MATHEMATICAL MODELLING OF BIOLOGICAL WASTEWATER TREATMENT OF OXIDATION POND AND CONSTRUCTED WETLAND SYSTEMS

AMIR SYAFIQ SYAMIN SYAH B AMIR HAMZAH

UNIVERSITI TEKNOLOGI MALAYSIA
MATHEMATICAL MODELLING OF BIOLOGICAL WASTEWATER TREATMENT OF OXIDATION POND AND CONSTRUCTED WETLAND SYSTEMS

AMIR SYAFIQ SYAMIN SYAH B AMIR HAMZAH

A thesis submitted in fulfilment of the requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

JULY 2017
To my beloved mother and father
ACKNOWLEDGEMENT

In preparing this thesis, I was in contact with many people, researchers, academician, and practitioners. They have contributed towards my understanding and thought. In particular, I wish to express my sincere appreciation to my main supervisor Assoc. Prof. Dr. Ali Hassan B. Mohamed Murid for his continuous support during my Ph.D study and related research, his patience, motivation, and immense knowledge. His guidance has helped me throughout my research and the writing of this thesis.

I would also like to give my biggest appreciation to my beloved parents Amir Hamzah B. Nordin and Hamidah Bt. Hassan for their pray and support since i was born until this moment.

In addition, I would also like to thank my co-supervisor Prof. Dr. Razman B. Salim for his support and assistance in strengthening my knowledge in this field. My sincere thanks also goes to Dr. Akbar Banitalebi from UTM Centre for Industrial and Applied Mathematics (UTM-CIAM) who has provided me an opportunity to learn many things especially in the field of optimization.

I would like to express my profound gratitude to the Director of UTM-CIAM, Prof. Dr. Zainal B. Abdul Aziz and the Head of Mathematical Sciences Department, Assoc. Prof. Dr. Rohanin B. Ahmad who had gave me the access to laboratory and research facilities.
I would also like to thank the Ministry of Higher Education Malaysia (MOHE) for the financial support through the research grant R.J130000.7809.4F637. Part of the research was undertaken while visiting the Mathematical Institute, University of Oxford, UK. I wish to thank the members of the institute for the pleasant mathematical atmosphere they offered. I would also gratefully acknowledge the help of Prof. Dr. Graham Sander on clarifying certain aspects of mathematical modeling of wetland system. This visit was supported by UTMLead, UTM and Graduate Employability Grants (STEM), UTM.

I wish to express my appreciation and thanks to all lecturers, laboratory technicians, and all administrative staffs of UTM-CIAM and the Department of Mathematical Sciences especially Shahliza, Siti Amirah, Norafidah, Farid, Liyana, and Tuan Mariam for their help and support. I would like to pass my thanks to all lecturers, Ph.D and Master students, and laboratory technicians of the UTM for their assistance.


Without their pray and support, i could never completed this thesis.
ABSTRACT

Wastewater treatment methods are intended to improve the quality of wastewater to prevent many health problems stemming from water sources. Among popular treatment methods are oxidation pond and constructed wetland (CW) treatment. There are some mathematical models for simulating oxidation pond process where some important parameters are considered such as bacteria (cleansing agent), pollutants and dissolved oxygen (DO). However, previous results did not provide good approximation of the required parameters. Meanwhile, for constructed wetland models, the stability analysis was rarely considered. However, the steady-state and bifurcation analyses are usually crucial in determining the reliability of the models that is under study. In this thesis, dynamic mathematical models are developed to allow simulation and prediction of the wastewater treatment process for both oxidation pond and CW case studies. The nonlinear system of ordinary differential equations (ODE) using multiple substrate limiting factors with interactive reactions and partial differential equations (PDE) using advection-diffusion-reaction equations are implemented for CW and oxidation pond, respectively. Water quality indexes considered in this study are chemical oxygen demand (COD), biochemical oxygen demand (BOD), ammonium nitrogen ($\text{NH}_4^+$), nitrate ($\text{NO}_3^-)$, and DO. For oxidation pond system, the input of microbe-based product (mPHO) is added to the model, whereas the effect of living plants (*Typha Angustifolia*) is introduced in the CW treatment system to mimic the natural behaviour of the wetland system. Since the models are nonlinear, coupled, and dynamic, computational algorithms with specific numerical methods are employed to simulate the dynamical behaviour of the system. Implicit Runge-Kutta method is selected for solving the ODE model. Whereas, for the PDE, the implicit Crank-Nicolson method is used. The process model built is then optimised using gradient-free optimisation method (least squares) algorithms `NonlinearModelFit` in *Mathematica* to identify the optimal solution for improving the efficiency of the simulation process. Stability, bifurcation, and numerical analyses are presented to illustrate the dynamical behaviour of the proposed model. Numerical results also revealed that the proposed models have good accuracy when compared to the experimental data. The two separate mathematical models for oxidation pond and constructed wetland, both are then applied to simulate a wastewater treatment site with pond-constructed wetland system. The combined mathematical model results in a further removal of COD as well as an increase of DO up to 94.1% and 97.4% respectively when compared to a single oxidation pond model.
Kaedah rawatan air sisa adalah bertujuan untuk meningkatkan kualiti air sisa bagi mengelakkan banyak masalah kesihatan yang berpunca daripada sumber air. Antara kaedah rawatan yang popular adalah kolam pengoksidasi dan tanah bencah yang dibina (CW). Terdapat beberapa model matematik untuk mensimulasikan proses kolam pengoksidasi di mana beberapa parameter penting diambil kira seperti bakteria (ejen pembersihan), pencemaran dan oksigen terlarut (DO). Walau bagaimanapun, keputusan yang sedia ada tidak memberikan aggaran yang baik bagi parameter yang diperlukan. Sementara itu, bagi model tanah bencah yang dibina, analisis kestabilan jarang dipertimbangkan. Walau bagaimanapun, analisis kestabilan dan pencabangan biasanya penting dalam menentukan kebolehpercayaan model yang dikaji. Dalam tesis ini, model matematik dinamik dibangunkan untuk membolehkan simulasi dan ramalan proses rawatan air sisa untuk kedua-dua kajian kes kolam pengoksidasi dan CW. Sistem tak linear persamaan pembezaan biasa (ODE) menggunakan faktor terhad pelbagai substrat dengan reaksi interaktif dan persamaan pembezaan separa (PDE) menggunakan persamaan alir lintang-penyebaran-reaksi diterapkan masing-masing untuk CW dan kolam pengoksidasi. Indeks kualiti air yang diukur dalam kajian ini ialah permintaan oksigen kimia (COD), permintaan oksigen biokimia (BOD), ammonia nitrogen (NH$_4^+$), nitrat (NO$_3$), dan DO. Untuk sistem kolam pengoksidasi, fungsi input produk berasaskan biologi (mPHO) ditambah pada model, manakala faktor tumbuhan hidup (Typha Angustifolia) diperkenalkan bagi sistem rawatan CW untuk menggambarkan sistem semula jadi tanah bencah. Memandangkan model yang dibina tak linear, terkait, dan dinamik, algoritma pengiraan dengan kaedah berangka yang khusus digunakan untuk mensimulasikan sifat dinamik sistem. Kaedah Runge-Kutta tersirat dipilih untuk menyelisikan model ODE. Bagi model PDE, kaedah Crank-Nicolson tersirat digunakan. Model proses yang dibina kemudiannya dioptimumkan dengan algoritma NonlinearModelFit dalam Mathematica iaitu kaedah pengoptimuman bebas kecerunan (kuasa dua terkecil) untuk mengenalpasti penyelesaian yang optimum bagi meningkatkan kecekapan proses simulasi. Kestabilan, pencabangan, dan analisis berangka dibentangkan untuk menggambarkan keadaan dinamik model yang dicadangkan. Keputusan berangka juga menunjukkan ketepatan yang baik apabila dibandingkan dengan data eksperimen. Dua model matematik yang berasingan untuk kolam pengoksidasi dan tanah bencah yang dibina kemudiannya kedua-duanya digunakan sekali untuk mensimulasikan proses rawatan air sisa dengan sistem kolam-tanah bencah. Model matematik gabungan menghasilkan penyingkiran COD serta peningkatan DO masing-masing sehingga 94.1% dan 97.4% berbanding model kolam pengoksidasi tunggal.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DECLARATION</td>
<td>ii</td>
<td></td>
</tr>
<tr>
<td>DEDICATION</td>
<td>iii</td>
<td></td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>iv</td>
<td></td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vi</td>
<td></td>
</tr>
<tr>
<td>ABSTRAK</td>
<td>vii</td>
<td></td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>viii</td>
<td></td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xi</td>
<td></td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xiv</td>
<td></td>
</tr>
<tr>
<td>LIST OF ABBREVIATIONS</td>
<td>xxi</td>
<td></td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>xxiii</td>
<td></td>
</tr>
<tr>
<td>LIST OF APPENDICES</td>
<td>xxvii</td>
<td></td>
</tr>
</tbody>
</table>

