneous depends only on its occurrence either at an interface or as single phase volume reaction (Bakr, 2011). Common real life applications of homogeneous reaction are the reaction between household gas and oxygen to get the flame and the reaction between acids and bases to produce salt and water. Another important application of a homogeneous chemical reaction is the formation of smog, which represents a first order chemical reaction where the reaction rate is directly proportional to species concentration.

The study of mixed convection flow with simultaneous heat and mass transfer under the influence of chemical reaction has attracted a considerable attention of researchers due to its widespread applications in engineering and industry. Specific applications of chemical reaction are found in design of chemical processing equipment, damage of crops, nuclear reactor safety, and solar collectors as well as in food processing and cooling towers. In addition, heat generation occurs when chemical reaction takes place between two species.

Further, the study of convective flow in heat absorbing fluids depends on thermal conditions. Generally, two types of temperature distributions are applied to thermal boundary conditions, namely, prescribed wall temperature and prescribed surface heat flux. There is another temperature distribution also applied to thermal boundary, known as convective/conjugate boundary condition. The need of heat transfer through convective boundary condition arises when heat transfer from the surface is proportional to local surface temperature. The existence of convective heat between the surface and surrounding fluid cannot be ignored while heating or cooling of the surface (Swapna *et al.*, 2015).

For the first time, Merkin (1994) pointed out four common types of heating processes, namely, (i) prescribed wall temperature distributions; (ii) prescribed surface heat flux; (iii) Newtonian heating; and (iv) convective/conjugate condition at

temperature wall. The study on convective boundary condition in porous medium has been carried out experimentally and numerically for several years because of its wide range applications in geothermal energy extraction, catalytic and chemical particle beds, petroleum processing, transpiration cooling, solid matrix, or micro porous heat exchangers, packed bed regenerators, heat transfer enhancement and micro-thrusters (Mahanta and Shaw, 2015). In the present study, convective conditions at temperature and concentration walls are also taken into account.

Moreover, the addition of velocity slip at wall is also an important factor to increase heat transfer in the fluid. The non-adherence of the fluid to a solid boundary is known as velocity slip. Slip flow normally occurs; either the size of flow system or the flow pressure is very small. A bulk of literature is dedicated to the flow with no slip condition. However, in several situations this condition fails to work. The need of partial slip often arise at stretching wall of several fluids like emulsions, suspensions, foams and polymer solutions are amongst the others. The term “partial slip” was first used by Beaver and Joseph (Beavers and Joseph, 1967). The behavior of slip flow is quite different from traditional flow because of its micro scale dimension (Turkyilmazoglu, 2012). The addition of slip condition is also efficient in the prediction of wall shear stress and heat transfer.

On the other hand, the study of non-Newtonian fluid over a stretching sheet has gained considerable attention due to its practical applications in several industries and numbers of technological processes, namely, glass manufacturing, geophysics, extrusion of plastic and rubber sheet, paper production, polymer processing and purification of crude oil. Crane (1970) was the first who investigated the flow caused by stretching sheet, whose velocity is varying linearly from the fixed point.

In the analysis of stretching sheet problems, stretching rate and rate of cooling are two main factors that influenced the mechanical properties of end product (Vyas and Ranjan, 2010). The desired thickness of polymer sheet from a dye in the manufacturing can be achieved by stretching the melt as it flows out from silt (Mahapatra *et al.*, 2011). The appropriate choice of heating/cooling fluid is important in stretching sheet because of its direct impact on rate of heat transfer (Pal and Mandal, 2015). A large body of literature is available on boundary layer flow of linear

stretching sheet; however, the stretching sheet velocity needs not to be linear (Gupta and Gupta, 1977). In view of this, researchers diverted their study to the boundary layer flow caused by exponentially, quadratically, and nonlinearly stretching sheet. The problem of boundary layer flow over nonlinearly stretching sheet also arises in several technical and industrial applications such as polymer sheet extrusion from dye, aerodynamic extrusion of plastic sheet and glass fiber production, among others (Jat *et al.*, 2014).

Another important and interesting study exists in boundary layer theory is the wedge flow, that is generated by pressure gradient. The wedge can be static (stationary) or moving in the fluid. Wedge is a piece of wood, metal or some other material which is thick at one end and thin at the other end, and is being used for the separation of two objects in contact. Falkner and Skan (1931) were the first who derived the equations for boundary layer flow over static wedge. They transformed the nonlinear equations by using similarity transformations and obtained similar solutions by taking the free stream velocity as power law function. For this reason, sometimes these equations are also called Falkner-Skan equations. The flow of heat transfer past a wedge has applications in several fields including aerodynamics, design of space craft, different types of transformers and generators and nuclear reactors (Prasad *et al.*, 2013).

Based on the discussions, two different geometries, namely, nonlinear stretching sheet and moving wedge are considered in this thesis. Further, steady and unsteady mixed convection flow of Casson and Casson nanofluid is analyzed in the presence of magnetic field, velocity slip condition, thermal radiation, chemical reaction and convective boundary conditions. Similarity transformations are used for the conversion of nonlinear partial differential equations into a set of nonlinear ordinary differential equations and then solved numerically by Keller-box method (Cebeci and Bradshaw, 1988). Numerical and graphical results are achieved by developing an algorithm in MATLAB software.

1.3 Problem Statement

Experts worldwide agree that heat and mass transfer is the most important factor in the analysis of boundary layer flow. A large body of research supports the key role of mixed convection flow of non-Newtonian fluids over stretching surfaces as well as wedge shaped bodies. On the other hand, the thermal conductivity of traditional fluids plays an important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Due to low thermal conductivity, the traditional fluids are unproductive and inadequate to fulfill today's cooling rate requirements. The reason is that desired quality of final product highly depends on heat transfer rate. Recent research reveals that thermal conductivity of nanofluids is comparatively higher than the base fluid. Present work will demonstrate the answers of following questions. How do the existing steady and unsteady Casson and Casson nanofluid models can be modified to analyze magnetohydrodynamic effects on flow over nonlinearly stretching sheet through porous medium? How does mixed convection flow will behave together with convective boundary conditions? How do the combined effects of thermal radiation and slip condition will affect the heat and mass transfer characteristics of Casson and Casson nanofluid? How does hydromagnetic flow of Casson fluid and Casson nanofluid over nonlinearly stretching sheet and moving wedge embedded in a porous medium be tackled by Keller-box method? How to develop an algorithm in MATLAB software that work for both steady and unsteady flow? Specifically, the problems considered in this study are as follow:

- (i) Steady and unsteady MHD mixed convection flow of Casson fluid over a nonlinearly stretching sheet saturated in a porous medium.
- (ii) Steady and unsteady MHD mixed convection flow of Casson nanofluid over a nonlinearly stretching sheet saturated in porous medium.
- (iii) Steady and unsteady MHD mixed convection flow of Casson fluid over a moving wedge saturated in porous medium.
- (iv) Steady and unsteady MHD mixed convection flow of Casson nanofluid over a moving wedge saturated in porous medium.

