Joint Power Allocation for DF Concatenated MIMO Successive Relaying Scheme under Network Power Constraints

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Graphic abstract

Abstract

Power efficiency is a vital consideration in wireless systems. In this paper, we propose a framework for efficient power allocation in decode and forward multiple input multiple output successive relaying systems under network power constraints. Our aim is to maximize the information rate at each link by an optimal power allocation scheme via the primal dual algorithm. Then, we jointly allocate power to the source and transmitting relay under network power constraints. The simulated results show that the proposed joint power allocation scheme under network power constraint can outperform the uniform power allocation under an aggregate power constraint.

Keywords: Power allocation; successive relaying; multiple input multiple output; decode and forward

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1.0 INTRODUCTION

Current and future wireless technologies rely on multiple-input multiple-output (MIMO) systems to improve reliability and increase system capacity [1], [2]. However, as wireless signal is propagated through space, the transmitted power attenuates exponentially with the distance. Hence, MIMO relays are viable options to further improve the communication system efficiency and extend communication coverage. Due to self-interference in full duplex relays, half duplex (HD) relays are generally used for analysis. However, HD relays suffer from half duplex constraint (HDC), i.e. the relay can only receive and transmit in orthogonal time slots or channels, leading to a loss of spectral efficiency of the systems. Several schemes have been proposed to recover the loss due to HDC [3]. Among these schemes are successive relaying (SR). In SR, two or more HD relays are used to forward the source message to the destination. The relays alternate between reception and transmission in orthogonal time slots so that the source can transmit new information in every time slot. Although several studies have focused on single antennas SR schemes [4], [5], recent technologies implement MIMO technology, therefore recent studies are focusing on MIMO SR relaying networks [6], [7].

When the direct link between the transmitting node and the user is weak or disconnected, two-path SR scheme can be implemented. However, several studies have shown that inter relay interference (IRI) can degrade the performance of SR [8], [9]. The IRI can be observed when one relay is forwarding its message to the destination while the other relay is receiving from the source, the relay in the forward mode becomes a source of interference to the receiving relay. Beamforming algorithms are proposed in [7], [9], for IRI cancellation. However, in [10], the authors have shown that, if IRI is properly managed, the user’s signal reliability can be improved. In [11], the IRI was used to aid the weaker source-relay link to decode its received message. The scheme shows about 33 percent loss in spectral efficiency, which make it not suitable for real time applications. In addition, the scheme is also designed for a single antenna system. In a MIMO SR scheme, managing IRI requires high degrees of freedom or large numbers of antennas which correspond to the numbers of interfering signals at the relay. Antenna selection can
be implemented at the relay transmitter to reduce the complexities of the SR system and the dimension of the IRI on the receiving relay [12].

In wireless cooperative network, power efficiency is a vital design considerations. Therefore to further improve the performance of the MIMO SR scheme, an efficient power allocation (PA) scheme is required. Several power allocation schemes have been studied for single relay cooperative systems. Amplify and forward relay is considered in [6] under individual power constraints (IPC) and aggregate power constraints (APC). In IPC, the constraints are considered over each of the antennas at the source and relay node where at each node the power rating of each antenna is constrained to its individual power rating. In APC, the constraints are considered at the nodes, therefore each node has an aggregate power it can transmit. Hybrid power allocation (HPA) can also be implemented in a MIMO relay network [13]. In hybrid power constraints, the IPC and APC can be jointly imposed on the source and relay transmit antennas. Although power allocation in amplify and forward (AF) relays have gained more interest due to the lower complexities compared to decode and forward (DF) relays, when the channel between the source and relay link is in good state, the DF relays can outperform the AF relays [13]. When implementing DF relay in SR scheme, the hybrid power allocation in [13], [14] cannot be directly implemented due to the presence of IRI. Also, in each time slot, the source and at least one relay are in transmission mode. Hence, while considering HPA at each nodes, a joint power allocation at the source and relay nodes is required. To the best of our knowledge, joint power allocation under hybrid power constraints (HPC), for DF MIMO SR scheme with concatenated IRI channel, has not been studied in any literature.

