

Calculation of the Modulation Transfer Function from the Square Wave Response Function data with an interactive curve fitting software

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Abstract

Two ways to calculate the modulation transfer function (MTF) of radiographic screen-film systems from the measured square wave response function (SWRF) data were investigated with an interactive curve fitting software. The measured SWRF data obtained by digitising a radiographic image of a bar pattern test object were fitted to a curve, and the fitted curve was used to calculate the MTF. Satisfactory MTF was obtained by using 12 terms in the calculation. A second version of the calculation included a correction for the normalization at 0.25 cycles/mm of the SWRF data. Measurements from a screen-film combination showed that the MTF of the first version was higher than the second by an average amount of 0.02 units for spatial range 0-3.5 cycles/mm, and on average the MTF of the first version was higher than the second by 10%. Both the SWRF data fitting and the MTF calculation were done within an interactive curve fitting software which made the calculation relatively easy to perform.

KEYWORDS: Medical radiography, modulation transfer function, screen-film system, square wave response function.

1. Introduction

The modulation transfer function (MTF) is commonly measured for an evaluation of the spatial resolution properties of screen-film systems [1]. There are at least three methods to measure the MTF namely, the slit, the square wave response function (SWRF), and the edge spread function methods.

In the SWRF method, a periodic square wave pattern of varying spatial frequency is used as an object and the contrast of the resulting image is determined experimentally. The ratios of the image contrast to the object contrast versus spatial frequency give the square wave response and these can be used to estimate the MTF using an equation quoted in a paper by Coltman [2]. We expand the equation up to the 15th term to obtain

$$R(u) = \frac{\pi}{4} \left[r(u) + \frac{r(3u)}{3} - \frac{r(5u)}{5} + \frac{r(7u)}{7} + \frac{r(11)}{11} - \frac{r(13u)}{13} - \frac{r(15u)}{15} - \frac{r(17u)}{17} + \frac{r(19u)}{19} + \frac{r(21u)}{21} + \frac{r(23u)}{23} - \frac{r(29u)}{29} + \frac{r(31r)}{31} - \frac{r(37u)}{37} - \frac{r(41u)}{41} \right] \quad (1)$$

where $r(u)$ and $R(u)$ are the square wave response

function and the MTF respectively, and u is spatial frequency.

Equation (1) implies that the obtainable modulation transfer factor values are restricted by the square wave response factors of a limited number of spatial frequencies available on the square wave pattern test object. This is further worsened by the fact that modulations of the highest spatial frequencies available in the test pattern cannot be determined due to noise interfering with the already small modulations. A way to deal with part of these problems is by fitting the measured square wave response factors to an analytic function, and using the resulting analytical function as the SWRF in Equation (1) to calculate the MTF. Morishita et al. [3], for example, employed the Fisher [4] equation for the purpose.

In this investigation, the measured SWRF data normalized at 0.25 cycles/mm were fitted to an analytical model [5] and the fitted analytical SWRF was used to calculate the MTF using 5 to 15 terms of Equation (1). The best number of terms to use was determined by examining the various MTF curves obtained.

It is important to have an accurately measured square wave response factor at the lowest spatial frequency possible, as the SWRF will be divided by the factor for normalization. Van Esch [6] estimated that using square wave response factor at 0.25 line pairs/mm instead of 0.025 line pairs/mm for the

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normalization will introduce systematic positive bias of 4% and 9% for sharp and unsharp screen-film systems respectively. For this, another version of the calculation considered correcting the normalization of the SWRF data that was done at 0.25 cycles/mm.

The purpose of this article is to investigate aspects regarding the calculation of the MTF from the SWRF data obtained from on site measurement in a hospital setting, using an interactive curve fitting software.