1.4 Objectives and Scope of Research

This study numerically investigates the steady and unsteady MHD mixed convection flow of Casson and Casson nanofluid over a nonlinearly stretching sheet and moving wedge in the presence of thermal radiation, chemical reaction, slip and convective boundary conditions. The highly nonlinear coupled partial differential equations are transformed into nonlinear ordinary differential equation with the help of suitable transformations and then solved using Keller-box method. The obtained results are displayed graphically and discussed in detail. In order to validate the present method, numerical results for skin friction and Nusselt number are compared with the previously published results. Following are the objectives of the present study:

- (i) To formulate steady and unsteady Casson and Casson nanofluid due to nonlinearly stretching sheet and moving wedge.
- (ii) To develop an algorithm in MATLAB software in order to get the solutions of all problems.
- (iii) To investigate the effects of pertinent parameters on velocity, temperature and concentration profiles as well as the variation of local skin friction, Nusselt and Sherwood numbers.

1.5 Scope of the Study

This study circles around steady and unsteady MHD mixed flow of Casson and Casson nanofluid. The flow is generated due to a nonlinear stretching sheet and a moving wedge. The first four problems explore steady and unsteady mixed convection flow of Casson and Casson nanofluid over a nonlinear stretching sheet. The next four problems, emphasis on steady and unsteady mixed convection flow of Casson and Casson nanofluid over a moving wedge. Further, the effects of MHD, porous medium, thermal radiation, chemical reaction, slip and convective boundary conditions are also taken into account, and are common in all eight problems. The proposed problems are solved numerically via Keller-box method. The Keller-box method is the implicit

finite difference scheme and is unconditionally stable. Since the governing equations are parabolic, therefore this method is chosen based on the fact that it is preferable method for the parabolic equations. The detail of this method can be found in (Cebeci and Bradshaw, 1988), (Sharidan, 2005) and (Sarif *et al.*, 2013). The numerical results are achieved via MATLAB algorithm and displayed graphically. The accuracy of the present method is compared with the results of available literature, and revealed in close agreement.

1.6 Significance of Study

The significance of the study is stated as follows:

- (i) Provide better theoretical understanding of Casson fluid over a nonlinearly stretching sheet and moving wedge filled with and without nanoparticles.
- (ii) Establishment of relationship between steady and unsteady Casson fluid and Casson nanofluid.
- (iii) Derivation of heat and mass transfer flow of Casson fluid involving buoyancy, chemical reaction and thermal radiation simultaneously.
- (iv) Enhance understanding of MHD mixed convection flow inside a porous medium.
- (v) Development of MATLAB code that will be capable of solving steady and unsteady flow of Casson and Casson nanofluid.
- (vi) The analysis in this thesis will serve as future reference for researchers on Casson fluid flow over a nonlinear stretching sheet and moving wedge with and without nanoparticles.

1.7 Thesis Outline

This thesis comprises of eight chapters. The brief introduction of research background along with a problem statement, objectives and scope of research and significance of research are presented in Chapter 1. In Chapter 2, an extensive

literature review related to the problems of interest as mentioned in the objectives and scope of the research is presented.

Chapter 3 shows the derivation of the governing equations of unsteady flow of Casson and nanofluid. The skin friction coefficient, Nusselt and Sherwood numbers are also derived in this chapter.

Chapter 4 analyzes the steady and unsteady flow of Casson fluid due to nonlinear stretching sheet. The effects of pertinent parameters on velocity, temperature and concentration profiles as well as friction factor, heat and mass transfer rates are displayed graphically and discussed. In Chapter 5, the steady and unsteady flow of Casson nanofluid due to nonlinear stretching sheet is discussed. This study is the extension of Chapter 4. In this chapter, the Casson fluid is taken as a base fluid for suspended nanoparticles. Further, Buongiorno's model is adopted for the analysis of flow fields. Besides the effects considered in the previous chapter, the influence of Brownian motion and thermophoresis parameters are also investigated.

Chapter 6 deals with the steady and unsteady electrically conducting mixed convection flow of Casson fluid over a moving wedge. Likewise, the pertinent parameters are kept similar to Chapter 4. The flow fields are analyzed and presented graphically. Chapter 7 reveals the steady and unsteady flow of Casson nanofluid due to moving wedge. This chapter is the extension of Chapter 6. The wedge flow is investigated for the Casson fluid in the presence of suspended nanoparticles. The effects of Brownian motion and thermophoresis parameters along with the parameters considered in Chapter 6 are interpreted in this chapter. Finally, Chapter 8 describes the brief summary of this research and suggestions for future research.

REFERENCES

- Abd El-Aziz, M. (2009). Radiation effect on the flow and heat transfer over an unsteady stretching sheet. *International Communications in Heat and Mass Transfer*, 36(5), 521–524.
- Ahmad, K., & Nazar, R. (2010). Magnetohydrodynamic three-dimensional flow and heat transfer over a stretching surface in a viscoelastic fluid. *Journal of science and technology*, 3(1), 1-14.
- Ahmed, N., Sarma, D., and Deka, H. (2012). MHD Mixed Convection and Mass Transfer from an Infinite Vertical Porous Plate with Chemical Reaction in Presence of a Heat Source. *Applied Mathematical Sciences*, 6(21), 1011–1020.
- Akbar, N. S., Ebaid, A., and Khan, Z. H. (2015). Numerical analysis of magnetic field effects on Eyring-Powell fluid flow towards a stretching sheet. *Journal of Magnetism and Magnetic Materials*, 382, 355–358.
- Alam, M. S., and Hossain, S. M. C. (2013). A new similarity approach for an unsteady two- dimensional forced convective flow of a micropolar fluid along a wedge. *Int. J. Appl. Math. and Mech.*, 9(14), 75–89.
- Ali, M. E., and Magyari, E. (2007). Unsteady fluid and heat flow induced by a submerged stretching surface while its steady motion is slowed down gradually. *International Journal of Heat and Mass Transfer*, 50(1-2), 188–195.
- Andersson, H. I., Aarseth, J. B., and Dandapat, B. S. (2000). Heat transfer in a liquid film on an unsteady stretching surface. *International Journal of Heat and Mass Transfer*, 43(1), 69–74.
- Animasaun, I. L. (2014). Effects of thermophoresis, variable viscosity and thermal conductivity on free convective heat and mass transfer of non-darcian MHD dissipative Casson fluid flow with suction and n^{th} order of chemical reaction. *Journal of the Nigerian Mathematical Society*, 34(1), 11–31.
- Anwar, M. I., Shafie, S., Khan, I., and Salleh, M. Z. (2012). Conjugate effects of radiation flux on double diffusive mhd free convection flow of a nanofluid over a power law stretching sheet. *International Scholarly Research Network*, 1–7.