![Figure 1 Proposed concatenated MIMO successive relaying scheme](image)

In this paper, we propose an optimal power allocation algorithm for concatenated MIMO SR (CM-SR) scheme, where the IRI and the source messages are jointly decoded and a superposition of the decoded messages are forwarded to the user in the next time slot to improve the reliability of the information, based on hybrid power constraints. Although, hybrid power constraints has been studied in [13] for a single MIMO DF relay network, we extend the work to CM-SR in the presence of IRI.

Notations: Throughout this paper, matrices and vector symbols are represented by uppercase and lowercase boldface respectively. $A^T$, $A^{-1}$ and $A^n$ represent transpose, inverse and Hermitian transpose of the matrix $A$. $A^*$ represents the pseudo inverse of $A$, $\| A \|_F$ represents the Frobenius norm, $\tr(A)$ and $[A]_{(i,j)}$ represent the trace and $(i, j)$-th element of matrix $A$. $E_a[A]$ the expectation $A$ of over the random variable $A$.

### 2.0 SYSTEM MODEL

#### 2.1 Transmission Protocol

We consider a wireless system with one source ($s$) one destination ($d$) and two DF HD relays ($r_1$) and ($r_2$) as shown in Figure 1. The numbers of antennas at $s$, $r_1$, $r_2$ and $d$, are denoted as $N_s$, $N_{r1}$, $N_{r2}$ and $N_d$ respectively. $N_t$ antennas that optimizes the $n$-th link are selected to transmit from the relay, the antenna selection process is implemented to reduce the dimension of the IRI on the receiving relay. Thereby receive antennas $N_r$ are assumed to correspond to the total number of received signals, i.e., $N_r \geq N_s + N_d$. We assume there is no direct link between the source and the destination, but the inter-relay link exits. Therefore all source codewords are transmitted to the destination though the two relays. Furthermore, the channel between the source and relay link is denoted as $H_{si} \in C^{N_r \times N_s}$, the channel between the relay to destination is denoted as $H_{dj} \in C^{N_t \times N_d}$ and the inter-relay link is denoted as $H_{rij} \in C^{N_r \times N_d}$ where $i$ denotes the identity of the receiving relay and $j = \{1,2\}$ and $i \neq j$ is the identity of the transmitting relay. All channel are assumed to be independent and identically distributed (i.i.d) Rayleigh fading process.

The noise vector at the relay and destination are denoted as $n_r \in C^{N_r \times 1}$ and $n_d \in C^{N_d \times 1}$ respectively. In addition, all the noise vectors are assumed to be complex Gaussian noise with zero mean and covariance $\sigma^2$ denoted as $CN(0, \sigma^2)$. The local channel state information (L-CSI) is also assumed to be known at each node, which can be acquired by channel training or through feedback channels. Hence, $s$ and $d$ can configure their beamforming matrices to achieve the best transmission performance.

In the CM-SR scheme, the source continues to transmit new message every time slot while two DF HD relays alternating between receive and transmit modes helps to forward the source message to the destination. However, at time slot $n$ the receiving relay jointly decodes the message from the source and inter-relay link, the decoded message is re-encoded using superposition coding and retransmitted to the destination at time slot $n + 1$. The aim of forwarding the
superposition codewords to the destination is to improve the reliability of the user’s information. Recall that sending multiple copies of the same message to the user improves the signal diversity against fading [15].

Assuming $N_t$ antennas are selected at the relay transmission phase a new $r$-d sub-channel matrix can be obtained as $H_{rs}$. The sub-channel can be decomposed by singular value decomposition (SVD), such that $H_{rs} = USW^H$, where $U$ and $W$ are unitary matrices and $S = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_{N_s}]$ is the nonzero diagonal element of the selected channel.