2. Materials and Methods

Type 53 PTW-Freiburg bar pattern (available from Facility for the Assessment of X-ray Imaging at Leeds University, The General Infirmary, Leeds) with 19 groups of line pairs of 0.05 mm lead thickness was used for the measurement. Each line pair group has four line pairs, except the first which has two. The groups are 0.25, 0.5, 0.6, 0.7, 0.85, 1.0, 1.2, 1.4, 1.7, 2.0, 2.4, 2.9, 3.5, 4.2, 5.0, 6.0, 7.0, 8.5, and 10 line pairs per mm. The bar pattern was placed on a Lanex Regular/T Mat G screen-film combination, and the screen-film combination was on a test rig 20 cm from the floor to reduce scatter. The exposures were made at 80 kVp with a 16 mm thick aluminium filter affixed to the X-ray tube window. The focus-to-film distance was 1.5 m. The set-up attempted to simulate the exit spectrum from human tissue in chest radiography, and the measurement was done in a hospital X-ray room using the facilities in the room. Thus the radiation quality was not similar to radiation quality used in standard MTF measurement.

The image of the bar pattern was scanned with a microdensitometer (Photoscan System P-1000, Optronics International Inc., Chelmsford, Massachusetts, USA) with a square aperture of side 12.5 mm, and sampling distance of 12.5 mm. From the image matrix, a computer program calculates the SWRF. Approximately 90 adjacent scan traces are averaged to give a one dimensional profile of the bar pattern image (optical density versus pixel position). The density values are converted to exposure values by the characteristic curve of the screen-film system. The modulation at each spatial frequency is determined, giving the SWRF. Finally, the SWRF is normalized by the square wave response factor at 0.25 line pairs/mm.

An MTF calculator was constructed using a curve fitting software which does a non-linear least square curve fitting via the Marquardt-Levenberg algorithm (see Appendix). Given a set of measured SWRF data (in the form of a text file containing two columns, with values of spatial frequency in the first column and values of normalized SWRF in the second), the calculator first fit the data to a model [5],

$$f(u) = \frac{c \cdot \exp(-\pi^2 u^2 / d) + a \cdot (1 + 4\pi^2 u^2 / b^2)^{-1}}{c + a} \quad (2)$$

where the parameters to be fitted are a , b , c , and d . The fitted curve was assumed as the SWRF, and the MTF was computed up to any required number of terms via Equation (1).

Another version of the calculation fits the normalized SWRF data to a modified equation

$$g(u) = e \cdot \left[\frac{c \cdot \exp(-\pi^2 u^2 / d) + a \cdot (1 + 4\pi^2 u^2 / b^2)^{-1}}{c + a} \right] \quad (3)$$

where the parameters to be fitted are a , b , c , d , and e . The analytical SWRF to be used in the MTF calculation via equation (1) was then taken as

$$f(u) = \frac{g(u)}{e} \quad (4)$$

This was thought would correct the SWRF that was normalized at 0.25 cycles/mm by extrapolating the fitted curve to zero frequency and normalizing the SWRF by the extrapolated SWRF value at the frequency.

3. Results and discussion

A set of SWRF data points measured from the screen-film combination was obtained. These SWRF data did not represent optimized data usually used for conventional MTF measurement, but represented data obtained by our measurement on site in a hospital X-ray room. As we only wanted to investigate the calculation aspect of the MTF, these data were enough for our purpose. Figure 1 and its close up (Fig. 2a and b) show the set of measured SWRF data points, its fit $[f(x)]$, and the various MTF curves computed by the calculator. For the curve fitting, the final fit parameters were $a = 0.146203$, $b = 7.70645$, $c = 0.0153656$, and $d = 14.1664$. No seeds constraining strategy for the curve fitting work was attempted, and we are not clear how the strategy and the problem of overparameterization might affect the result. This needs further investigation. In this work only convergence as offered by the software and visual inspection of the data against the fitted curve were used as criteria for a good fit. The MTF curves obtained by using different number of terms in Equation (1) are $mtf5(x)$, $mtf6(x)$, ... , $mtf15(x)$, where they are the MTF calculated up to the 5th, 6th, ..., 15th term of the equation respectively, and x is the spatial frequency u . We used x instead of u in the software environment because the software accepts only x as the independent variable. Figure 2a and b clearly show that the MTF curves at low spatial frequencies fluctuate with the number of terms used to calculate them.

Table I gives the MTF values at zero frequency for different number of terms used in Equation (1).