- Anwar, M. I., Sharidan, S., Khan, I., and Salleh, M. Z. (2014). Magnetohydrodynamic and radiation effects on stagnation-point flow of nanofluid towards a nonlinear stretching sheet. *Indian Journal of Chemical Technology*, 21, 199–204.
- Aurangzaib, Kasim, A. R. M., Mohammad, N. F., & Shafie, S. (2012). Effect of thermal stratification on MHD free convection with heat and mass transfer over an unsteady stretching surface with heat source, Hall current and chemical reaction. *International Journal of Advances in Engineering Sciences and Applied Mathematics*, 4(3), 217-225.
- Awais, M., Malik, M. Y., Bilal, S., Salahuddin, T., & Hussain, A. (2017). Magnetohydrodynamic (MHD) flow of Sisko fluid near the axisymmetric stagnation point towards a stretching cylinder. *Results in Physics*, 7, 49-56.
- Bachok, N., Ishak, A., and Pop, I. (2012). Unsteady boundary-layer flow and heat transfer of a nanofluid over a permeable stretching/shrinking sheet. *International Journal of Heat and Mass Transfer*, 55(7-8), 2102–2109.
- Bakr, A. A. (2011). Effects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. *Communications in Nonlinear Science and Numerical Simulation*, 16(2), 698–710.
- Bayley, F. J., Owen, J., and Turner, A. (1972). *Heat Transfer*. London: Thomas Nelson and Sons Ltd.
- Beavers, G. S., and Joseph, D. D. (1967). Boundary conditions at a naturally permeable wall. *Journal of Fluid Mechanics*. 30(01), 197-207.
- Bég, O. A., Khan, M. S., Karim, I., Alam, M. M., and Ferdows, M. (2013). Explicit numerical study of unsteady hydromagnetic mixed convective nanofluid flow from an exponentially stretching sheet in porous media. *Applied Nanoscience*, 4(8), 943–957.
- Bég, O. A., and Makinde, O. D. (2011). Viscoelastic flow and species transfer in a Darcian high-permeability channel. *Journal of Petroleum Science and Engineering*, 76(3-4), 93–99.
- Bég, O. A., Maleque, K. A., and Islam, M. N. (2012). Modelling of Ostwald-deWaele non-Newtonian flow over a rotating disk in a non-Darcian porous medium. *Int. J. Appl. Math. Mech*, 8(13), 46–67.

- Beg, O. A., Takhar, H., Bharagava, R., Rawat, S., and Prasad, V. (2008). Numerical study of heat transfer of a third grade viscoelastic fluid in non-Darcian porous media with thermophysical effects. *Physica Scripta*, 77(6), 1–11.
- Bhattacharyya, S., Pal, A., and Pop, I. (1998). Unsteady mixed convection on a wedge in a porous medium S. *Int. Comm. Heat Mass Transfer*, 25(5), 743–752.
- Buongiorno, J. (2006). Convective Transport in Nanofluids. *ASME J. of Heat Transfer*, 128(3), 240–250.
- Casson, N. (1959). A flow equation for pigment-oil suspensions of the printing ink type. In: Mill, C.C., Ed., *Rheology of Disperse Systems*, Pergamon Press, Oxford, 84–104.
- Cebeci, T., and Bradshaw, P. (1988). *Physical and computational aspects of convective heat transfer* (1st ed.). New York,: Springer New York.
- Chamkha, A., Aly, A., and Mansour, M. (2010). Similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects. *Chemical. Engineering Communications*, 197(6), 846–858.
- Chamkha, A. J., Abbasbandy, S., Rashad, a. M., and Vajravelu, K. (2012). Radiation Effects on Mixed Convection over a Wedge Embedded in a Porous Medium Filled with a Nanofluid. *Transport in Porous Media*, 91(1), 261–279.
- Chamkha, A. J., and Rashad, A. M. (2014). MHD forced convection flow of a nanofluid adjacent to a non-isothermal wedge. *Computational Thermal Sciences*, 6(1), 27–39.
- Chamkha, A. J., Rashad, A. M., EL-Hakiem, M. A., and Abdou, M. M. M. (2011). Non-similar solutions for mixed convective boundary layer flow a non-Newtonian fluid over a wedge embedded in a porous medium filled with a nanofluid. *International Journal of Microscale and Nanoscale Thermal and Fluid Transport Phenomena*, 2(4), 323–341.
- Chamkha, A. J., Rashad, A. M., and Gorla, R. S. R. (2014). Non-similar solutions for mixed convection along a wedge embedded in a porous medium saturated by a non-Newtonian nanofluid: Natural convection dominated regime. *International Journal of Numerical Methods for Heat and Fluid Flow*, 24(7), 1471–1486.
- Choi, S. U. S., and Eastman, J. A. (1995). Enhancing thermal conductivity of fluids with nanoparticles. *ASME-Publications-Fed*, 231, 99-106.

- Cortell, R. (2007). Viscous flow and heat transfer over a nonlinearly stretching sheet. *Applied Mathematics and Computation*, 184(2), 864–873.
- Cortell, R. (2008). Effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. *Physics Letters A*, 372(5), 631–636.
- Cortell, R. (2014). Fluid flow and radiative nonlinear heat transfer over a stretching sheet. *Journal of King Saud University - Science*, 26(2), 161–167.
- Crane, L. J. (1970). Flow past a stretching plate. *Zeitschrift für angewandte Mathematik und Physik (ZAMP)*, 21(4), 645–647.
- Das, B., and Batra, R. L. (1993). Secondary flow of a Casson fluid in a slightly curved tube. *International Journal of Non-Linear Mechanics*, 28(5), 567–577.
- Das, K. (2014). Nanofluid flow over a non-linear permeable stretching sheet with partial slip. *Journal of the Egyptian Mathematical Society*, 23(2), 451–456.
- Das, K., Duari, P. R., and Kundu, P. K. (2014a). Nanofluid flow over an unsteady stretching surface in presence of thermal radiation. *Alexandria Engineering Journal*, 53(3), 737–745.
- Das, K., Duari, P. R., and Kundu, P. K. (2014b). Numerical simulation of nanofluid flow with convective boundary condition. *Journal of the Egyptian Mathematical Society*, 23(0), 435–439.
- Davidson, P. A. (2001). *An introduction to magnetohydrodynamics* (Vol. 25). Cambridge university press.
- Elbashbeshy, E. M. A, and Bazid, M. A. A. (2004). Heat transfer over an unsteady stretching surface. *Heat and Mass Transfer*, 41(1), 1–4.
- El-dabe, N. T., Ghaly, A. Y., Rizkallah, R. R., and Ewis, K. M. (2015). Numerical solution of mhd boundary layer flow of non-newtonian casson fluid on a moving wedge with heat and mass transfer and induced magnetic field. *Journal of Applied Mathematics and Physics*, 3, 649–663.
- Falkner, V. M., and Skan, S. W. (1931). Some approximate solutions of the boundary-layer equations. *Philosophical Magazine.*, 12, 865–896.
- Gaffar, S. A., Prasad, V. R., and Keshava Reddy, E. (2015). Mixed convection boundary layer flows of a non-Newtonian Jeffrey's fluid from a non-isothermal wedge. *Ain Shams Engineering Journal*, 8(2), 145-162.
- Ganapathirao, M., Ravindran, R., & Momoniat, E. (2015). Effects of chemical reaction, heat and mass transfer on an unsteady mixed convection boundary layer