Without loss of generality, we assume that, the codeword to be transmitted is first divided into $l$ symbols. Furthermore, at time slot $n$ the source transmits $x_n$ to $r_1$, while $r_2$ transmit $x_{n+1}$ to $d$ and $r_1$. The signal received at $r_1$ and $d$ at time slot $n$ can be expressed as

$$y_{r_1} = H_{sr_1}B_n x_n + H_{r_1r_2}W_n x_{n+2} + n_{r_1}$$

(1)

$$y_{dr_2} = H_{dr_2}W_n x_{n+2} + n_{dr_2}$$

(2)

Where $x_n = [x_1, x_2, \ldots, x_{N_s}]^T$ is the independent transmitted signal vector from $s$. Similarly $x_{n+2} = x_{(n-1)} + x_{(n-2)}$ is the transmitted signal vector from the relay $j$, $\Theta = \text{diag}[\theta_1, \theta_2, \ldots, \theta_{N_s}]$ and $V = \text{diag}[v_1, v_2, \ldots, v_{N_s}]$ are the nonnegative power scaling factor at the source and relay respectively.

$B_n$ is the transmit beamforming matrix at the source, in which $b_k = [b_{1k}, b_{2k}, \ldots, b_{N_bk}]$, $\forall k \in N_s$.

$W_n = [w_1, w_2, \ldots, w_{N_ww}]^T$ is the transmit beamforming matrix at the relay in which $w_k = [w_1, w_2, \ldots, w_{N_{ww}}]$ $\forall k \in N_s$ where $s = L_2, \ldots$ since each relay transmit only half of the entire source transmission length.

At the relay, zero forcing (ZF) receivers $G^\dagger$ are applied to the relay received signal, to spatially separate the composite source and inter-relay channel into independent orthogonal streams. The ZF criteria is given as:

$$G^\dagger H_c = I_{(N_s+N_s)}$$

(3)

where

$$H_c = \begin{bmatrix} H_{sr_1} \\ H_{r_1r_2} W_n \end{bmatrix}$$

The input-output relationship at the relay node can be expressed as

$$G^\dagger y_{r_1} = G^\dagger (H_{sr_1}B_n x_n + H_{r_1r_2}W_n x_{n+2}) + G^\dagger n_{r_1}$$

(4)

$$\tilde{y}_{r_1} = B_n \Theta \tilde{x}_n + W_n \tilde{x}_{n+2} + \tilde{n}_{r_1}$$

(5)

The input-output relationship at the destination can be expressed as

$$y_{dr_2} = U^H_2 H_{dr_2} W_n \tilde{x}_{n+2} + \tilde{n}_{dr_2}$$

(6)

$$\tilde{y}_{dr_2} = U^H_2 W_n \tilde{x}_{n+2} + \tilde{n}_{dr_2}$$

(7)

$$\tilde{y}_{dr_2} = S_n \tilde{x}_n + \tilde{n}_{dr_2}$$

(8)

Due to the symmetrical nature of the proposed scheme, at time slot $n+1$, s transmits new symbols to $r_2$ while forwards its decoded message to $d$, the signal received at $r_2$ and $d$ can be expressed as

$$y_{r_2} = H_{sr_2}B_n \tilde{x}_n + H_{r_1r_2}W_n \tilde{x}_{n+2} + n_{r_2}$$

(9)

$$y_{dr_1} = H_{dr_1}W_n \tilde{x}_n + n_{dr_1}$$

(10)

The input-output relationship for equation (9) and (10) is similar to (5) and (8) respectively.