## 1 INTRODUCTION

1.1 Motivation

1.1.1 Wastewater

1.1.2 Oxidation Pond

1.1.3 Constructed Wetland

1.2 Background of the Problem

1.3 Problem Statement

1.4 Objectives of the Research

1.5 Scope of the Research

1.6 Significance of the Research

1
1.7 Thesis Organization 14

2 LITERATURE REVIEW 16

2.1 Introduction 16

2.2 Wastewater Treatment Process Modelling 16

2.2.1 Mathematical Models of River and Pond 17

2.2.2 Mathematical Models of Constructed Wetland 23

2.3 Activated Sludge Process 26

3 METHODOLOGY 30

3.1 Introduction 30

3.2 Mathematical Modelling 30

3.3 Nondimensionalisation 41

3.4 Steady State Solution 45

3.5 Parameter Estimation 51

3.6 One-Dimensional Conservation Equation 54

3.7 Coefficient of Determination ($R^2$) 56

3.8 Numerical Implementation (Implicit Runge-Kutta Method) 57

3.9 Numerical Implementation (Implicit Crank-Nicolson Method) 59

3.10 Application of mPHO at Taman Timur Oxidation Pond 64

3.11 Horizontal Subsurface Flow Constructed Wetland 68

4 OXIDATION POND SYSTEM 71

4.1 Introduction 71

4.2 Mathematical Model 1: Three Competing Species Model 71

4.3 Mathematical Model 2: Coupled-Reaction Equations Model 86

4.4 Partial Differential Equation Models 95

4.4.1 Mathematical Model 3: Advection-Reaction Equation Model 97
4.4.2 Nondimensionalisation of PDE Model 102

4.4.3 Mathematical Model 4: Advection-Diffusion-Reaction Equation Model (Transport of Pollutant) 105

5 CONSTRUCTED WETLAND SYSTEM 118

5.1 Introduction 118

5.2 Mathematical Model 1: Nonlinear Ordinary Differential Equations Model 118

5.3 Mathematical Model 2: The Simplified Model 131

5.4 Mathematical Model 3: Dimensionless Model 133

5.5 Steady State Analysis 136

6 POND-CONSTRUCTED WETLAND SYSTEM 144

6.1 Introduction 144

6.2 Pond-Constructed Wetland Model 147

7 CONCLUSION 150

7.1 Summary of the Research 150

7.2 Recommendation for Further Study 152

REFERENCES 154

Appendices A - C 163-175
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE NO.</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Chronology of Major Conclusions Made about WWTP Between 2011-Present Year</td>
<td>21</td>
</tr>
<tr>
<td>2.2</td>
<td>Chronology of Major Conclusions Made about WWTP Between 2011-Present Year</td>
<td>22</td>
</tr>
<tr>
<td>3.1</td>
<td>Constitutive Mechanisms for Transport Phenomena [65]</td>
<td>34</td>
</tr>
<tr>
<td>3.2</td>
<td>Initial Layout for Routh Table</td>
<td>51</td>
</tr>
<tr>
<td>3.3</td>
<td>Conditions of State Variable (C)</td>
<td>61</td>
</tr>
<tr>
<td>3.4</td>
<td>Treatment Schedule for mPHO Application at Taman Timor Oxidation Pond (Phase 1) [3]</td>
<td>66</td>
</tr>
<tr>
<td>3.5</td>
<td>Treatment Schedule for mPHO Application at Taman Timor Oxidation Pond (Phase 2) [3]</td>
<td>67</td>
</tr>
<tr>
<td>4.1</td>
<td>Variables and Parameters Used in Mathematical Model 1</td>
<td>72</td>
</tr>
<tr>
<td>4.2</td>
<td>Process Rates Involved in Mathematical Model 1</td>
<td>73</td>
</tr>
<tr>
<td>4.3</td>
<td>Population of <em>E. coli</em> from Experimental Data at CP1 and CP2 (MISG 2014 Report [3])</td>
<td>73</td>
</tr>
<tr>
<td>4.4</td>
<td>Population of <em>Coliform</em> from Experimental Data at CP1 and CP2 (MISG 2014 Report [3])</td>
<td>74</td>
</tr>
<tr>
<td>4.5</td>
<td>Population of PSB from Experimental Data at CP1 and CP2 (MISG 2014 Report [3])</td>
<td>74</td>
</tr>
<tr>
<td>4.6</td>
<td>List of Parameters in Mathematical Model 1</td>
<td>79</td>
</tr>
<tr>
<td>4.7</td>
<td>Calculation of Cost and Amount of mPHO that can be Saved for Considered Schedule</td>
<td>85</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.8</td>
<td>The Variables and Parameters Used in Mathematical Model 2</td>
<td>87</td>
</tr>
<tr>
<td>4.9</td>
<td>Process Rates Involved in Mathematical Model 2</td>
<td>88</td>
</tr>
<tr>
<td>4.10</td>
<td>The Amount of COD from Experimental Data at CP1 and CP2 (MISG 2014 Report [3])</td>
<td>88</td>
</tr>
<tr>
<td>4.11</td>
<td>The Amount of Dissolved Oxygen from Experimental Data at CP1 and CP2 (MISG 2014 Report [3])</td>
<td>89</td>
</tr>
<tr>
<td>4.12</td>
<td>List of Parameters in Mathematical Model 2</td>
<td>93</td>
</tr>
<tr>
<td>4.13</td>
<td>The Variables and Parameters Used in Mathematical Model 3</td>
<td>97</td>
</tr>
<tr>
<td>4.14</td>
<td>Description of State Variables Used in Mathematical Model 4</td>
<td>106</td>
</tr>
<tr>
<td>4.15</td>
<td>Process Rates</td>
<td>106</td>
</tr>
<tr>
<td>4.16</td>
<td>Parameter Values Used in the Mathematical Model 4</td>
<td>107</td>
</tr>
<tr>
<td>4.17</td>
<td>Values of Calibrated Constants and Values Specified in Literature</td>
<td>112</td>
</tr>
<tr>
<td>5.1</td>
<td>Description of State Variables in Model 1</td>
<td>120</td>
</tr>
<tr>
<td>5.2</td>
<td>State Variables and Parameter Values Used in Model 1</td>
<td>121</td>
</tr>
<tr>
<td>5.3</td>
<td>All Process Rates Involved in Model 1</td>
<td>122</td>
</tr>
<tr>
<td>5.4</td>
<td>The Amount of COD from Experimental Data at CP1 and CP2 (Chew [15])</td>
<td>125</td>
</tr>
<tr>
<td>5.5</td>
<td>The Amount of BOD from Experimental Data at CP1 and CP2 (Chew [15])</td>
<td>125</td>
</tr>
<tr>
<td>5.6</td>
<td>The Amount of $\text{NH}_4^+$ from Experimental Data at CP1 and CP2 (Chew [15])</td>
<td>126</td>
</tr>
<tr>
<td>5.7</td>
<td>The Amount of $\text{NO}_3^-$ from Experimental Data at CP1 and CP2 for Planted Case (Chew [15])</td>
<td>126</td>
</tr>
<tr>
<td>5.8</td>
<td>Values of Calibrated Constant and Values Specified in Literature</td>
<td>126</td>
</tr>
<tr>
<td>5.9</td>
<td>Description of Parameters Used in the Mathematical Model 3</td>
<td>135</td>
</tr>
</tbody>
</table>
6.1 The Variables and Parameters Used in Pond-Constructed Wetland Model

