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GRAVITATIONAL SEARCH ALGORITHM: $R$ IS BETTER THAN $R^2$?

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ABSTRACT

Gravitational Search Algorithm (GSA) is a metaheuristic population-based optimization algorithm inspired by the Newtonian law of gravity and law of motion. Ever since it was introduced in 2009, GSA has been employed to solve various optimization problems. Despite its superior performance, GSA has a fundamental problem. It has been revealed that the force calculation in GSA is not genuinely based on the Newtonian law of gravity. Based on the Newtonian law of gravity, force between two masses in the universe is inversely proportional to the square of the distance between them. However, in the original GSA, $R$ is used instead of $R^2$. In this paper, the performance of GSA is re-evaluated considering the square of the distance between masses, $R^2$. The CEC2014 benchmark functions for real-parameter single objective optimization problems are employed in the evaluation. An important finding is that by considering the square of the distance between masses, $R^2$, significant improvement over the original GSA is observed provided a large gravitational constant should be used at the beginning of the optimization process.

Keywords: gravitational search algorithm, newtonian law of gravity, law of motion.

INTRODUCTION

Gravitational Search Algorithm (GSA) has been firstly introduced by Rashedi et al. in 2009 [1]. It is a metaheuristic population-based optimization algorithm which is inspired by the Newtonian law of gravity and law of motion. In GSA, fitness is translated into mass and interaction between agents is simulated based on the Newtonian Law of Gravity and Law of Motion.

However after three years GSA was introduced, Gauci et al. [2] has found an inconsistency used of gravitational formulation in GSA. They have proved theoretically that GSA was indeed not genuinely based on Newtonian law of gravity. Specifically, in the calculation of force, distance $R$ is employed instead of $R^2$. However, the main reason of this has been explained in the first paper of GSA. The original author stated in [1], original gravitational formulation was not used because of poor experimental result.

Therefore, in this paper, we re-evaluate the performance of standard GSA with distance $R$, using CEC2014 benchmark dataset. Then, we propose the use of square of distance $R^2$, in the calculation of force which we denoted as GSAR2. We also investigate the performance of GSAR2 algorithm by varying the value of initial gravitational constant, $G_0$. The performance is then analyzed statistically.

GRAVITATIONAL SEARCH ALGORITHM

In GSA, agents are considered as an object and their performance are expressed by their masses. The position of particle is corresponding to the solution of the problem. Consider a population consisted $N$ quantity of agents, so the position of $i$th agent can be presented by:

$$X_i = \left(x_i^1 \ldots x_i^d \ldots x_i^N\right) \text{ for } i = 1, 2, ..., N$$  (1)

The mass of $i$th particle at time $t$ is derived from Eqn. (2) and Eqn. (3), denoted as $M_i(t)$.

$$m_i(t) = \frac{\text{fit}_i(t) - \text{worst}(t)}{\text{best}(t) - \text{worst}(t)}$$  (2)

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)}$$  (3)

where $N$ is a population size, $m_i(t)$ is an intermediate variable in agent mass calculation, $\text{fit}_i(t)$ is the fitness value of $i$th agent at time $t$, $\text{best}(t)$ and $\text{worst}(t)$ denote the best and the worst fitness value of the population at time $t$. The best and the worst fitness for the case of minimization problem are defined as follows;

$$\text{best}(t) = \min_{j \in \{1,...,N\}} \text{fit}_j(t)$$
$$\text{worst}(t) = \max_{j \in \{1,...,N\}} \text{fit}_j(t)$$  (4)

whereas for maximization problem,

$$\text{best}(t) = \max_{j \in \{1,...,N\}} \text{fit}_j(t)$$
$$\text{worst}(t) = \min_{j \in \{1,...,N\}} \text{fit}_j(t)$$  (5)

At specific time “$t$”, the force acting on agent “$i$” from agent “$j$” in $d$th dimension can be represented as the following:
\[ F_{ij}^d(t) = G(t) \frac{M_p(t) \times M_j(t)}{R_{ij}(t)^2} (x_j^d(t) - x_i^d(t)) \]  