### 2.2 Power Constrains

In this section, we formulate the power allocation problem. We assume that, the source node is under an aggregate power constraint $P_s$ while the relay is also under an aggregate power constraint $Pr$. Also, each of the multiple antennas at the source and at the relay are under individual power constraint. The aggregate power constraint at the source can be expressed as

$$E[x_r^H x_r] = E[(B_n \tilde{x}_n)^H B_n \Theta \tilde{x}_n] = \text{tr}[B_n \tilde{x}_n] - \left|\Theta\right|^2 \leq P_s$$

(11)

Assuming a network power constraints of $P_t$ then the aggregate power constraint at the relay transmitter can be expressed as

$$E[x_{r_2}^H x_{r_2}] = \text{tr}[(SVS_r)^H (SVS_r)] = \left|V_r\right|^2 \leq Pr$$

(12)

where

$$Pr = P_t - Ps$$

(13)

Consider the transmitted sequence from the $k$-th antenna at the source under individual power constraint, the transmitted sequence can be expressed as

$$s_k = b_{k1} \theta_1 x_1 + b_{k2} \theta_2 x_2 + \ldots + b_{kL} \theta_L x_L$$

$$= \sum_i b_{ki} \theta_i x_i$$

The average transmitted power from the $k$-th antenna can be expressed as

$$E[x_k^H x_k] = \left|b_k \theta_k\right|^2 \leq Ps$$

(14)

where $v = \frac{P_{ki}}{Ps}$ is the individual power constraints at the relay node. Similarly the average power transmitted from the $i$-th antenna at the relay can be expressed as

$$E[x_i^H x_i] = \left|v_i \theta_i\right|^2 \leq Pr$$

(15)

From (11) it can be observed that the power transmitted from the $i$-th antenna under individual power constraints is a fraction of the aggregate power at the source node. Similarly from (12), the
power transmitted by the \( j \)-th antenna at the relay under individual power constraints is a fraction of the aggregate power at the relay. For example, assuming \( \nu = 0.2 \), then the individual power of each antenna at the source cannot exceed 20 percent of the aggregate source transmit power. However, at time slot \( n \) the source and one of the relays are in transmission mode. Hence, there is a need to jointly optimize the transmit power allocation at the source and relay node. This requires the joint-HPA at the source and relay transmitting nodes under network power constraints. The hybrid power constraints requires that (13), (14) and (15) simultaneously hold.

### 2.3 Problem Formulation

From (3), the per stream signal to noise ratio at \( r1 \) withZF receiver can be expressed as

\[
\gamma_v = \frac{\rho}{N \left| \mathbf{H}^H \mathbf{H}_c \right|^{-1}} \quad \forall v \in N \tag{16}
\]

where \( N = N_s + N_r \) and \( \rho = \frac{1}{\sigma_n^2} \). The instantaneous capacity at the relay can be expressed as

\[
C_{r_1} = \sum_{i=1}^{N_r} \log_2 \left( 1 + \gamma_{v_i,j} \right) \tag{17}
\]

Similarly the instantaneous capacity at the destination from \( r2 \) can be expressed as

\[
C_{d_{r_2}} = \log_2 \det \left( \mathbf{I} + \frac{1}{\sigma_n^2} \mathbf{S}_2 \mathbf{V}_2 \right) \tag{18}
\]

\[
= \sum_{k=1}^{N_t} \log_2 \left( 1 + \frac{1}{\sigma_n^2} \mathbf{v}_2,k^H \mathbf{v}_2,k \right). \tag{19}
\]

Equation (17) and (18), also holds for \( r1 \) and \( d \) respectively at time slot \( n + 1 \). In the CM-SR scheme the end to end capacity is expressed as [16]

\[
C_e = \frac{L}{2(L+1)} \left( R_{r_1} + R_{r_2} \right) \tag{20}
\]

where

\[
R_{r_1} = \min \left\{ C_{r_1}, C_{d_{r_1}} \right\}
\]

\[
R_{r_2} = \min \left\{ C_{r_2}, C_{d_{r_2}} \right\}
\]

Hence, the power allocation problem can be formulated as

\[
\left( \mathbf{P}_{s}, \mathbf{P}_{r}, \mathbf{B}, \mathbf{W} \right) = \arg \max_{\mathbf{P}_{s}, \mathbf{P}_{r}, \mathbf{B}, \mathbf{W}} C_e \tag{21}
\]

\[
s.t. \ E[\|\mathbf{y}_i\|^2] + E[\|\mathbf{y}_j\|^2] = \mathbf{P}_t. \tag{22}
\]