- flow over a wedge with heat generation/absorption in the presence of suction or injection. *Heat and Mass Transfer*, 51(2), 289-300.
- Ganapathirao, M., Ravindran, R., and Momoniat, E. (2015). Effects of chemical reaction, heat and mass transfer on an unsteady mixed convection boundary layer flow over a wedge with heat generation/absorption in the presence of suction or injection. *International Journal of Heat and Mass Transfer*, 51, 289–300.
- Ghiaasiaan, S. M. (2011). *Convective heat and mass transfer*. Cambridge University Press.
- Gorla, R. S. R., Chamkha, A. J., and Rashad, A. M. (2011). Mixed convective boundary layer flow over a vertical wedge embedded in a porous medium saturated with a nanofluid: Natural Convection Dominated Regime. *Nanoscale Research Letters*, 6(1), 207.
- Gupta, A., and Gupta, P. (1977). Heat and mass transfer on a stretching sheet with suction or blowing. *The Canadian Journal of Chemical Engineering*, 55(6), 744–746.
- Harris, S. D., Ingham, D. B., and Pop, I. (2008). Unsteady heat transfer in impulsive Falkner-Skan flows: Constant wall heat flux case. *Acta Mechanica*, 201(1-4), 185–196.
- Hassani, M., Tabar, M. M., Nemat, H., Domairry, G., and Noori, F. (2011). An analytical solution for boundary layer flow of a nanofluid past a stretching sheet. *International Journal of Thermal Sciences*, 50(11), 2256–2263.
- Hayat, T., Abbas, Z., Pop, I., & Asghar, S. (2010). Effects of radiation and magnetic field on the mixed convection stagnation-point flow over a vertical stretching sheet in a porous medium. *International Journal of Heat and Mass Transfer*, 53(1), 466-474.
- Hayat, T., Hussain, M., Nadeem, S., and Mesloub, S. (2011). Falkner-Skan wedge flow of a power-law fluid with mixed convection and porous medium. *Computers and Fluids*, 49(1), 22–28.
- Hayat, T., Imtiaz, M., and Alsaedi, A. (2016a). Unsteady flow of nanofluid with double stratification and magnetohydrodynamics. *International Journal of Heat and Mass Transfer*, 92, 100–109.
- Hayat, T., Javed, T., and Abbas, Z. (2009). MHD flow of a micropolar fluid near a stagnation-point towards a non-linear stretching surface. *Nonlinear Analysis: Real World Applications*, 10(3), 1514–1526.

- Hayat, T., Shafiq, A., Imtiaz, M., & Alsaedi, A. (2016b). Impact of melting phenomenon in the Falkner–Skan wedge flow of second grade nanofluid: A revised model. *Journal of Molecular Liquids*, 215, 664–670.
- Hayat, T., Waqas, M., Shehzad, S. A., & Alsaedi, A. (2016c). Stretched flow of Carreau nanofluid with convective boundary condition. *Pramana*, 86(1), 3–17.
- Hirsch, C. (2007). *Numerical computation of internal and external flows: The fundamentals of computational fluid dynamics*. Butterworth-Heinemann.
- Hossain, A. M., Bhowmick, S., and Gorla, R. S. R. (2006). Unsteady mixed-convection boundary layer flow along a symmetric wedge with variable surface temperature. *International Journal of Engineering Science*, 44(10), 607–620.
- Hsiao, K. (2010). Mixed convection with radiation effect over a nonlinearly stretching sheet. *World Academy of Science, Engineering and Technology*, 4(2), 338–342.
- Hsiao, K. L. (2011). MHD mixed convection for viscoelastic fluid past a porous wedge. *International Journal of Non-Linear Mechanics*, 46(1), 1–8.
- Hsiao, K. L. (2016). Stagnation electrical MHD nanofluid mixed convection with slip boundary on a stretching sheet. *Applied Thermal Engineering*, 98, 850–861.
- Hussain, T., Shehzad, S. A., Alsaedi, A., Hayat, T., and Ramzan, M. (2015). Flow of Casson nanofluid with viscous dissipation and convective conditions: A mathematical model. *Journal of Central South University*, 22(3), 1132–1140.
- Ibrahim, S. M., Lorenzini, G., Kumar, V. P., and Raju, C. S. K. (2017). Influence of chemical reaction and heat source on dissipative MHD mixed convection flow of a Casson nanofluid over a nonlinear permeable stretching sheet. *International Journal of Heat and Mass Transfer*, 111, 346–355.
- Ibrahim, W., and Shankar, B. (2013). MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions. *Computers and Fluids*, 75, 1–10.
- Ishak, A., Nazar, R., and Pop, I. (2006). Moving wedge and flat plate in a micropolar fluid. *International Journal of Engineering Science*, 44(18-19), 1225–1236.
- Ishak, A., Nazar, R., and Pop, I. (2007). Falkner-Skan equation for flow past a moving wedge with suction or injection. *Journal of Applied Mathematics and Computing*, 25(1-2), 67–83.
- Ishak, A., Nazar, R., and Pop, I. (2009). MHD boundary-layer flow of a micropolar fluid past a wedge with constant wall heat flux. *Communications in Nonlinear Science and Numerical Simulation*, 14(1), 109–118.

- Jalilpour, B., Jafarmadar, S., Ganji, D. D., Shotorban, A. B., and Taghavifar, H. (2014). Heat generation/absorption on MHD stagnation flow of nanofluid towards a porous stretching sheet with prescribed surface heat flux. *Journal of Molecular Liquids*, 195, 194–204. <https://doi.org/10.1016/j.molliq.2014.02.021>
- Jaluria, Y. (1980). *Natural Convection Heat and Mass Transfer*. Pergamon Press Oxford.
- James, M., Mureithi, E. W., and Kuznetsov, D. (2015). Effects of variable viscosity of nanofluid flow over a permeable wedge embedded in saturated porous medium with chemical reaction and thermal radiation MSC: *International Journal of Advances in Applied Mathematics and Mechanics*, 2(3), 101–118.
- Jat, R. N., Chand, G., and Rajotia, D. (2014). MHD Heat and Mass Transfer for Viscous flow over Nonlinearly Stretching Sheet in a Porous Medium. *Thermal Energy and Power Engineering*, 3(1), 191–197.
- Kandasamy, R., Loganathan, P., and Arasu, P. P. (2011). Scaling group transformation for MHD boundary-layer flow of a nanofluid past a vertical stretching surface in the presence of suction/injection. *Computers and Fluids*, 52(1), 15–21.
- Kandasamy, R., and Muhaimin, I. (2015). Impact of thermal stratification on unsteady hiemenz non- darcy copper nanofluid flow over a porous wedge in the presence of magnetic field due to solar radiation (Green) Energy. *Chemical and Process Engineering Research*, 36, 31–46.
- Kandasamy, R., Muhaimin, I., and Khamis, A. B. (2009). Thermophoresis and variable viscosity effects on MHD mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction. *Heat Mass Transfer*, 45, 703–712.
- Kandasamy, R., Muhaimin, I., Khamis, A. B., and Roslan, R. Bin. (2013). Unsteady Hiemenz flow of Cu-nanofluid over a porous wedge in the presence of thermal stratification due to solar energy radiation: Lie group transformation. *International Journal of Thermal Sciences*, 65(January), 196–205.
- Kandasamy, R., Muhaimin, I., and Rosmila, A. K. (2014). The performance evaluation of unsteady MHD non-Darcy nanofluid flow over a porous wedge due to renewable (solar) energy. *Renewable Energy*, 64, 1–9.
- Kasmani, R. M., Sivasankaran, S., Bhuvaneshwari, M., and Siri, Z. (2016). Effect of chemical reaction on convective heat transfer of boundary layer flow in nanofluid