145
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE NO.</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Relationship between the Organic Carbon in Sewage [2]</td>
<td>4</td>
</tr>
<tr>
<td>1.2</td>
<td>Aerial View of Oxidation Pond, Tampoi, Johor Bahru [3]</td>
<td>4</td>
</tr>
<tr>
<td>1.3</td>
<td>Physical Condition of Oxidation Pond, Tampoi, Johor Bahru [3]</td>
<td>5</td>
</tr>
<tr>
<td>1.4</td>
<td>Beneficial Microbe-Based Product (mPHO) Produced by J-Biotech Company [3]</td>
<td>5</td>
</tr>
<tr>
<td>1.5</td>
<td>(a) Location of CP1 (influent) (b) Location of CP2 (effluent) [3]</td>
<td>5</td>
</tr>
<tr>
<td>1.6</td>
<td>(a) Phragmites Australis (b) Glyceria Maxima (c) Phalaris Arundinacea (d) Cattails (Typha Angustifolia) [8]</td>
<td>7</td>
</tr>
<tr>
<td>1.7</td>
<td>Putrajaya Lake and Wetland System, Putrajaya, Malaysia [12]</td>
<td>7</td>
</tr>
<tr>
<td>2.1</td>
<td>Mathematical Models of Wastewater Treatment Process from the Beginning until 2010. [19, 24–37]</td>
<td>18</td>
</tr>
<tr>
<td>2.2</td>
<td>Michaelis-Manten Kinetics [39]</td>
<td>20</td>
</tr>
<tr>
<td>2.3</td>
<td>Nitrogen Cycle in Constructed Wetland [8]</td>
<td>24</td>
</tr>
<tr>
<td>2.4</td>
<td>Activated Sludge Process for Treating Wastewater [41]</td>
<td>27</td>
</tr>
<tr>
<td>2.5</td>
<td>Schematic Diagram of Eutrophication Process</td>
<td>28</td>
</tr>
<tr>
<td>2.6</td>
<td>Schematic Diagram of Nitrogen Cycle</td>
<td>28</td>
</tr>
<tr>
<td>3.1</td>
<td>Overview of the Mathematical Modelling Process [65]</td>
<td>33</td>
</tr>
<tr>
<td>3.2</td>
<td>Model Validation Methodology [65]</td>
<td>34</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.3</td>
<td>Phase Portrait Diagram of Equation (3.24) with $c_1 = 1$ and $c_2 = 1$</td>
<td>47</td>
</tr>
<tr>
<td>3.4</td>
<td>Illustration of the Parameter Estimation Process</td>
<td>54</td>
</tr>
<tr>
<td>3.5</td>
<td>One Dimensional Pipe of Uniform Diameter</td>
<td>54</td>
</tr>
<tr>
<td>3.6</td>
<td>Dimensional Pipe of Cross Section Area of Length ($\Delta x$)</td>
<td>55</td>
</tr>
<tr>
<td>3.7</td>
<td>Sampling and Study Location at Taman Timor Oxidation Pond [3]</td>
<td>65</td>
</tr>
<tr>
<td>3.8</td>
<td>Layout of The Horizontal Flow Subsurface Constructed Wetland Units</td>
<td>69</td>
</tr>
<tr>
<td>3.9</td>
<td>Actual Image of Reactor</td>
<td>69</td>
</tr>
<tr>
<td>3.10</td>
<td>Horizontal Sections of Subsurface Constructed Wetlands</td>
<td>69</td>
</tr>
<tr>
<td>4.1</td>
<td>The Graphs of <em>E. coli</em> in the Oxidation Pond at CP1 and CP2 (refer Table 4.3)</td>
<td>75</td>
</tr>
<tr>
<td>4.2</td>
<td>The Graphs of <em>Coliform</em> in the Oxidation Pond at CP1 and CP2 (refer Table 4.4)</td>
<td>75</td>
</tr>
<tr>
<td>4.3</td>
<td>The Graphs of PSB in the Oxidation Pond at CP1 and CP2 (refer Table 4.5)</td>
<td>75</td>
</tr>
<tr>
<td>4.4</td>
<td>Graphical Comparison on the Population of <em>E. coli</em> at CP2. The Blue Curve Represents the Numerical Results and Red Dots are the Experimental Data ($R^2 = 0.7954$)</td>
<td>79</td>
</tr>
<tr>
<td>4.5</td>
<td>Graphical Comparison on the Population of <em>Coliform</em> at CP2 ($R^2 = 0.7913$)</td>
<td>80</td>
</tr>
<tr>
<td>4.6</td>
<td>Graphical Comparison on the Population of PSB at CP2 ($R^2 = 0.6541$)</td>
<td>80</td>
</tr>
<tr>
<td>4.7</td>
<td>Graphical Comparison on the Population of <em>E. coli</em> at CP2. The Blue Curve is Based on the Simulation of Applying 10 Litres Per Day of mPHO</td>
<td>81</td>
</tr>
<tr>
<td>4.8</td>
<td>Graphical Comparison on the Population of <em>Coliform</em> at CP2 (10 Litres Per Day of mPHO)</td>
<td>81</td>
</tr>
</tbody>
</table>
4.9 Graphical Comparison on the Population of PSB at CP2 (10 Litres Per Day of mPHO) 82
4.10 Graphical Comparison on the Population of *E. coli* at CP2 (30 Litres Per Day of mPHO) 82
4.11 Graphical Comparison on the Population of *E. coli* at CP2 (30 Litres Per Day of mPHO) 82
4.12 Graphical Comparison on the Population of PSB at CP2 (30 Litres Per Day of mPHO) 83
4.13 Graphical Comparison on the Population of *E. coli* at CP2 (50 Litres Per Day of mPHO) 83
4.14 Graphical Comparison on the Population of *Coliform* at CP2 (50 Litres Per Day of mPHO) 83
4.15 Graphical Comparison on the Population of PSB at CP2 (50 Litres Per Day of mPHO) 84
4.16 Graphical Comparison on the Population of *E. coli* at CP2 (without mPHO) 84
4.17 Graphical Comparison on the Population of *Coliform* at CP2 (without mPHO) 84
4.18 (a) Graph of Relative Error between Applying 30 Liters and 50 Liters of mPHO for *E. coli* Relative to $X_{2i}(30)$ (b) Graph of Relative Error between Applying 30 Liters and 50 Liters of mPHO for *E. coli* Relative to $X_{2i}(50)$ 85
4.19 (a) Graph of Relative Error between Applying 30 Liters and 50 Liters of mPHO for *Coliform* Relative to $X_{3i}(30)$ (b) Graph of Relative Error between Applying 30 Liters and 50 Liters of mPHO for *Coliform* Relative to $X_{3i}(50)$ 85
4.20 The Graphs of COD from CP1 and CP2 for 70 Days (refer Table 4.10) 87
4.21 The Graphs of DO from CP1 and CP2 for 70 Days (refer Table 4.11) 89
4.22 Graphical Comparison between Experimental Data and Simulated Model 2 for PSB ($R^2 = 0.6531$) 94
4.23 Graphical Comparison between Experimental Data and Simulated Model 2 for Coliform ($R^2 = 0.7754$) 94
4.24 Graphical Comparison between Experimental Data and Simulated Model 2 for COD ($R^2 = 0.8472$) 94
4.25 Graphical Comparison between Experimental Data and Simulated Model 2 for DO ($R^2 = 0.7127$) 95
4.26 3D Graph of PSB from Mathematical Model 3 99
4.27 Contour Plot of PSB from Mathematical Model 3 99
4.28 3D Graph of COD from Mathematical Model 3 100
4.29 Contour Plot of COD from Mathematical Model 3 100
4.30 Graphical Comparison between Experimental Data and Simulated Model 3 for PSB. The Blue Curve Represents the Numerical Results and Red Dots are the Experimental Data ($R^2 = 0.62$) 101
4.31 Graphical Comparison between Experimental Data and Simulated Model 3 for COD. The Blue Curve Represents the Numerical Results and Red Dots are the Experimental Data ($R^2 = 0.85$) 101
4.32 3D Graph of PSB Based on Mathematical Model 4 113
4.33 Contour Plot of PSB Based on Mathematical Model 4 113
4.34 3D Graph of COD Based on Mathematical Model 4 114
4.35 Contour Plot of COD Based on Mathematical Model 4 114
4.36 3D Graph of DO Based on Mathematical Model 4 115
4.37 Contour Plot of DO Based on Mathematical Model 4 115
4.38 Graphical Comparison between Experimental Data and Simulated Model 4 for PSB ($R^2 = 0.90$) 116
4.39 Graphical Comparison between Experimental Data and Simulated Model 4 for COD ($R^2 = 0.86$) 116
4.40 Graphical Comparison between Experimental Data and Simulated Model 4 for DO ($R^2 = 0.92$) 117