(6)

where \( M_p(t) \) is the passive gravitational mass of agent “i”, \( M_j(t) \) is the active gravitational mass of agent “j”, \( G(t) \) is the gravitational constant, \( \varepsilon \) is a small constant, and \( R_{ij}(t) \) is the Euclidean distance between agent “i” and “j”. The distance is calculated using a standard formula as follow:

\[ R_{ij}(t) = \left| X_i(t), X_j(t) \right| \]  

(7)

while gravitational constant, \( G(t) \), is defined as a decreasing function of time, which is set to \( G_0 \) at the beginning and decreases exponentially towards zero with lapse of time [3].

\[ G(t) = G_0 \times e^{-\alpha \frac{t}{t_{max}}} \]  

(8)

To give a stochastic characteristic to GSA, the total force acted on the system divided by mass of inertia [3], which is shown in the following formula.

\[ F_i^d(t) = \sum_{j=1}^{N} \text{rand}_i \times F_{ij}^d(t) \]  

(9)

where rand\(_i\) is a random number in the interval of [0,1].

According to law of motion, the current velocity of any mass is equal to the sum of the force exerted from other agents;

\[ V_i^d(t + 1) = V_i^d(t) + a_i^d(t) \]  

(10)

where \( a_i^d(t) \) is the active gravitational mass of agent i “i”

\[ a_i^d(t) = \frac{F_i^d(t)}{M_i(t)} \text{ for } M_{ai} = M_{pi} = M_{ti} \]

Therefore, the new agent velocity and position are calculated using these equations:

\[ V_i^d(t + 1) = \text{rand}_i \times V_i^d(t) + a_i^d(t) \]  

(11)

\[ x_i^d(t + 1) = x_i^d(t) + V_i^d(t + 1) \]  

(12)

Finally, the next iteration is executed until the maximum number of iterations, \( t_{max} \), is reached. In summary, the principle of standard GSA is shown in Figure-1.

**GSA IS NOT GENUINELY FOLLOWS THE NEWTONIAN GRAVITATIONAL LAW**

Newton stated that “Every particle in the universe attract every other particle with a force that is directly proportional to the square of the distance between them” [4]. This definition is known as gravitational force and it is formulated as:

\[ F = G \frac{M_1 \times M_2}{R^2} \]  

(13)

In GSA, the calculation of force is also based on this equation. However, as shown in Eqn. (6), distance \( R \) is used as the denominator instead of \( R^2 \). Let \( \varepsilon = 0 \), then

\[ F_{ij}^d(t) = G(t) \frac{M_p(t) \times M_j(t)}{R_{ij}(t)^2} (x_j^d(t) - x_i^d(t)) \]  

(14)

since \( R_{ij}(t) = x_j^d(t) - x_i^d(t) \), therefore,

\[ F_{ij}^d(t) = G(t) \times M_p(t) \times M_j(t) \]  

(15)

which clearly shows that the force \( F_{ij}(t) \) is not influenced by the distance between agent i and j. Thus, the original GSA is not genuinely follows the Newtonian gravitational law.

In this paper, we follow genuinely the Newtonian gravitational law and use the square of distance, \( R^2 \), in the calculation of force. The performance of GSAR2 with different value of initial gravitational constant is investigated as well.

**EXPERIMENTS**

The parameter setting for all experiments is tabulated in Table-1. Different value of \( G_0, G_0 = 10^0 \) until \( G_0 = 10^6 \) were tested in experiments for GSAR2.

In this study, 30 standard benchmark functions from CEC2014 test functions [5] shown in Table-2 were used throughout the experiment. These benchmark functions consist of the shifted, rotated, expanded and combined classical test function. They are categorized into three four groups; unimodal, multimodal, hybrid, and composite function.

**RESULT AND DISCUSSION**

Convergence curves of the two variations of GSA, which is the original GSA and GSA that employs square of distance between masses (GSAR2), are shown in Figure-2 to Figure-5. For GSAR2, \( G_0=10^0 \) is used. These results show that generally better performance can be obtained even though square of distance between masses is used.