- over a wedge with heat generation / absorption and suction. *Journal of Applied Fluid Mechanics*, 9(1), 379–388.
- Kays, W. M., Crawford, M., and Weigand, B. (2005). *Convective Heat and Mass Transfer* (4th ed.). New Jersey: McGraw-Hill.
- Kechil, S. A., and Hashim, I. (2009). Flow and diffusion of chemically reactive species over a nonlinearly stretching sheet immersed in a porous medium. *Journal of Porous Media*, 12(11), 1053–1063.
- Khalili, S., Tamim, H., Khalili, A., and Rashidi, M. M. (2015). Unsteady convective heat and mass transfer in pseudoplastic nanofluid over a stretching wall. *Advanced Powder Technology*, 26(5), 1319–1326.
- Khan, M., Karim, I., Islam, M., and Wahiduzzaman, M. (2014). MHD boundary layer radiative, heat generating and chemical reacting flow past a wedge moving in a nanofluid. *Nano Convergence*, 1(1), 20.
- Khan, W., Gul, T., Idrees, M., Islam, S., Khan, I., & Dennis, L. C. C. (2016a). Thin Film Williamson Nanofluid Flow with Varying Viscosity and Thermal Conductivity on a Time-Dependent Stretching Sheet. *Applied Sciences*, 6(11), 334.
- Khan, U., Ahmed, N., Mohyud-Din, S. T., and Bin-Mohsin, B. (2016b). Nonlinear radiation effects on MHD flow of nanofluid over a nonlinearly stretching/shrinking wedge. *Neural Computing and Applications*, 1–10.
- Khan, W. A., and Gorla, R. S. R. (2010). Mixed convection of power-law fluids along a vertical wedge with convective boundary condition in a porous medium. *Journal of Mechanical Science and Technology*, 24(9), 1919–1925.
- Khan, W. A., Hamad, M. A. A., and Ferdows, M. (2012). Heat transfer analysis for Falkner-Skan boundary layer nanofluid flow past a wedge with convective boundary condition considering temperature-dependent viscosity. *Journal of Nanoengineering and Nanosystems*, 227(1), 19–27.
- Khan, W. A., and Pop, I. (2010). Boundary-layer flow of a nanofluid past a stretching sheet. *International Journal of Heat and Mass Transfer*, 53(11-12), 2477–2483.
- Khan, W. A., and Pop, I. (2013). Boundary layer flow past a wedge moving in a nanofluid. *Mathematical Problems in Engineering*.
- Kumara, B. C. P., Ramesh, G. K., Chamkha, A. J., and Gireesha, B. J. (2015). Stagnation-point flow of a viscous fluid towards a stretching surface with

- variable thickness and thermal radiation. *International Journal of Industrial Mathematics*, 7(1), 77–85.
- Kumari, M., Takhar, H. S., and Nath, G. (2001). Mixed convection flow over a vertical wedge embedded in a highly porous medium. *Heat and Mass Transfer*, 37, 139–146.
- Kuo, B. L. (2005). Heat transfer analysis for the Falkner-Skan wedge flow by the differential transformation method. *International Journal of Heat and Mass Transfer*, 48(23-24), 5036–5046.
- Kuznetsov, A. V., and Nield, D. A. (2010). The onset of double-diffusive nanofluid convection in a layer of a saturated porous medium. A revised model. *Transport in Porous Medium*, 85, 941–951.
- Lavanya, S., and Chenna, D. K. (2014). Magnetic field and Radiation effects on MHD Free convection heat and mass transfer flow through a porous medium with chemical reaction. *Int. Journal of Applied Sciences and Engineering Research*, 3(4), 850–868.
- Laxmi, T. V., and Shankar, B. (2016). Effect of Nonlinear Thermal Radiation on Boundary Layer Flow of Viscous Fluid over Nonlinear Stretching Sheet with Injection / Suction. *Journal of Applied Mathematics and Physics*, (4), 307–319.
- Leal, L. (1992). *Laminar Flow and Convective Transport Process: Scaling Principles and Asymptotic Analysis*. London:Heinemann.
- Lee, S., Choi, S. U. S., Li, S., and Eastman, J. A. (1999). Measuring thermal conductivity of fluids containing oxide nanoparticles. *Journal of Heat Transfer*, 121(2), 280–289.
- Lin, H. T., and Lin, L. K. (1987). Similarity solutions for laminar forced convection heat transfer from wedges to fluids of any Prandtl number. *International Journal of Heat and Mass Transfer*, 30(6), 1111–1118.
- Lok, Y. Y., Amin, N., and Pop, I. (2003). Unsteady boundary layer flow of a micropolar fluid near the rear stagnation point of a plane surface. *International Journal of Thermal Sciences*, 42(11), 995–1001.
- Mabood, F., Abdel-Rahman, R. G., and Lorenzini, G. (2016). Numerical study of unsteady jeffery fluid flow with magnetic field effect and variable fluid properties. *Journal of Thermal Science and Engineering Applications*, 8(4), 041003.