5.1 Conceptual Diagram of HSSF Model Illustrating the Coupling of Eco-Physiological Process Through Carbon and Nitrogen Pathways 119

5.2 (a) Graph of $M_B$ from Simulated Model 1 (Planted) (b) Graph of $M_B$ from Simulated Model 1 (Unplanted) 124

5.3 (a) Graphical Comparison between Experimental Data and Simulated Model 1 for COD (Planted, $R^2 = 0.7945$) (b) Graphical Comparison between Experimental Data and Simulated Model 1 for COD (Unplanted, $R^2 = 0.9964$) 127

5.4 (a) Graphical Comparison between Experimental Data and Simulated Model 1 for BOD (Planted, $R^2 = 0.8574$) (b) Graphical Comparison between Experimental Data and Simulated Model 1 for BOD (Unplanted, $R^2 = 0.6306$) 127

5.5 (a) Graphical Comparison between Experimental Data and Simulated Model 1 for $NH_4^+$ (Planted, $R^2 = 0.6954$) (b) Graphical Comparison between Experimental Data and Simulated Model 1 for $NH_4^+$ (Unplanted, $R^2 = 0.8137$) 127

5.6 (a) Graphical Comparison between Experimental Data and Simulated Model 1 for $NO_3$ (Planted, $R^2 = 0.8993$) (b) Graphical Comparison between Experimental Data and Simulated Model 1 for $NO_3$ (Unplanted, $R^2 = 0.9591$) 128

5.7 (a) Graph of DO from Simulated Model 1 (Planted) (b) Graph of DO from Simulated Model 1 (Unplanted) 128

5.8 (a) Graph of Relative Error between Planted and Unplanted Cases for COD Relative to Planted Case (b) Graph of Relative Error between Planted and Unplanted Cases for COD Relative to Unplanted Case 128
5.9 (a) Graph of Relative Error between Planted and Unplanted Cases for BOD Relative to Planted Case (b) Graph of Relative Error between Planted and Unplanted Cases for BOD Relative to Unplanted Case 129

5.10 (a) Graph of Relative Error between Planted and Unplanted Cases for NH$_4^+$ Relative to Planted Case (b) Graph of Relative Error between Planted and Unplanted Cases for NH$_4^+$ Relative to Unplanted Case 129

5.11 (a) Graph of Relative Error between Planted and Unplanted Cases for NO$_3$ Relative to Planted Case (b) Graph of Relative Error between Planted and Unplanted Cases for NO$_3$ Relative to Unplanted Case 129

5.12 (a) Plot of the Error Magnitudes for Model 1 as Iterations Progress for Planted Case (20$^{\text{th}}$ iteration, 20.3891) (b) Plot of the Error Magnitudes for Model 1 as Iterations Progress for Unplanted Case (20$^{\text{th}}$ iteration, 8.0729) 130

5.13 Graph of Typha Angustifolia from Simulated Model 1 130

5.14 (a) Graphs of $M_B$, DO, and Typha Angustifolia for Extended Days up to One Year (planted) (b) Graphs of $M_B$ and DO for Extended Days up to One Year (Unplanted) 130

5.15 (a) Graphs of COD, BOD, NH$_4^+$, and NO$_3$ for Extended Days up to One Year (planted) (b) Graphs of COD, BOD, NH$_4^+$, and NO$_3$ for Extended Days up to One Year (Unplanted) 131

5.16 (a) Graph of $M_B$ from Simulated Model 2 (Planted) (b) Graph of $M_B$ from Simulated Model 2 (Unplanted) 132

5.17 (a) Graphical Comparison between Experimental Data and Simulated Model 2 for COD (Planted, $R^2 = 0.7952$) (b) Graphical Comparison between Experimental Data and Simulated Model 2 for COD (Unplanted, $R^2 = 0.9967$) 133
5.18  (a) Graph of DO from Simulated Model 2 (Planted) (b) Graph of DO from Simulated Model 2 (Unplanted)

5.19  Steady-State Diagram of Model 3 Against $c_3$ with $c_1$ = 1, $c_2$ = 4, $c_5$ = 1, $c_4$ = 2 for $M_B$

5.20  Steady-State Diagram of Model 3 Against $c_3$ for COD

5.21  Steady-State Diagram of Model 3 Against $c_3$ for DO

5.22  Time Plots of $m$, $s$, and $x$ with Initial Conditions (1, 1, 1) for $c_1$ = 1, $c_2$ = 4, $c_3$ = 0.5, $c_4$ = 2, and $c_5$ = 1

5.23  Time Plots of $m$, $s$, and $x$ for $c_1$ = 1, $c_2$ = 4, $c_3$ = 1.5, $c_4$ = 2, and $c_5$ = 1

6.1  Framework of the Pond-Constructed Wetland Model

6.2  (a) Graphical Comparison between ADR Model of Oxidation Pond and PCW Model for COD at Discharged Point (b) Graph Percentage Removal of COD by PCW Model at Discharged Point