### 3.0 JOINT SOURCE RELAY POWER ALLOCATION

The end to end capacity is limited by the minimum of \( S_r \) and \( d_r \) links. To further improve the power efficiency of the system, optimal power allocation should be jointly carried out at the source and relay. Furthermore, it can be observed that the capacity at the relay is a function of the Hybrid power at the source and transmitting relay due to the inter-relay link, which can be expressed as \( C_r (\mathbf{P}_s, \mathbf{P}_r) \), while the corresponding capacity at the destination is a function of the Hybrid power at the transmitting relays and can be expressed as \( C_d (\mathbf{P}_r) \).

Define

\[
\begin{align*}
A_1 &= \left[ p_{1}, p_{2}, \ldots, p_{N_s} \right]^T, & \text{in which} \ p_v = \left[ b_{1,v}^2, b_{2,v}^2, \ldots, b_{N_s,v}^2 \right]^T, \forall v \in N_s. \\
A_2 &= \left[ q_1, q_2, \ldots, q_{N_r} \right]^T, \text{ in which} \ q_v = \left[ w_{1,v}^2, w_{2,v}^2, \ldots, w_{N_r,v}^2 \right]^T, \forall v \in N_r. \\
c &= \left[ c_{1}, c_{2}, \ldots, c_{N_s} \right]^T = \left[ \theta_1^2, \theta_2^2, \ldots, \theta_{N_s}^2 \right]^T, \\
d &= \left[ d_{1}, d_{2}, \ldots, d_{N_r} \right]^T = \left[ \nu_1^2, \nu_2^2, \ldots, \nu_{N_r}^2 \right]^T, \\
b_1 &= [p_{1}, p_{2}, \ldots, p_{N_s}]^T \text{ and } b_2 = [p_{1}, p_{2}, \ldots, p_{N_r}]^T. \end{align*}
\]

The optimization problem can be expressed as

\[
\left\{ c^*, d^* \right\} = \arg \max_{c_i, d_i} C_e. \tag{23}
\]

\[
s.t. \ A_1 c = A_2 d, A_2 d = \mathbf{F}_c + F_d d = h. \tag{24}
\]

where \( c^* \) denote the component wise inequalities, \( c^* \) is the optimum value of \( c \) from the source, while \( d^* \) is the optimum value of \( d \). Using the dual decomposition, the partial Lagrangian can be expressed as

\[
L(c, d, \lambda) = g^T c + g_2^T d + \lambda^T (F_c + F_d c - h) = \left( F_1^T \lambda + R_1 \right)^T c + \left( F_2^T \lambda + R_2 \right)^T d - \lambda^T h. \tag{25}
\]

The dual function in (21) can be expressed as

\[
q(r) = \inf_{c, d} \left\{ L(c, d, \lambda) | A_1 c = A_2 d, A_2 d = h \right\}. \tag{26}
\]

The solution to (22) can be achieved by maximizing

\[
\max_{c, d} q(r)
\]

\[
s.t. \ \lambda > 0 \tag{27}
\]

The subgradient of (21) can be expressed as

\[
l(t) = -F_1^T c(t) + F_2^T d(t) + h, \tag{28}
\]

the iteration is denoted as

\[
\lambda(t + 1) = \left[ \lambda(t) - \zeta(t) l(t) \right], \tag{29}
\]

where \( \zeta(t) \) is the iteration variable and \( \zeta(t) \) is the step to generate the convergence [17]. From (27), we can observe that the master algorithm adjust the Lagrangian multiplier which regulates the separate solutions of the source and relay transmit power in (26). Two slave optimization problem can be achieved by substituting (20) into (23) and taking the partial derivative with respect to \( \theta_k \) and \( \nu_j \), which is given by