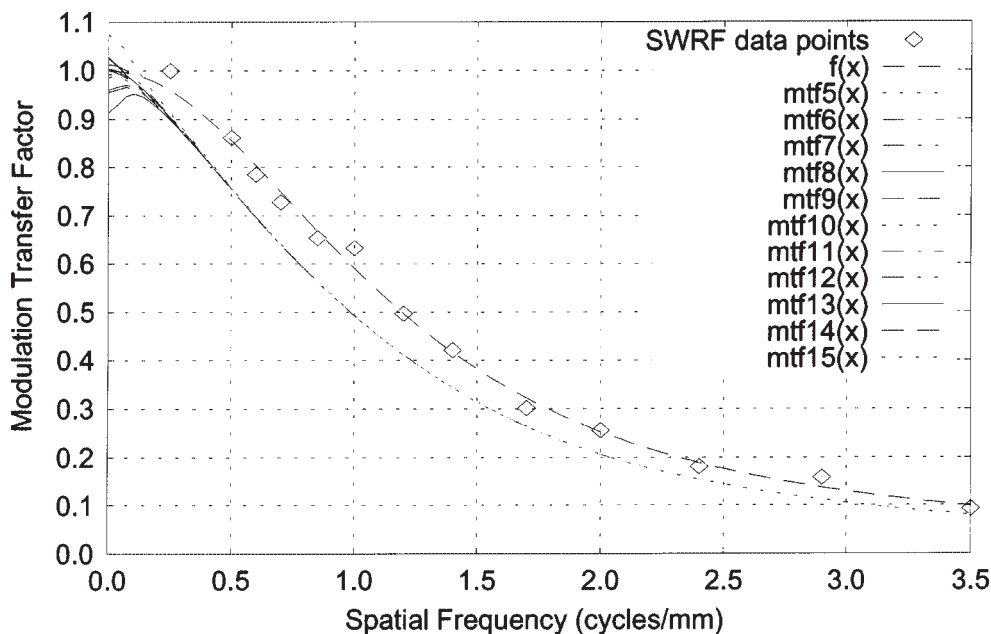


Fig. 1 – A set of measured SWRF data points, its fit $[f(x)]$, and the MTF curves computed by the calculator using 5 to 15 terms of Equation (1) $[mtf5(x), mtf6(x), \dots, mtf15(x)]$.

The normalized MTF at zero frequency is unity, therefore the curve which has zero frequency MTF value that is closest to unity is the best curve. Table I also gives the ranking in terms of the value which is closest to unity. Hence, the best curve is $mtf12(x)$, followed by $mtf14(x)$, $mtf10(x)$, $mtf6(x)$, $mtf15(x)$, $mtf13(x)$, $mtf11(x)$, $mtf7(x)$, $mtf9(x)$, $mtf5(x)$ and $mtf8(x)$. Thus when 5 to 15 terms of Equation (1) were considered, a satisfactory estimate of the MTF could be obtained by using 12 terms.

An implication of the above result is that a systematic error might be introduced if one uses a small number of terms and ‘normalizes’ the MTF obtained via Equation (1) by dividing it with the zero frequency MTF, i.e., the normalized MTF is taken as $R(u)/R(0)$. For example, if 9 terms of Equation (1) are used, the value of $mtf9(0)$ is 0.95608 and the systematic error is a positive increase in the MTF of $(1-0.95608)/0.95608 \times 100\% = 4.6\%$ at all spatial frequencies. It is clear that Equation (1) must be used in its absolute form, division by the zero frequency MTF must be avoided. When a small number of terms are used, only the MTFs at spatial frequencies less than about 0.1 cycle/mm are prone to error.

In Figure 3 the measured SWRF data points, its fit according to Equation (2) $[f(x)]$, and the MTF computed using the calculator up to the 12th term $[mtf12(x)]$ are shown once again for clarity. A curve described by Equation (2) is inherently normalized at zero spatial frequency (its value is 1.0 at zero spatial frequency). Thus the fitted curve $f(x)$ runs under the normalized SWRF point (0.25, 1.0). These are results of the first version of the calculation.