6.3  (a) Graphical Comparison between ADR Model of Oxidation Pond and PCW Model for DO at Discharged Point (b) Graph of DO Increase Percentage from PCW Model at Discharged Point

6.4  Graphical Comparison between ADR Model of Oxidation Pond and PCW Model for $M_B$ at Discharged Point
## LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>UTM</td>
<td>Universiti Teknologi Malaysia</td>
</tr>
<tr>
<td>COD</td>
<td>Chemical Oxygen Demand</td>
</tr>
<tr>
<td>BOD&lt;sub&gt;5&lt;/sub&gt;</td>
<td>5-day Biochemical Oxygen Demand</td>
</tr>
<tr>
<td>DO</td>
<td>Dissolved Oxygen</td>
</tr>
<tr>
<td>NH&lt;sub&gt;4&lt;/sub&gt;&lt;sup&gt;+&lt;/sup&gt;</td>
<td>Ammoniacal Nitrogen</td>
</tr>
<tr>
<td>NO&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Nitrate</td>
</tr>
<tr>
<td>E. coli</td>
<td>Esrichia Coli</td>
</tr>
<tr>
<td>PSB</td>
<td>Phototrophic Bacteria</td>
</tr>
<tr>
<td>ODEs</td>
<td>Ordinary Differential Equations</td>
</tr>
<tr>
<td>PDEs</td>
<td>Partial Differential Equations</td>
</tr>
<tr>
<td>CW</td>
<td>Constructed Wetland</td>
</tr>
<tr>
<td>HSSF</td>
<td>Horizontal Sub-surface Flow</td>
</tr>
<tr>
<td>ADR</td>
<td>Advection-Diffusion-Reaction</td>
</tr>
<tr>
<td>TOC</td>
<td>Total Organic Carbon</td>
</tr>
<tr>
<td>H&lt;sub&gt;2&lt;/sub&gt;SO&lt;sub&gt;4&lt;/sub&gt;</td>
<td>Sulphuric Acid</td>
</tr>
<tr>
<td>mPHO</td>
<td>Microbe-Based Product produced by J-Biotech</td>
</tr>
<tr>
<td>WEPA</td>
<td>Water Environment Partnership in Asia</td>
</tr>
<tr>
<td>PO&lt;sub&gt;4&lt;/sub&gt;&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>Phosphates</td>
</tr>
<tr>
<td>NO&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Nitrates</td>
</tr>
<tr>
<td>CO&lt;sub&gt;2&lt;/sub&gt;</td>
<td>Carbon dioxide</td>
</tr>
<tr>
<td>K&lt;sub&gt;2&lt;/sub&gt;Cr&lt;sub&gt;2&lt;/sub&gt;O&lt;sub&gt;7&lt;/sub&gt;</td>
<td>Potassium Dichromate</td>
</tr>
<tr>
<td>C&lt;sub&gt;6&lt;/sub&gt;H&lt;sub&gt;6&lt;/sub&gt;</td>
<td>Benzene</td>
</tr>
<tr>
<td>CH&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Toluene</td>
</tr>
<tr>
<td>J-Biotech</td>
<td>Johor Biotechnology &amp; Biodiversity Corporation</td>
</tr>
</tbody>
</table>
nm - nanometer
μm - micrometer
**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ℜ</td>
<td>The set of real numbers</td>
</tr>
<tr>
<td>ℜ^n</td>
<td>Real coordinate space of n dimensions</td>
</tr>
<tr>
<td>J</td>
<td>Jacobian Matrix</td>
</tr>
<tr>
<td>MB</td>
<td>Mixed Culture bacteria</td>
</tr>
<tr>
<td>α</td>
<td>Saturated oxygen concentration</td>
</tr>
<tr>
<td>ε</td>
<td>Material porosity for constructed wetland</td>
</tr>
<tr>
<td>A</td>
<td>Cross section area of oxidation pond</td>
</tr>
<tr>
<td>v_p</td>
<td>Volume of the oxidation pond (2,864,139 litres of wastewater)</td>
</tr>
<tr>
<td>m_0</td>
<td>Amount of PSB in mPHO (1.91 \times 10^{-9} CFU / 100 ml)</td>
</tr>
<tr>
<td>X_1(t)</td>
<td>Amount of PSB in the pond where t varies from initial time up to 35 days</td>
</tr>
<tr>
<td>X_2(t)</td>
<td>Amount of E. coli in the pond (MPN / 100 ml)</td>
</tr>
<tr>
<td>X_3(t)</td>
<td>Amount of Coliform in the pond (MPN / 100 ml)</td>
</tr>
<tr>
<td>X_p(t)</td>
<td>Input wastewater containing pathogens (E. coli and Coliform) from CP1 (290,304 litres day^{-1})</td>
</tr>
<tr>
<td>X_m(t)</td>
<td>Scheduled amount of mPHO applied at the oxidation pond (litre / day)</td>
</tr>
<tr>
<td>X_1_i(t)</td>
<td>Input of PSB added in the oxidation pond at CP1 (MPN / 100 ml)</td>
</tr>
<tr>
<td>X_1_e(t)</td>
<td>Existing of PSB in the oxidation pond at CP1 (MPN / 100 ml)</td>
</tr>
<tr>
<td>X_2_i(t)</td>
<td>Input of E. coli added in the oxidation pond at CP1 (MPN / 100 ml)</td>
</tr>
<tr>
<td>X_3_i(t)</td>
<td>Input of Coliform added in the oxidation pond at CP1 (MPN / 100 ml)</td>
</tr>
</tbody>
</table>
\( k_1 \) - Growth rate of PSB (1.000045 day\(^{-1}\))

\( k_2 \) - Mortality rate of PSB (4.8 \times 10^{-5} \text{ day}^{-1})

\( k_3 \) - Predation rate on \( E. \ coli \) that increases the number of PSB

\( k_4 \) - Predation rate on \( Coliform \) that increases the number of PSB (3.2 \times 10^{-6} \text{ day}^{-1})

\( k_5 \) - Input rate of PSB at the influent (CP1) (0.0152 day\(^{-1}\))

\( k_6 \) - Growth rate of \( E. \ coli \)

\( k_7 \) - Mortality rate of \( E. \ coli \)

\( k_8 \) - Predation rate that decreases the number of \( E. \ coli \)

\( k_9 \) - Input rate of \( E. \ coli \) at the influent (CP1)

\( k_{10} \) - Growth rate of \( Coliform \) (0.0081 day\(^{-1}\))

\( k_{11} \) - Mortality rate of \( Coliform \) (1.6 \times 10^{-7} \text{ day}^{-1})

\( k_{12} \) - Predation rate that decreases the number of \( Coliform \) (2.7 \times 10^{-5} \text{ day}^{-1})

\( k_{13} \) - Input rate of \( Coliform \) at the influent (CP1) (0.0275 day\(^{-1}\))

\( c_1 \) - Degradation rate coefficient for COD

\( c_2 \) - Half saturation coefficient concentration for COD decay

\( c_3 \) - Input rate of COD at the influent (CP1)

\( c_4 \) - Mass transfer rate for oxygen from air to water

\( c_5 \) - Input rate of oxygen at the influent (CP1)

\( M(t) \) - Concentration of PSB in the pond (mg/litre) where \( t \) varies from initial time up to 70 days