- Mahanta, G., and Shaw, S. (2015). 3D Casson fluid flow past a porous linearly stretching sheet with convective boundary condition. *Alexandria Engineering Journal*, 54(3), 653–659.
- Mahapatra, T. R., Nandy, S. K., Vajravelu, K., and Van Gorder, R. a. (2011). Stability analysis of fluid flow over a nonlinearly stretching sheet. *Archive of Applied Mechanics*, 81(8), 1087–1091.
- Mahdy, A. (2016). Unsteady MHD slip flow of a non-Newtonian Casson fluid due to stretching sheet with suction or blowing effect. *Journal of Applied Fluid Mechanics*, 9(2), 785–793.
- Mahdy, A., and Chamkha, A. (2015). Heat transfer and fluid flow of a non-Newtonian nanofluid over an unsteady contracting cylinder employing Buongiorno's model. *International Journal of Numerical Methods for Heat and Fluid Flow*, 25(4), 703–723.
- Mahdy, A., & Ahmed, S. E. (2015). Thermosolutal Marangoni boundary layer magnetohydrodynamic flow with the Soret and Dufour effects past a vertical flat plate. *Engineering Science and Technology, an International Journal*, 18(1), 24–31.
- Maity, S., Singh, S. K., and Kumar, A. V. (2016). Unsteady three dimensional flow of Casson liquid film over a porous stretching sheet in the presence of uniform transverse magnetic field and suction/injection. *Journal of Magnetism and Magnetic Materials*, 419, 292–300.
- Makinde, O. D., and Aziz, A. (2011). Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. *International Journal of Thermal Sciences*, 50(7), 1326–1332.
- Makinde, O. D., Khan, W. A., and Khan, Z. H. (2013). Buoyancy effects on MHD stagnation point flow and heat transfer of a nanofluid past a convectively heated stretching/shrinking sheet. *International Journal of Heat and Mass Transfer*, 62(1), 526–533.
- Malvandi, A., Hedayati, F., and Ganji, D. D. (2014). Slip effects on unsteady stagnation point flow of a nanofluid over a stretching sheet. *Powder Technology*, 253, 377–384.
- Martin, M. J., and Boyd, I. D. (2010). Falkner-Skan Flow over a Wedge with Slip Boundary Conditions. *Journal of Thermophysics and Heat Transfer*, 24(2), 263–270.

- Masuda, H., Ebata, A., and Teramae, K. (1993). Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles. Dispersion of Al₂O₃, SiO₂ and TiO₂ ultra-fine particles. *Netsu Bussei*, 7, 227–233.
- Medikare, M., Joga, S., and Chidem, K. K. (2016). MHD stagnation point flow of a Cfluid over a nonlinearly stretching sheet with viscous dissipation. *American Journal of Computational Mathematics*, 6(01), 37-48.
- Merkin, J. H. (1994). Natural-convection boundary-layer flow on a vertical surface with Newtonian heating. *International Journal of Heat and Fluid Flow*, 15(5), 392-398.
- Mohamad, R. B., Kandasamy, R., and Muhaimin, I. (2013). Enhance of heat transfer on unsteady Hiemenz flow of nanofluid over a porous wedge with heat source/sink due to solar energy radiation with variable stream condition. *Heat and Mass Transfer*, 49(9), 1261–1269.
- Mohiddin, S. G., Prasad, V. R., Varma, S. V. K., Program, A. E., Group, E. S., and Building, S. (2010). Numerical study of unsteady free convective heat and mass transfer in a Walters-B viscoelastic flow along a vertical cone. *International Journal of Advances in Applied Mathematics and Mechanics*, 6(15), 88–114.
- Monica, M., Sucharitha, J., and Kumar, C. K. (2016). Stagnation point flow of a Williamson fluid over a nonlinearly stretching sheet with thermal radiation. *American Chemical Science Journal*, 13(4), 1–8.
- Muhaimin, I., Kandasamy, R., Khamis, A. B., and Roslan, R. (2013). Effect of thermophoresis particle deposition and chemical reaction on unsteady MHD mixed convective flow over a porous wedge in the presence of temperature-dependent viscosity. *Nuclear Engineering and Design*, 261, 95–106.
- Mukhopadhyay, S. (2009). Effect of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium. *International Journal of Heat and Mass Transfer*, 52(13-14), 3261–3265.
- Mukhopadhyay, S. (2010). Effects of Slip on Unsteady Mixed Convective Flow and Heat Transfer Past a Stretching Surface. *Chinese Physics Letters*, 27(12), 124401.
- Mukhopadhyay, S. (2013a). Casson fluid flow and heat transfer over a nonlinearly stretching surface. *Chinese Physics B*, 22(7), 074701.

- Mukhopadhyay, S. (2013b). Effects of thermal radiation on Casson fluid flow and heat transfer over an unsteady stretching surface subjected to suction/blowing. *Chinese Physics B*, 22(11), 114702.
- Mukhopadhyay, S., and Bhattacharyya, K. (2012). Unsteady flow of a Maxwell fluid over a stretching surface in presence of chemical reaction. *Journal of the Egyptian Mathematical Society*, 20(3), 229–234.
- Mukhopadhyay, S., De, P. R., Bhattacharyya, K., and Layek, G. C. (2013a). Casson fluid flow over an unsteady stretching surface. *Ain Shams Engineering Journal*, 4(4), 933–938.
- Mukhopadhyay, S., and Mandal, I. C. (2014). Boundary layer flow and heat transfer of a Casson fluid past a symmetric porous wedge with surface heat flux. *Chinese Physics B*, 23(4), 044702.
- Mukhopadhyay, S., Mondal, I. C., and Chamkha, A. J. (2013b). Casson fluid flow and heat transfer past a symmetric wedge. *Heat Transfer-Asian Research*, 42(8), 665–675.
- Mustafa, M., Hayat, T., and Alsaedi, A. (2013). Unsteady Boundary Layer Flow of Nanofluid Past an Impulsively Stretching Sheet. *Journal of Mechanics*, 29(3), 423–432.
- Mustafa, M., and Khan, J. A. (2015). Model for flow of Casson nanofluid past a non-linearly stretching sheet considering magnetic field effects. *AIP Advances*, 5(7), 077148.
- Mutlag, A. A., Uddin, M. J., Hamad, M. A. A., & Ismail, A. I. M. (2013). Heat transfer analysis for falkner-skan boundary layer flow past a stationary wedge with slips boundary conditions considering temperature-dependent thermal conductivity. *Sains Malaysiana*, 42(6), 855-862.
- Nadeem, S., Haq, R. U., Akbar, N. S., and Khan, Z. H. (2013). MHD three-dimensional Casson fluid flow past a porous linearly stretching sheet. *Alexandria Engineering Journal*, 52(4), 577–582.
- Nadeem, S., and Hussain, S.T. (2016). Analysis of MHD Williamson nanofluid flow over a heated surface. *Journal of Applied Fluid Mechanics*. 9, 729–739.
- Nandeppanavar, M. M., and Siddalingappa, M. N. (2013). Effect of viscous dissipation and thermal radiation on heat transfer over a non-linearly stretching sheet through porous medium. *International Journal of Applied Mechanics and Engineering*, 18(2), 461–474.