Figure 4 shows the same set of measured SWRF data points, its modified fit according to Equation

(3) $[g(x)]$, its corrected SWRF according to Equation (4) $[f(x)]$, and the MTF computed using the calculator up to the 12th term $[mtf12(x)]$. For this modified fit, the final fit parameters were $a = 5.99709$, $b = 5.10619$, $c = -2.32603$, $d = 5.21272$, and $e = 1.07624$. Here the fitted curve $g(x)$ runs over point (0.25, 1.0) and has a value greater than 1.0 at zero frequency. It is not clear whether it is valid to extrapolate the SWRF by assuming a model of SWRF as per Equation (3), and it is less clear how to attack this problem. The ‘corrected’ SWRF $[f(x)]$ was obtained by dividing $g(x)$ with e . These are results of the second version of the calculation.

Figure 5 compares the MTF obtained via the first and the second versions of the calculation. For the spatial frequency range 0-3.5 cycles/mm, the MTF of the first version was higher than the second by an average amount of 0.02 units, and on average the MTF of the first version was higher than the second by 10%. Results of Morishita et. al. [3], obtained from four institutions by the SWRF method are also shown in the figure. Their results are higher than ours, and this might be due to different X-ray radiation qualities used in the measurement. Further study is needed to validate our work by determining the MTF using radiation qualities of the standard MTF measurement and comparing it with published MTF results.

Droegge and Morin [7] have suggested a method to determine the MTF of CT scanners that relies upon the measurement of the standard deviation of the pixel values within the image of cyclic bar patterns. This method has not been extended to conventional screen-film radiography due to the fact that relatively large and many different areas of film have to be digitized by the microdensitometer before