\( C(t) \) - Concentration of \( Coliform \) in the pond (mg/litre)

\( P(t) \) - Concentration of COD in the pond (mg/litre)

\( X(t) \) - Concentration of DO in the pond (mg/litre)

\( U(t) \) - Amount of mPHO applied according to J-Biotech schedule (litre/day)
\[ M_i(t) - \text{Input of PSB added in the pond at CP1 (mg/litre)} \]
\[ C_i(t) - \text{Input of } \textit{Coliform} \text{ added in the pond at CP1 (mg/litre)} \]
\[ P_i(t) - \text{Input of COD added in the pond at CP1 (mg/litre)} \]
\[ X_i(t) - \text{Input of DO added in the pond at CP1 (mg/litre)} \]
\[ m_0 - \text{Concentration of PSB in one litre of mPHO (19.1 mg/litre)} \]
\[ S - \text{Saturated oxygen concentration (10 mg/litre)} \]
\[ v_s - \text{Average amount of incoming sewage (290,304 litres/day)} \]
\[ v_p - \text{Volume of the pond (2,864,139 litres of wastewater)} \]
\[ p_0 - \text{Concentration of COD in one m}^3 \text{ of sewage from input CP1 (50.0 g m}^{-3} \text{)} \]
\[ \mu_b - \text{Maximum growth rate for PSB (31 m}^2 \text{ day}^{-1} \text{)} \]
\[ D_M - \text{Diffusion coefficient of PSB in the } x \text{ direction (50 m}^2 \text{ day}^{-1} \text{)} \]
\[ D_P - \text{Diffusion coefficient of COD in the } x \text{ direction (50 m}^2 \text{ day}^{-1} \text{)} \]
\[ D_X - \text{Diffusion coefficient of DO in the } x \text{ direction (50 m}^2 \text{ day}^{-1} \text{)} \]
\[ k_L - \text{Mass transfer rate for oxygen from air to water (4 m}^2 \text{ day}^{-1} \text{)} \]
\[ k_p - \text{Half saturation oxygen demand concentration for COD decay (560 g m}^{-3} \text{)} \]
\[ k_m - \text{Half-saturation coefficient for growth of PSB (1.7} \times 10^{-4} \text{ g m}^{-3} \text{)} \]
\[ L - \text{Length of the HSSF constructed wetland} \]
\[ W - \text{Width of the constructed wetland} \]
\[ D - \text{Depth of reed bed of the constructed wetland} \]
\[ w - \text{Pore water volume} \]
\[ v - \text{Constructed wetland holding volume} \]
\[ \epsilon - \text{Material porosity} \]
\[ \beta - \text{Decay coefficient for mixed} \]
\[ b_P - \text{Decay coefficient for living plants} \]
\[ \alpha - \text{Saturated oxygen concentration} \]
\[ \mu - \text{Maximum growth rate for mixed} \]
\( q_i \) - Input flow rate of wastewater
\( q_o \) - Output flow rate of wastewater
\( k_l \) - Mass transfer rate of oxygen from atmosphere to soil
\( k_C \) - Half-saturation coefficient for \( M_B \) growth on COD
\( k_B \) - Half-saturation coefficient for \( M_B \) growth on BOD
\( k_A \) - Half-saturation coefficient for \( M_B \) growth on \( NH_4^+ \)
\( k_N \) - Half-saturation coefficient for \( M_B \) growth on \( NO_3^- \)
\( k_{pua} \) - Rate of plant uptake on \( NH_4^+ \) \( (0.0036 \text{ m day}^{-1}) \)
\( k_{pun} \) - Rate of plant uptake on \( NO_3^- \) \( (0.0044 \text{ m day}^{-1}) \)
\( k_{pa} \) - Half-saturation coefficient for plant uptake on \( NH_4^+ \) \( 0.0620 \text{ g m}^{-3} \)
\( k_{pn} \) - Half-saturation coefficient for plant uptake on \( NO_3^- \) \( 0.0440 \text{ g m}^{-3} \)
\( S_{Ci}(t) \) - Input of COD from the inlet source that varies with time (CP1)
\( S_{Bi}(t) \) - Input of BOD from the inlet source that varies with time (CP1)
\( S_{Ai}(t) \) - Input of \( NH_4^+ \) from the inlet source that varies with time (CP1)
\( S_{Ni}(t) \) - Input of \( NO_3^- \) from the inlet source that varies with time (CP1)
\( w_d \) - Calculated as \( \frac{w}{L \cdot W \cdot \epsilon} \)
\( c_1 \) - Ratio of decay rate for \( M_B \) to the water volume
\( c_2 \) - Ratio of maximum specific growth for \( M_B \) and saturated oxygen to the water volume
\( c_3 \) - Input of pollutant source (COD) from CP1
\( c_4 \) - Ratio of saturated oxygen to the half-saturation coefficient for growth of \( M_B \)
\( c_5 \) - Ratio of mass transfer rate for oxygen to the water volume
# LIST OF APPENDICES

<table>
<thead>
<tr>
<th>APPENDIX</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>List of Publications</td>
<td>163</td>
</tr>
<tr>
<td>B</td>
<td>Oxidation Pond System</td>
<td>165</td>
</tr>
<tr>
<td>C</td>
<td>Constructed Wetland System</td>
<td>170</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Motivation

Mathematical modelling is an important and well-known field of study which has led to the enrichment of science and technology. Fields that require mathematical modelling include medicine, ecology, biology, finance, and economics. This has further encouraged many researchers to develop new models to fulfill the demand arose from these fields. These new mathematical models are expected to aid in analyzing and solving the problems encountered by the fields mentioned. A mathematical model is a simplified version of the real world process employing the tools of mathematics such as statistics, probability theory, graph theory, and differential equations. These mathematical methods help in understanding the nature of problems that cannot be clearly interpreted through phenomenological observation. Sometimes, one needs to develop a new method to solve the problems or modify the standard previous methods that have been successful. Challenges should be taken as a motivation for researchers to mathematically model the current problems and facilitate it to be understood by the public.
1.1.1 Wastewater

One of the main problems widely concerned is the issue of environmental pollution. Development without a systematic planning is like ignoring the sustainability of environment. There are a huge number of wastes being produced daily as the result of human activities; for instance, solid waste, hazardous waste, wastewater (sewage and surface runoff), and radioactive waste. Wastewater can be classified into several types including industrial waste, municipal waste, food waste, and sewage from houses. This kind of wastes has to be carefully treated to ensure that there would be no harm to human and the environment. If untreated wastewater is allowed to accumulate in the river followed by the processing of decomposed organic material, it can lead to water pollution. Additionally, untreated wastewater usually contains numerous pathogens or diseases caused by microorganisms.

Severe pollution has become our main concern that leads to the production of a mathematical model that is able to preserve and conserve the environment to run smoothly, thus helping the development of human capital. The execution of wastewater treatment process depends on symbiotic relationships of biological organism found in a system. Therefore, understanding the ecological system is very crucial to construct the so-called symbiotic relationship and function related to wastewater treatment processes.

1.1.2 Oxidation Pond

Oxidation pond techniques have become very popular among small communities due to their low construction and operating costs [1]. The construction and maintenance costs of this treatment are inexpensive compared to other recognized treatment systems including microbial fuel cell (MFC), membrane bioreactor (MBR), and rotating biological contactor (RBC). The core procedure of an oxidation pond
treatment process is the degradation of contaminants and organic matter in two conditions; where oxygen is present (aerobic) or absence (anaerobic). At each stage, existing microorganisms are used to breakdown either organic or inorganic substances of influent and to reduce organic material into more concise forms, which are carbon dioxide, water, and cell biomass.