\[
\frac{\partial L}{\partial \theta_k} = \frac{\Delta_k}{1 + \Delta_k \theta_k + \sum_{i=1}^{N_s} y_{k,i}} \tag{30}
\]
\[
\frac{\partial L}{\partial \eta_j} = \frac{y_j}{1 + y_j v_j} - \sum_{k=1}^{Nd} \lambda_k \left| s_k \right|^2 - \lambda_{(Nd+1)},
\]
where \(z_k = h_k^* / \sigma_{nr}^2\) and \(y_j = \frac{s_j^2}{\sigma_n^2}\). From [17], we get
\[
\theta_k = \left\lceil \frac{1}{\sum_{i=1}^{N_S \times N_t} \lambda_i \left| s_i \right|^2 + \lambda_{i+1}} - \frac{1}{z_k} \right\rceil^+.
\]
However, the equations are related to one another. Hence, an iterative algorithm can be used to calculate one variable at a time while fixing the other variable. In addition, without loss of generality, we assume that
\[
C_r (P_s, P_r) > C_d (P_r).
\]
To obtain the optimum \(P_s\) and \(P_r\), some root finding method can be applied [18], therefore we have the following algorithm.

**Algorithm:** search algorithm to solve (27)
1. Initialize \(P_s=P_1/2, P_r=P_h-P_1\)
2. solve the master problem in (22) by using the subgradient to update \(\lambda\)
3. until \(\lambda(t+1) = \left[ \lambda(t) - \zeta(t) \right] i(t) \leq t\)
4. Update (25) and (26)
5. While \(C_r (P_s, P_r) - C_d (P_r) \geq \varepsilon\) do
6. if \(C_r (P_s, P_r) > C_d (P_r)\) then
7. \(P_s=P_s - \mu\) where \(\mu\) is a steps by which the power is decreased.
8. else
9. \(P_r=P_r - \mu\)
10. endif
11. Output \(C_r (P_s, P_r)\)
12. end while
13. Output \(C_r (P_s, P_r)\)

**4.0 DISCUSSION OF RESULTS**

In this section, we present the Monte-Carlo simulation results to evaluate the performance of the proposed joint power allocation scheme in concatenated MIMO SR scheme. The joint power allocation scheme under network power constraint is compared with the uniform power constraint at each node. In the uniform power constraint, the power at each node is uniformly distributed within each antenna. We assume that slow faded channel where the channel distributions remain constant throughout the transmission length. In addition we assume that the selected transmit antennas can be used for transmission throughout the transmission length as long as the channel conditions do not change. In this simulation, we consider the following antenna configurations \(N_S=N_d=2\), while \(N_r=4, N_t=2\). It is also assumed that the \(P_s=P_r=1\) and the network power

\[
P_l=2.\text{ All nodes are assumed to have the same thermal noise coefficients, i.e.} \sigma_{nr}^2 = \sigma_n^2.
\]

Figure 2 shows the result when uniform power allocation is implemented at each node while each node is under an aggregate power constraint, i.e.

\[
P_s=P_r=1.\text{ The result is compared with the joint-HPA under NPC with a margin of } \varepsilon = 0.2. \text{ It can be observed that the optimal system capacity can further be improved. The improvement is also observed in Figure 3, where the outage probability of the optimal joint power allocation is compared with the uniform power allocation. In addition, the optimal power allocation scheme improves with increasing SNR.}

Finally, by jointly allocating the power to the source and relay nodes, the network power can be efficiently utilized assuming network power constraints.
5.0 CONCLUSIONS

This paper proposed an optimal joint power allocation scheme under network power constraints for a concatenated MIMO SR scheme. Although, an optimal antenna selection scheme which maximizes the relay destination link is implemented in the design, the joint power allocation scheme is observed to further improve the capacity of the system. This is due to the efficient allocation of power to the transmit nodes. Furthermore, the outage probability is also improved, thereby making the system even more reliable. Finally, the simulations show that the joint power allocation scheme under network power constraints can perform better than the uniform power allocation scheme.

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