Analysis of convergence curves of GSAR2 with different \( G_0 \) are shown in Figure-6 to Figure-10. These figures show that solutions can be improved faster and convergence rate is better if smaller \( G_0 \) is used. However, there is no guarantee that small \( G_0 \) produces better result.
Figure-1. General principle of standard GSA.

Table-1. Parameter setting used in all experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of agents, $N$</td>
<td>100</td>
</tr>
<tr>
<td>Number of iterations, $t$</td>
<td>2000</td>
</tr>
<tr>
<td>Number of dimensions, $D$</td>
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</tr>
<tr>
<td>Number of runs, $N_{\text{run}}$</td>
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<tr>
<td>Search range</td>
<td>$[100,-100]$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure-2. Convergence curve for function 3, in which, $G_0=10^2$ is used for original GSA and $G_0=10^9$ is used for GSAR2.

Figure-3. Convergence curve for function 6, in which, $G_0=10^2$ is used for original GSA and $G_0=10^9$ is used for GSAR2.

Table-2. CEC 2014 benchmark functions.

<table>
<thead>
<tr>
<th>Function Type</th>
<th>Function ID</th>
<th>Function Description</th>
<th>Ideal Fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unimodal Function</td>
<td>F1</td>
<td>Rotated High Conditioned Elliptic Function</td>
<td>100</td>
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<tr>
<td></td>
<td>F2</td>
<td>Rotated Bent Cigar Function</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>F3</td>
<td>Rotated Discus Function</td>
<td>500</td>
</tr>
<tr>
<td>Simple Multimodal Function</td>
<td>F4</td>
<td>Shifted and Rotated Rosenbrock’s Function</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>F5</td>
<td>Shifted and Rotated Ackley’s Function</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>F6</td>
<td>Shifted and Rotated Weierstrass Function</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>F7</td>
<td>Shifted and Rotated Griewank’s Function</td>
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<tr>
<td></td>
<td>F8</td>
<td>Shifted Rastrigin’s Function</td>
<td>800</td>
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<tr>
<td></td>
<td>F9</td>
<td>Shifted and Rotated Rastrigin’s Function</td>
<td>900</td>
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<tr>
<td></td>
<td>F10</td>
<td>Shifted Schwefel’s Function</td>
<td>1000</td>
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<tr>
<td></td>
<td>F11</td>
<td>Shifted and Rotated Schwefel’s Function</td>
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<tr>
<td></td>
<td>F12</td>
<td>Shifted and Rotated Katsuura Function</td>
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</tr>
<tr>
<td></td>
<td>F13</td>
<td>Shifted and Rotated HappyCat Function</td>
<td>1300</td>
</tr>
<tr>
<td>Hybrid Function</td>
<td>F14</td>
<td>Shifted and Rotated HGBat Function</td>
<td>1400</td>
</tr>
<tr>
<td></td>
<td>F15</td>
<td>Shifted and Rotated Expanded Griewank’s plus Rosenbrock’s Function</td>
<td>1500</td>
</tr>
<tr>
<td></td>
<td>F16</td>
<td>Shifted and Rotated Expanded Scaffer’s F6 Function</td>
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</tr>
<tr>
<td></td>
<td>F17</td>
<td>Hybrid Function 1 (N=3)</td>
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<tr>
<td></td>
<td>F18</td>
<td>Hybrid Function 2 (N=3)</td>
<td>1800</td>
</tr>
<tr>
<td></td>
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<td>Hybrid Function 3 (N=4)</td>
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<td>Hybrid Function 4 (N=4)</td>
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<tr>
<td></td>
<td>F21</td>
<td>Hybrid Function 5 (N=5)</td>
<td>2100</td>
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<tr>
<td></td>
<td>F22</td>
<td>Hybrid Function 5 (N=5)</td>
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</tr>
<tr>
<td>Composite Function</td>
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<td>Composition Function 1 (N=5)</td>
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<tr>
<td></td>
<td>F24</td>
<td>Composition Function 2 (N=3)</td>
<td>2400</td>
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<tr>
<td></td>
<td>F25</td>
<td>Composition Function 3 (N=3)</td>
<td>2500</td>
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<td></td>
<td>F26</td>
<td>Composition Function 4 (N=3)</td>
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<tr>
<td></td>
<td>F27</td>
<td>Composition Function 5 (N=3)</td>
<td>2700</td>
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<td></td>
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<td>Composition Function 6 (N=5)</td>
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</tr>
<tr>
<td></td>
<td>F29</td>
<td>Composition Function 7 (N=3)</td>
<td>2900</td>
</tr>
<tr>
<td></td>
<td>F30</td>
<td>Composition Function 8 (N=3)</td>
<td>3000</td>
</tr>
</tbody>
</table>
According to inferential statistic, hypothesis testing can be used to obtain inferences about one or more algorithms from given sample. This can be achieved by defining two types of hypothesis, the null hypothesis $H_0$ and the alternative hypothesis $H_1$. The null hypothesis is a statement of no effect or no difference, whereas the alternative hypothesis represents significant difference between algorithms.