Oxidation pond chosen for the pilot scale study is an exposed oxidation pond located at Taman Timor Oxidation Pond, Tampoi, Johor. This pond was chosen because it has been experimentally studied by J-Biotech for three months period to observe the effects of microbe-based product (mPHO) in treating sewage. Nonetheless, there is no specific study done on this product until now. Briefly, the size of this pond is estimated about 1,909 square metres with a depth of 1.5 metres and total water volume of 2,864.13 cubic metres or 2,864,125.13 litres (refer to Figure 1.2 and Figure 1.3). However, the input and output flow rate of wastewater as well as the volume of rain may not change the volume of water in the pond as the wastewater is discharged due to overflow. In order to intensify the effectiveness of oxidation pond technique and to accelerate the population of Phototrophic bacteria (PSB) in the pond, mPHO containing mainly PSB have been added regularly within three months period of study between 13 November 2013 to 12 February 2014.

The product mPHO is made from selected species of PSB (refer Figure 1.4) manufactured by J-Biotech. About 1,375 litres of mPHO were applied to the pond throughout three months of treatment. Samples were collected at two points, which are CP1 (influent and application of mPHO) and CP2 (effluent) (refer to Figure 1.5). Comparison of data taken at both points CP1 and CP2 demonstrated that mPHO has a good effect in reducing the concentration of pathogenic bacteria (E. coli and Coliform), BOD, COD and other pollutants as the PSB and dissolved oxygen (DO) concentration increases.
Figure 1.1  Relationship between the Organic Carbon in Sewage [2]

Figure 1.2  Aerial View of Oxidation Pond, Tampoi, Johor Bahru [3]
Figure 1.3  Physical Condition of Oxidation Pond, Tampoi, Johor Bahru [3]

Figure 1.4  Beneficial Microbe-Based Product (mPHO) Produced by J-Biotech Company [3]

Figure 1.5  (a) Location of CP1 (influent) (b) Location of CP2 (effluent) [3]
1.1.3 Constructed Wetland

Constructed wetland system can be considered as a secondary or tertiary treatment facility for treating wastewater originated from the residential, municipal and industrial areas [4]. Besides playing an important role in wastewater treatment process to remove contaminants including organic matter and inorganic matter (based on COD removal and BOD removal), it is also helpful in maintaining the landscape that preserve the natural habitats of flora and fauna [5–7]. Wetlands treatment is defined as a treatment system using the aquatic root system of cattails, reeds, and similar plants to treat wastewater applied to either above or below the soil surface [8–10].

This treatment system acts as a filter to remove excess nutrients in the form of carbon and nitrogen from its source. The top layer of constructed wetland is planted with various types of plant, while the roots are allowed to develop deep and extensive roots that can penetrate the filter media. In fact, it can also help to develop porous throughout the land, allowing the wastewater to seep below the soil surface. At the root of the plant, there are fixed surfaces on which bacteria can attach and perform the breakdown of organic matter [11]. The vegetation provides an air flow to the root zone transporting an amount of oxygen. This environment will help aerobic bacteria to grow while maximizing the degradation process.

However, the primary role of vegetation is to maintain permeability in the filter and to provide habitats for microorganisms. Nutrients and organic material are absorbed and degraded by the dense microbial population. Unlike oxidation pond, constructed wetland system usually treats some sort of wastewater known as leachate. Leachate can be identified as any contaminated liquid generated from water permeating through a solid waste disposal site moving into subsurface regions. As these wastes are compacted or chemically react, bound water is discharged as leachate [10]. Therefore, landfill leachate treatment has been perceived as an essential part of solid waste management.
**Figure 1.6** (a) Phragmites Australis (b) Glyceria Maxima (c) Phalaris Arundinacea (d) Cattails (Typha Angustifolia) [8]

**Figure 1.7** Putrajaya Lake and Wetland System, Putrajaya, Malaysia [12]
Constructed wetlands can be planted with a number of adapted, emergent wetlands plant species. *Typha Angustifolia*, which belongs to the Typhaceae family, was selected as the subject of this study (refer Figure 1.6). It is an erect, perennial freshwater aquatic herb that can grow three or more meters in height. The linear cattail leaves are thick, ribbon-like structures with a spongy cross-section exhibiting air channels. The subterranean steam rises from thick rhizomes [13]. This plant has been selected for various reasons. One of those is that it is among the most common wetland plants available in the region (refer Figure 1.7). Besides, *typha* types of plant have been extensively studied in the Europe as suitable species of vegetation in constructed wetlands [14].

According to the study by Chew [15], the removal efficiency of nutrient from landfill leachate in the form of ammoniacal nitrogen and nitrate by *Typha Angustifolia* ranges from 42.6%–88.9%. Meanwhile, the removal of BOD and COD ranging from 62.6%–72.8% and 64.5%–85.7%, respectively.

1.2 Background of the Problem

This study aims to explain the biological processes that underpin the wastewater treatment system by showing how the bacteria deal with the pollutant in the sewage. Basically, there are three major processes involved in the treatment plant, which are biodegradation of pollutant, the decreased of oxygen levels, and the cleansing of wastewater. Wastewater can be divided into two types, which are the one produced by human and the other resulting from industrial activities. According to Fakhrul-Razi *et al.* [16], sewage is considered as the largest contributor of organic pollution to water resources around the world. In addition, the largest proportion (64.4%) of total waste in Malaysia is also contributed by sewage, followed by animal husbandry wastes (32.6%), agricultural resources (1.7%) and lastly industrial waste (1.3%) in terms of biochemical oxygen demand (BOD) load. If the wastewater is not well treated and directly discharged into the environment, water-borne diseases will be spread.
It all started around the early twentieth when many researchers are trying to design an environmental friendly system utilizing biological treatment. This biological treatment was constructed to preserve the environment and to treat the wastewater. Since then, the treatment system has become the foundation of many wastewater treatment systems worldwide. The treatment method involves retaining bacteria naturally present in high concentration or population of wastewater treatment plant. It comprises several types of bacteria and protozoa found in treatment plant collectively referred as activated sludge [2]. The essence of the treatment is that bacteria break down organic carbon as a source of energy and food. As a result, bacteria can grow while the wastewater is being cleansed. Treated sewage at treatment plant is usually safe to be discharged into rivers or sea. Although the idea of applying bacteria into this treatment looks simple, the process is actually more complex considering many parameters that affect the treatment system. These include the changes in composition of bacteria, external factors such as weather, temperature and sunlight for an exposed treatment plant, as well as the changes in sewage passing through the treatment plant.

Industrial wastewater containing toxic chemicals at very high concentration may also affect the treatment process as the bacteria are only able to slowly degrade the pollutant. This toxic shock may inhibit the growth of bacteria, resulting in the untreated effluent discharged by the treatment plant to the environment. In this case, treatment plant will become malfunction until the dead bacteria are replaced with the new bacterial seeds.