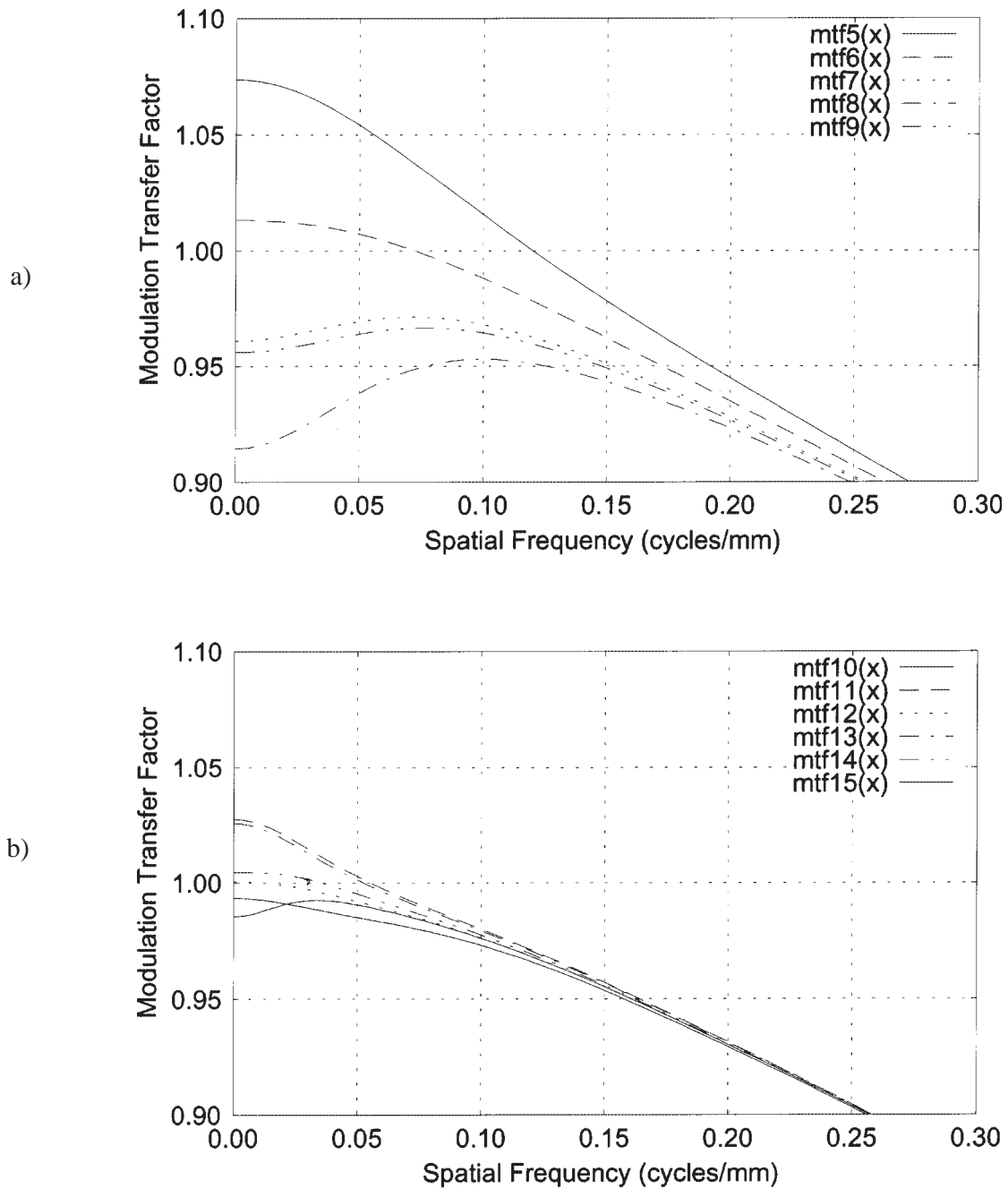


Fig. 2 – A close up of Figure 1 showing the MTF curves at low spatial frequencies fluctuate with the number of terms used in the calculation. For convenience, the first 5 curves [mtf5(x), mtf6(x), ..., mtf9(x)] are shown in (a), while the remaining [mtf10(x), mtf11(x), ..., mtf15(x)] are in (b). The best result was obtained by using 12 terms of Equation (1) [mtf12(x)].

the standard deviation of pixel values of the areas could be calculated. With the advent of CCD film scanners a study in this direction is worth trying, however, the typical resolution of a CCD film scanner is around 50 microns as opposed to 10 microns for a microdensitometer. Furthermore, in Type 53 PTW-Freiburg bar pattern, the bars of different line pairs per mm are arranged one after another. This makes it very difficult to select an area at the high frequency end of the pattern for the calculation of the standard deviation.

We believe the first version is appropriate when

one has the square wave response factor at a relatively low spatial frequency, for example at 0.025 cycles/mm, obtained from the bar pattern test object. The second version would be useful in one needs to correct the SWRF data due to unavailability of a SWRF value at a low enough spatial frequency from the bar pattern test object.

4. Conclusions

Two ways to calculate the MTF of a radiographic screen-film combination from the measured SWRF

Table I – Results of the calculation using different number of terms in Equation (1).

Number of terms used in Equation (1)	Modulation transfer function value at zero	Ranking of modulation transfer function value at zero frequency that is closest to unity
5	mtf5(0) = 1.07372	10th
6	mtf6(0) = 1.0133	4th
7	mtf7(0) = 0.960942	8th
8	mtf8(0) = 0.914742	11th
9	mtf9(0) = 0.956079	9th
10	mtf10(0) = 0.993479	3rd
11	mtf11(0) = 1.02763	7th
12	mtf12(0) = 1.00054	1st
13	mtf13(0) = 1.02588	6th
14	mtf14(0) = 1.00465	2nd
15	mtf15(0) = 0.985497	5th