Friedman’s test is an omnibus test which can be used to carry out these types of comparison. It allows us to detect differences considering the global set of algorithms. Once Friedman’s test rejects the null hypothesis, we can proceed with a post-hoc test in order to find the concrete pairwise comparisons which produce differences.
<table>
<thead>
<tr>
<th>Function</th>
<th>$G_{a,10^1}$</th>
<th>$G_{a,10^2}$</th>
<th>$G_{a,10^3}$</th>
<th>$G_{a,10^4}$</th>
<th>$G_{a,10^5}$</th>
<th>$G_{a,10^6}$</th>
<th>$G_{a,10^7}$</th>
<th>$G_{a,10^8}$</th>
<th>$G_{a,10^9}$</th>
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<tr>
<td>1</td>
<td>87.53612.54</td>
<td>85.63481.53</td>
<td>83.53213.39</td>
<td>81.5301.18</td>
<td>79.5301.18</td>
<td>77.5301.18</td>
<td>75.5301.18</td>
<td>73.5301.18</td>
<td>71.5301.18</td>
</tr>
<tr>
<td>2</td>
<td>2.827.234.11</td>
<td>2.737.341.11</td>
<td>2.647.451.11</td>
<td>2.557.561.11</td>
<td>2.467.671.11</td>
<td>2.377.781.11</td>
<td>2.287.891.11</td>
<td>2.197.001.11</td>
<td>2.107.111.11</td>
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<tr>
<td>4</td>
<td>19.488.541.1</td>
<td>18.398.648.2</td>
<td>17.298.757.3</td>
<td>16.198.867.4</td>
<td>15.098.976.5</td>
<td>13.998.987.6</td>
<td>12.898.998.7</td>
<td>11.798.999.8</td>
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<td>67.835.857.89</td>
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<td>62.835.852.89</td>
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<td>2.744.391.0</td>
<td>2.644.381.0</td>
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</table>

Table 3. Friedman test result for variant of $G_0$ value.
Table-4. G0 variant for post-hoc comparison using Holm procedure.