Normally, the composition of effluents discharged to receiving waters is monitored by the national environment agencies. For example in Malaysia, the water quality standard must be in agreement with the Water Environment Partnership in Asia (WEPA). The legislation is concerned with the prevention of pollution and therefore sets concentration limits on dissolved organic carbon as biochemical oxygen demand (BOD), chemical oxygen demand (COD), nitrogen and phosphates (PO$_4^{3-}$) that can cause eutrophication if excessive [2].
Most wastewaters largely comprised organic carbon either in solution or particulate matter. Extremely small particle ranging from one nanometer (nm) to 100 micrometers (μm) will remain in colloidal suspension and become adsorbed to the activated sludge during treatment. It is quite straightforward to experimentally measure the amount of organic carbon in the sewage. There are two types of different measurement namely combustion and chemical oxidation, which can be described as total organic carbon (TOC) and chemical oxygen demand (COD), respectively. TOC is calculated by the accumulation of carbon dioxide (CO₂) produced by combustion at a very high temperature. Meanwhile for COD, the sample is heated in strong sulphuric acid (H₂SO₄) containing potassium dichromate (K₂Cr₂O₇). The oxidized carbon is determined by the amount of dichromate used in the reaction and the result is expressed in unit of oxygen [2].

However, both measurements have their own weaknesses, since they overestimate the organic carbon compounds that cannot be broken down biologically. Conversely, some aromatic compounds including benzene (C₆H₆), toluene (CH₃) and pyridine (C₅H₅N) are only partly oxidized in the procedure. Overall, TOC and COD will overestimate the carbon that can be removed by activated sludge. The more accurate method that can be used to determine the biodegradable carbon is the 5-day biological oxygen demand (BOD₅) (refer to Figure 1.1). This method is used to measure the oxygen uptake over a 5-day period by a small seed of bacteria that are confined in the dark, in a bottle containing the wastewater. During this time, the biodegradable organic carbon was taken up, and a corresponding decrease in the dissolved oxygen can be observed as some carbons were used for the respiration of the bacteria. The values obtained for BOD₅ are always lower than those for COD by two reasons. The first reason is that activated sludge bacteria cannot chemically degrade some of the compounds oxidized in the COD test, while the second one is that some of the carbon removed during the BOD test is not oxidized, but ends up in a new bacterial biomass. Thus, it can be stated that BOD is the measurement of biodegradable carbon that is actually oxidized by the bacteria [17].
The mathematical models of the activated sludge process from previous studies will be discussed in Chapter 2. This section also includes relevant mathematical background and concepts necessary for modelling the process and interpret the solutions. The modelling equations as well as constitutive relations represent the physical effects that are present in the process. Basically, solving these mathematical models, requires the discretization in both space and time.

There are many studies done using ordinary differential equation models for constructed wetland including that conducted by Rousseau [18]. However, it was found that these studies have not considered the stability analysis of the models including the steady-state and bifurcation analyses, which are crucial in determining the reliability and importance of the proposed models. Therefore, a model is proposed introducing a more simplified form for mass transport of oxygen through plant roots which is the difference between saturated oxygen concentration and the current concentration of dissolved oxygen (DO) compared to exponential term used by Rousseau [18].

Meanwhile, for oxidation pond model, the advection-diffusion-reaction equation model proposed by Pimpunchat et al. [19] is extended to three state variables in this study, which are phototrophic bacteria (PSB), chemical oxygen demand (COD) and DO. On top of that, it has been discovered that there is no available simulation published on the combined treatment systems (pond-constructed wetland) despite the experimental studies done [20–23]. Thus, this present study is conducted simulating the aforementioned system to act as a reference in improving the efficiency of wastewater treatment system.
1.3 Problem Statement

The sources of wastewater can be either from the industrial or non-industrial area. The major source of pollution comes from the non-industrial parts and the waste produced by human contributes the largest part of the non-industrial pollution. If it cannot be well handled, many problems will arise including epidemics. Hence, pollution level should be maintained at a very low level or at least controllable. Wastewater with human sources can be efficiently treated by oxidation pond. In addition, wetland system can effectively control industrial wastewater; for instance, the wastewater discharging from construction sites. Currently, there are several mathematical models available for simulating oxidation pond process where some important parameters are considered such as bacteria (cleansing agent), pollutants and dissolved oxygen (DO). However, previous results did not provide good approximation on the required parameters. Moreover, stability analysis was rarely considered for constructed wetland models. However, the steady-state and bifurcation analyses are usually crucial in determining the reliability of the models that is under study. Thus, dynamic mathematical models are developed in this study to allow the simulation and prediction of wastewater treatment process for both oxidation pond and CW case studies. Furthermore, the nonlinear system of ordinary differential equations (ODE) using multiple substrate limiting factors with interactive reactions and partial differential equations (PDE) using advection-diffusion-reaction equations are implemented for CW and oxidation pond, respectively.

1.4 Objectives of the Research

The objectives of this research are as follows:

1. To develop a mathematical model for wastewater treatment process of an oxidation pond with microbe-based product (mPHO) produced by Johor Biotechnology & Biodiversity Corporation (J-Biotech).
2. To develop a mathematical model for horizontal subsurface flow constructed wetlands system with vegetation type (*Typha Angustifolia*) based on the study given in Chew [15].

3. To construct numerical simulation and analysis of the models for validation.

4. To combine wastewater treatment processes by using pond-constructed wetland system.

### 1.5 Scope of the Research

This research is divided into two major parts, which are oxidation pond treatment process using microbe-based product in liquid form (mPHO) and horizontal subsurface flow constructed wetland system using plant type (*Typha Angustifolia*).

### 1.6 Significance of the Research

The significance of this research are as follows:

1. This study emphasises the ability of mathematical modelling to facilitate the process of wastewater treatment system using oxidation pond, which has become an important treatment procedure in Malaysia governed by Indah Water Consortium (IWK).

2. This study provides a mathematical model to understand the wastewater treatment process of constructed wetland and allow it to predict the output if the model is used for a long period.

3. The mathematical models are able to help the preservation and conservation of environment to run smoothly in the sense that it can save a lot of maintenance cost as well as being more efficient.
1.7 Thesis Organization

This thesis is organized as follows:

The first chapter explains in depth on the issue of water pollution, which has become our main concern. It includes motivation, the background of study, problem statement, objectives, significance as well as the scope of study to be carried out.

Chapter 2 reviews the biological processes related to water treatment process as other studies have obtained relevant methods to treat wastewater in the past until the present such as that conducted by Rousseau [18], Pimpunchat et al. [19] and Wang et al. [23].

In Chapter 3, the proposed solutions are discussed in detail. It covers the construction of model, parameter estimation and the method for nondimensionalization of the model.

Chapter 4 presents the oxidation pond problems involving ODE as well as the PDE models. This chapter presents four types of different models. The first model is three competing species model, which includes three types of bacterium known as E.coli, Coliform, and PSB. The second model is the coupled-reaction equations model, which includes COD, DO, PSB, and Coliform. The other two are the PDE models comprising advection-reaction equations and advection-diffusion-reaction equations models for competing species and transport of pollutant, respectively.

Chapter 5 presents the constructed wetland problems. In this chapter, three models are constructed based on the ODE model. The first model is the nonlinear ordinary differential equations model consisting six state variables. The second model is considered as the simplified model for the purpose of model analysis. Thus, the state variables for the model were reduced to only three variables including DO,
mixed culture bacteria (cleansing agent) and COD. Lastly, the dimensionless model is analysed to show the behaviour of the proposed model.

Chapter 6 presents the simulation of pond-constructed wetland system. The simulation was carried out by combining an advection-diffusion-reaction equations model for oxidation pond with the simplified model for constructed wetland.

Finally, Chapter 7 summarizes the study with a conclusion, re-stating the contributions as well as some suggestions for future studies.
REFERENCES


86. Howison, S. Practical Applied Mathematics: Modelling, Analysis, Approximation. United Kingdom: Cambridge University Press. 2005