- Nandy, S. K., and Mahapatra, T. R. (2013). Effects of slip and heat generation/absorption on MHD stagnation flow of nanofluid past a stretching/shrinking surface with convective boundary conditions. *International Journal of Heat and Mass Transfer*, 64, 1091–1100.
- Nazar, R., Amin, N., Filip, D., and Pop, I. (2004). Unsteady boundary layer flow in the region of the stagnation point on a stretching sheet. *International Journal of Engineering Science*, 42(11-12), 1241–1253.
- Noor, N. F. M., Haq, R. U., Nadeem, S., and Hashim, I. (2015). Mixed convection stagnation flow of a micropolar nanofluid along a vertically stretching surface with slip effects. *Meccanica*, 50(8), 2007–2022.
- Oyelakin, I. S., Mondal, S., and Sibanda, P. (2016). Unsteady Casson nanofluid flow over a stretching sheet with thermal radiation, convective and slip boundary conditions. *Alexandria Engineering Journal*, 55(2), 1025–1035.
- Ozisik, M.N. (1985). *Heat Transfer: A Basic Approach*. New York: McGraw- Hill Book Company.
- Pal, D., and Hiremath, P. S. (2010). Computational modeling of heat transfer over an unsteady stretching surface embedded in a porous medium. *Meccanica*, 45(3), 415–424.
- Pal, D., and Mandal, G. (2015). MHD convective stagnation-point flow of nanofluids over a non-isothermal stretching sheet with induced magnetic field. *Meccanica*, 50(8), 2023-2035.
- Pal, D., and Mondal, H. (2009). Influence of temperature-dependent viscosity and thermal radiation on MHD forced convection over a non-isothermal wedge. *Applied Mathematics and Computation*, 212(1), 194–208.
- Pal, D., Mandal, G., & Vajravalu, K. (2016). Soret and Dufour effects on MHD convective radiative heat and mass transfer of nanofluids over a vertical non-linear stretching/shrinking sheet. *Applied Mathematics and Computation*, 287, 184-200
- Pandey, A. K., and Kumar, M. (2016). Effect of viscous dissipation and suction/injection on MHD nanofluid flow over a wedge with porous medium and slip. *Alexandria Engineering Journal*, 55(4), 3115–3123.
- Pop, I., and Na, T. Y. (1996). Unsteady flow past a stretching sheet. *Mechanics Research Communications*, 23(4), 413–422.

- Porwal, P., and Badshah, V. H. (2012). Analysis of Steady Blood Flow with Casson Fluid along an Inclined Plane Influenced by the Gravity Force. *International Journal of Theoretical and Applied Sciences*, 4(2), 76–81.
- Pramanik, S. (2014). Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation. *Ain Shams Engineering Journal*, 5(1), 205–212.
- Prandtl, L. (1904). On fluid motions with very small friction. *Verhldg*, 3, 484-91.
- Prasad V, R. (2013). Flow and heat transfer of Casson Fluid from a horizontal circular cylinder with partial slip in non-darcy porous medium. *Journal of Applied and Computational Mathematics*, 02(02).
- Prasad, K. V., Datti, P. S., and Vajravelu, K. (2013). MHD mixed convection flow over a permeable non-isothermal wedge. *Journal of King Saud University - Science*, 25(4), 313–324.
- Prasad, K. V., Santhi, S. R., & Datti, P. S. (2012). Non-Newtonian power-law fluid flow and heat transfer over a non-linearly stretching surface. *Applied Mathematics*, 3(05), 425-435.
- Prasad, V., Vasu, B., Anwar, B., and Parshad, R. (2011). Unsteady free convection heat and mass transfer in a Walters-B viscoelastic flow past a semi-infinite vertical plate: A numerical study. *Thermal Science*, 15(suppl. 2), 291–305.
- Rahman, A. T. M. M., Alam, M. S., Alim, M. A., and Chowdhury, M. K. (2013). Unsteady MHD forced convective heat and mass transfer flow along a wedge with variable electric conductivity and thermophoresis. *Procedia Engineering*, 56, 531–537.
- Rahman, M. M., and Eltayeb, I. A. (2013). Radiative heat transfer in a hydromagnetic nanofluid past a non-linear stretching surface with convective boundary condition. *Meccanica*, 48(3), 601–615.
- Raju, C. S. K., and Sandeep, N. (2016a). MHD slip flow of a dissipative Casson fluid over a moving geometry with heat source/sink: A numerical study. *Acta Astronautica*, 133, 436–443.
- Raju, C. S. K., and Sandeep, N. (2016b). Nonlinear radiative magnetohydrodynamic Falkner-Skan flow of Casson fluid over a wedge. *Alexandria Engineering Journal*, 55(3), 2045–2054.

- Rana, P., and Bhargava, R. (2012). Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: A numerical study. *Communications in Nonlinear Science and Numerical Simulation*, 17(1), 212–226.
- Rashidi, M. M., Ali, M., Freidoonimehr, N., Rostami, B., and Hossain, M. A. (2015). Mixed convective heat transfer for MHD viscoelastic fluid flow over a porous wedge with thermal radiation. *Advances in Mechanical Engineering*, 6, 735939.
- Rashidi, M. M., Rastegari, M. T., Asadi, M., and Bég, O. A. (2012). a Study of Non-Newtonian Flow and Heat Transfer Over a Non-Isothermal Wedge Using the Homotopy Analysis Method. *Chemical Engineering Communications*, 199(2),
- Rawat, S., Kapoor, S., and Bhargava, R. (2016). MHD flow heat and mass transfer of micropolar fluid over a nonlinear stretching sheet with variable micro inertia density, heat flux and chemical reaction in a non-darcy porous medium. *Journal of Applied Fluid Mechanics*, 9(1), 321-331.
- Reddy, P. S., Reddy, P. S., Chamkha, A., & Chamkha, A. (2017). Heat and mass transfer analysis in natural convection flow of nanofluid over a vertical cone with chemical reaction. *International Journal of Numerical Methods for Heat & Fluid Flow*, 27(1), 2-22.
- Roy, N.C., and Hossain, M.A. (2017). Unsteady magnetohydrodynamic mixed convection flow of micropolar fluid past a permeable sphere. *Journal of Thermophysics and Heat Transfer*. 31, 686-699.
- Sahoo, B., and Poncet, S. (2011). Flow and heat transfer of a third grade fluid past an exponentially stretching sheet with partial slip boundary condition. *International Journal of Heat and Mass Transfer*, 54(23-24), 5010–5019.
- Sajid, M., Ahmad, I., Hayat, T., and Ayub, M. (2009). Unsteady flow and heat transfer of a second grade fluid over a stretching sheet. *Communications in Nonlinear Science and Numerical Simulation*, 14, 96–108.
- Sarif, N. M., Salleh, M. Z., and Nazar, R. (2013). Numerical solution of flow and heat transfer over a stretching sheet with Newtonian heating using the Keller box method. *Procedia Engineering*, 53(iv), 542–554.
- Sattar, M. A. (2011). A local similarity transformation for the unsteady two-dimensional hydrodynamic boundary layer equations of a flow past a wedge. *Int. J. Appl. Math. and Mech*, 7(1), 15-28.
- Shang, D. (2011). *Heat and Mass Transfer. Theory of heat transfer with forced convection film flows*. Springer Science and Business Media.