<table>
<thead>
<tr>
<th>i</th>
<th>algorithm</th>
<th>$z = (R_i - R_j)_{SE}$</th>
<th>$p$</th>
<th>Holm</th>
</tr>
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<tbody>
<tr>
<td>103</td>
<td>GO1 vs. GO9</td>
<td>9.122334</td>
<td>0</td>
<td>0.00475</td>
</tr>
<tr>
<td>109</td>
<td>GO1 vs. GO8</td>
<td>9.066664</td>
<td>0</td>
<td>0.00481</td>
</tr>
<tr>
<td>103</td>
<td>GO1 vs. GO10</td>
<td>9.066664</td>
<td>0</td>
<td>0.00485</td>
</tr>
<tr>
<td>102</td>
<td>GO2 vs. GO9</td>
<td>8.651387</td>
<td>0</td>
<td>0.0049</td>
</tr>
<tr>
<td>101</td>
<td>GO2 vs. GO8</td>
<td>8.515916</td>
<td>0</td>
<td>0.00496</td>
</tr>
<tr>
<td>100</td>
<td>GO2 vs. GO10</td>
<td>8.515916</td>
<td>0</td>
<td>0.005</td>
</tr>
<tr>
<td>99</td>
<td>GO1 vs. GO7</td>
<td>8.487049</td>
<td>0</td>
<td>0.00505</td>
</tr>
<tr>
<td>99</td>
<td>GO1 vs. GO13</td>
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<td>0</td>
<td>0.00511</td>
</tr>
<tr>
<td>97</td>
<td>GO2 vs. GO7</td>
<td>7.986301</td>
<td>0</td>
<td>0.00515</td>
</tr>
<tr>
<td>95</td>
<td>GO3 vs. GO8</td>
<td>7.986306</td>
<td>0</td>
<td>0.0051</td>
</tr>
<tr>
<td>94</td>
<td>GO2 vs. GO9</td>
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<td>0</td>
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<tr>
<td>94</td>
<td>GO2 vs. GO11</td>
<td>7.794229</td>
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<td>0.00532</td>
</tr>
<tr>
<td>92</td>
<td>GO3 vs. GO6</td>
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<tr>
<td>92</td>
<td>GO3 vs. GO15</td>
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<tr>
<td>91</td>
<td>GO1 vs. GO8</td>
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<td>0.00549</td>
</tr>
<tr>
<td>90</td>
<td>GO1 vs. GO11</td>
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<td>0</td>
<td>0.00556</td>
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<tr>
<td>87</td>
<td>GO2 vs. GO9</td>
<td>7.044973</td>
<td>0</td>
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<tr>
<td>87</td>
<td>GO2 vs. GO12</td>
<td>6.754998</td>
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<td>84</td>
<td>GO3 vs. GO15</td>
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<td>84</td>
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<td>82</td>
<td>GO2 vs. GO12</td>
<td>6.610061</td>
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<tr>
<td>81</td>
<td>GO1 vs. GO4</td>
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<td>81</td>
<td>GO1 vs. GO13</td>
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<td>0.00617</td>
</tr>
<tr>
<td>79</td>
<td>GO3 vs. GO12</td>
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<td>0.00639</td>
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<td>GO3 vs. GO15</td>
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<td>0</td>
<td>0.00639</td>
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<td>77</td>
<td>GO4 vs. GO11</td>
<td>6.081045</td>
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<td>0.00641</td>
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<td>77</td>
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<tr>
<td>74</td>
<td>GO5 vs. GO12</td>
<td>5.46055</td>
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<td>GO5 vs. GO15</td>
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<tr>
<td>72</td>
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<td>0.00675</td>
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<td>GO6 vs. GO15</td>
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<td>GO1 vs. GO13</td>
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<td>0</td>
<td>0.00714</td>
</tr>
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<td>GO1 vs. GO14</td>
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<td>0</td>
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<tr>
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<td>0.0082</td>
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Table-3 shows the overall experimental results for Friedman procedure obtained in this study. For the case of GSAR2, value of G0=109 provides the best average ranking among others. These results were subjected to post-hoc test using Holm procedures and the results are shown in Table-4. According to Holm’s procedure, hypothesis that have an adjusted p-value less or equal to 0.001887 are rejected.

Table-5. GSA Original vs GSAR2 with G0=109 Wilcoxon test result.

![Table-5](image)

In other words, G0=101, G0=102, G0=103, G0=1013, G0=1014, and G0=1015 are significantly different compared to G0=109 which was highlighted in Table-4. The rest of G0 value has no significant difference between each other. However, based on the average ranking, result of G0=109 is chosen for the comparison with the original GSA in pairwise Wilcoxon test. According to the result of the Wilcoxon test shown in Table-5, by using p-value equal to 0.05, the Z-value obtained is -2.931. Based on normal distribution curve it shows p-value for -2.932 is equal to 0.00338 which is smaller than 0.05. So it can be concluding the GSAR2 not only better than the original GSA in terms of performance, but also significant difference exists between these two algorithms.

**CONCLUSIONS**

The original GSA algorithm is not genuinely follows the Newtonian gravitational law. In this paper, by correcting the force of calculation in original GSA and investigating various initial gravitational constants G0,
GSAR2 has been proposed. It is found that the GSAR2 not only superior to the original GSA, but most importantly, GSAR2 follows more closely to the Newtonian gravitational law.

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