- Sharada, K., & Shankar, B. (2015). MHD Mixed Convection Flow of a Casson Fluid over an Exponentially Stretching Surface with the Effects of Soret, Dufour, Thermal Radiation and Chemical Reaction. *World Journal of Mechanics*, 5(09), 165.
- Sharidan, S. (2005). *Mathematical Modelling of g-Jitter Induced Free Convection*. Universiti Teknologi Malaysia.
- Sharidan, S., Mahmood, T., and Pop, I. (2006). Similiarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet. *Int. J. of Applied Mechanics and Engineering*, 11(3), 647–654.
- Shen, M., Wang, F., and Chen, H. (2015). MHD mixed convection slip flow near a stagnation-point on a nonlinearly vertical stretching sheet. *Boundary Value Problems*, 2015(1), 1-15.
- Shit, G. C., Haldar, R., and Ghosh, S. K. (2015). Convective heat transfer and MHD viscoelastic nanofluid flow induced by a stretching sheet. *International Journal of Applied and Computational Mathematics*.
- Singh, P. J., Roy, S., and Ravindran, R. (2009). Unsteady mixed convection flow over a vertical wedge. *International Journal of Heat and Mass Transfer*, 52(1-2), 415–421.
- Singh, S. (2011). Clinical Significance of Aspirin on Blood Flow through Stenotic Blood Vessels. *Journal of Biomimetics, Biomaterials, and Tissue Engineering*, 10(m), 17–24.
- Su, X., Zheng, L., Zhang, X., & Zhang, J. (2012). MHD mixed convective heat transfer over a permeable stretching wedge with thermal radiation and ohmic heating. *Chemical Engineering Science*, 78, 1-8.
- Subba Rao, A., Ramachandra Prasad, V., Bhaskar Reddy, N., and Anwar Bég, O. (2015). Heat transfer in a casson rheological fluid from a semi-infinite vertical plate with partial slip. *Heat Transfer-Asian Research*, 44(3), 272–291.
- Sulochana, C., Ashwinkumar, G. P., & Sandeep, N. (2016). Similarity solution of 3D Casson nanofluid flow over a stretching sheet with convective boundary conditions. *Journal of the Nigerian Mathematical Society*, 35(1), 128-141.
- Sumalatha, C., and Bandari, S. (2015). Effects of radiations and heat source / sink on a Casson fluid flow over nonlinear stretching sheet. *World Journal of Mechanics*, 5, 257–265.

- Swapna, G., Kumar, L., Rana, P., and Singh, B. (2015). Finite element modeling of a double-diffusive mixed convection flow of a chemically-reacting magneto-micropolar fluid with convective boundary condition. *Journal of the Taiwan Institute of Chemical Engineers*, 47, 18–27.
- Tripathi, D., Bég, O. A., and Curiel-Sosa, J. L. (2012). Homotopy semi-numerical simulation of peristaltic flow of generalised Oldroyd-B fluids with slip effects. *Computer Methods in Biomechanics and Biomedical Engineering*, (May 2012), 37–41.
- Tsai, R., Huang, K. H., and Huang, J. S. (2008). Flow and heat transfer over an unsteady stretching surface with non-uniform heat source. *Meccanica*, 35, 1340–1343.
- Turkylmazoglu, M. (2012). Exact analytical solutions for heat and mass transfer of MHD slip flow in nanofluids. *Chemical Engineering Science*, 84, 182–187.
- Turkylmazoglu, M. (2013). Unsteady convection flow of some nanofluids past a moving vertical flat plate with heat transfer. *Journal of Heat Transfer*, 136(3), 031704.
- Vajravelu, K. (2001). Viscous flow over a nonlinearly stretching sheet. *Applied Mathematics and Computation*, 124, 281–288.
- Vajravelu, K., and Cannon, J. R. (2006). Fluid flow over a nonlinearly stretching sheet. *Applied Mathematics and Computation*, 181(1), 609–618.
- Vatankhah, A. R. (2011). Analytical solutions for Bingham plastic fluids in laminar regime. *Journal of Petroleum Science and Engineering*, 78(3-4), 596–600.
- Vyas, P., and Ranjan, A. (2010). Dissipative MHD boundary- layer flow in a porous medium over a sheet stretching nonlinearly in the presence of radiation. *Applied Mathematical Sciences*, 4(63), 3133–3142.
- Wang, C. Y. (1990). Liquid film on an unsteady stretching surface. *Quarterly of Applied Mathematics*, 48(4), 601–610.
- Watanabe, T. (1990). Thermal boundary layers over a wedge with uniform suction or injection in forced flow. *Acta Mechanica*, 83(3-4), 119–126.
- Watkins, C. B. (1975). Unsteady heat transfer in impulsive Falkner-Skan flows. *Int. J. Heat and Mass Transfer*, 19, 395–403.
- Welty, J. R., Wicks, C. E., Wilson, R. E., and Rorrer, R. L. (2009). *Fundamentals of momentum, heat, and mass transfer*. 5th edition. Jhon Wiley & Sons Inc.

- Wong, K. V., and De Leon, O. (2010). Applications of nanofluids: current and future. *Advances in Mechanical Engineering*.
- Xu, H., Liao, S. J., and Pop, I. (2006). Series solution of unsteady boundary layer flows of non-Newtonian fluids near a forward stagnation point. *Journal of Non-Newtonian Fluid Mechanics*, 139(1-2), 31–43.
- Xu, H., and Pop, I. (2008). Homotopy analysis of unsteady boundary-layer flow started impulsively from rest along a symmetric wedge. *ZAMM Zeitschrift Fur Angewandte Mathematik Und Mechanik*, 88(6), 507–514.
- Xuan, Y., and Li, Q. (2000). Heat transfer enhancement of nanofluids. *International Journal of Heat and Fluid Flow*, 21(1), 58–64.
- Xuan, Y., and Roetzel, W. (2000). Conceptions for heat transfer correlation of nanofluids. *International Journal of Heat and Mass Transfer*, 43(19), 3701–3707.
- Yacob, N. A., Ishak, A., Nazar, R., and Pop, I. (2011). Falkner-Skan problem for a static and moving wedge with prescribed surface heat flux in a nanofluid. *International Communications in Heat and Mass Transfer*, 38(2), 149–153.
- Yao, B. (2009). Series solution of the temperature distribution in the Falkner–Skan wedge flow by the homotopy analysis method. *European Journal of Mechanics-B/Fluids*, 28(5), 689–693.
- Yazdi, M. H., Abdullah, S., Hashim, I., and Sopian, K. (2011). Slip MHD liquid flow and heat transfer over non-linear permeable stretching surface with chemical reaction. *International Journal of Heat and Mass Transfer*, 54(15-16), 3214–3225.
- Yih, K. A. (1999). MHD forced convection flow adjacent to a non-isothermal wedge. *International Communications in Heat and Mass Transfer*, 26(6), 819–827.
- You, X., Xu, H., and Pop, I. (2010). Homotopy analysis of unsteady heat transfer started impulsively from rest along a symmetric wedge. *International Communications in Heat and Mass Transfer*, 37(1), 47–51.