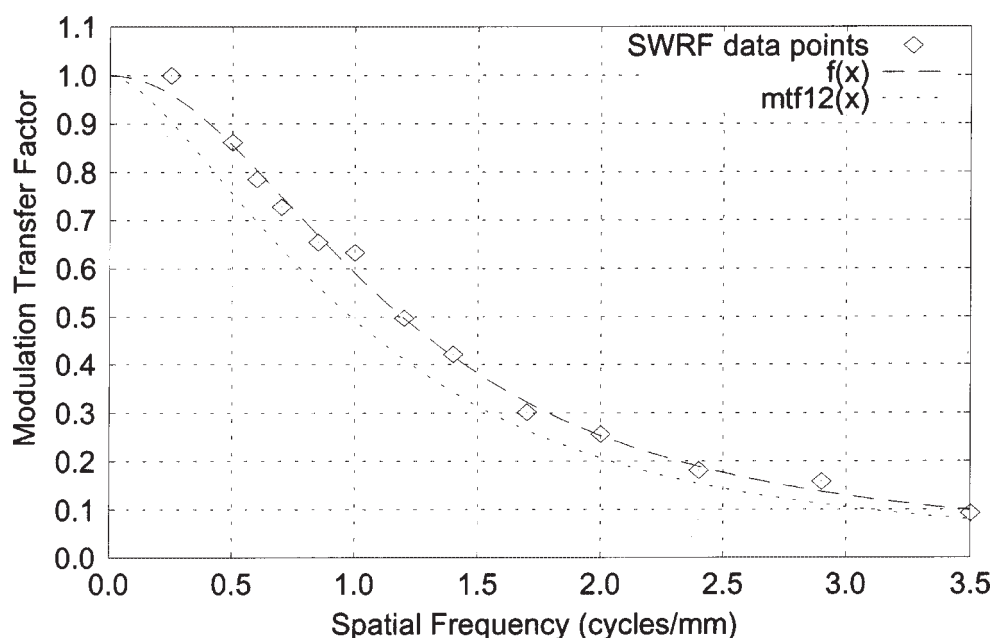


Fig. 3 – The results of the first version of the calculation showing the set of measured SWRF data points, its fit [f(x)], and the MTF computed by the calculator using 12 terms of Equation (1) [mtf12(x)].

data in a hospital environment were investigated. The measured SWRF data were fitted to a curve and the fitted curve gave the analytical SWRF to be used in the calculation of the MTF up to 15 terms. The MTF curves at low spatial frequencies (less than about 0.1 cycle/mm) fluctuated with the number of terms used in the calculation, and the best result was obtained by using 12 terms. A second version of the calculation corrected for the normalization of the SWRF data that was done at 0.25 cycles/mm. The results from measured SWRF data a screen-film combination showed that the MTF of the first version was higher than the second by an average amount of 0.02 units for spatial

frequency 0-3.5 cycles/mm, and on average the MTF of the first version was higher than the second by 10%. Both the SWRF data fitting and the MTF calculation were done within an interactive curve fitting software which made the calculation relatively easy to perform.

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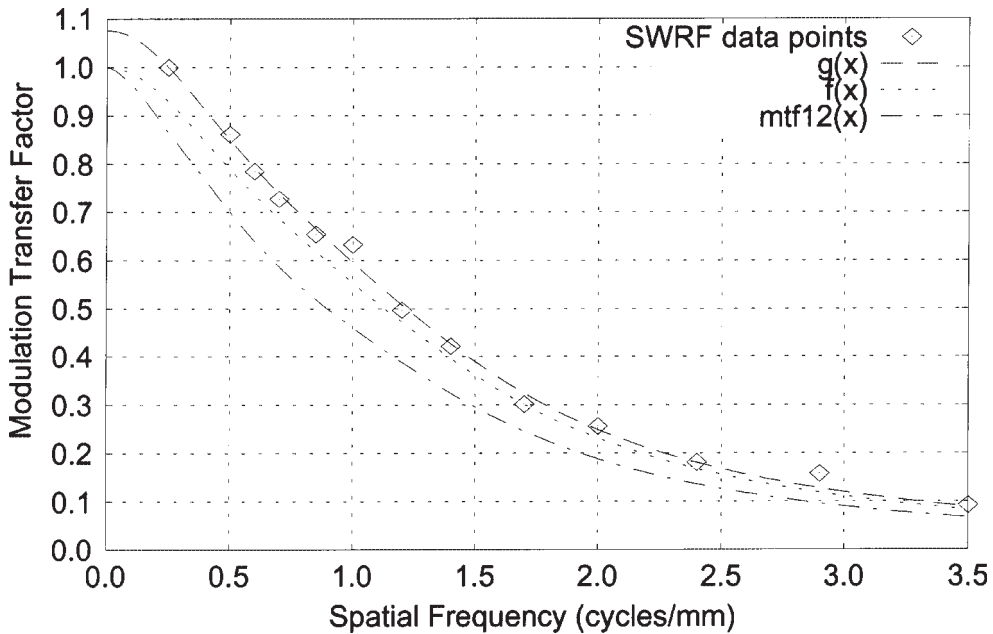


Fig. 4 – The results of the second version of the calculation showing the set of measured SWRF data points, its modified SWRF fit [$g(x)$], its corrected SWRF [$f(x)$], and the MTF computed by the calculator using 12 terms of Equation (1) [$mtf12(x)$].

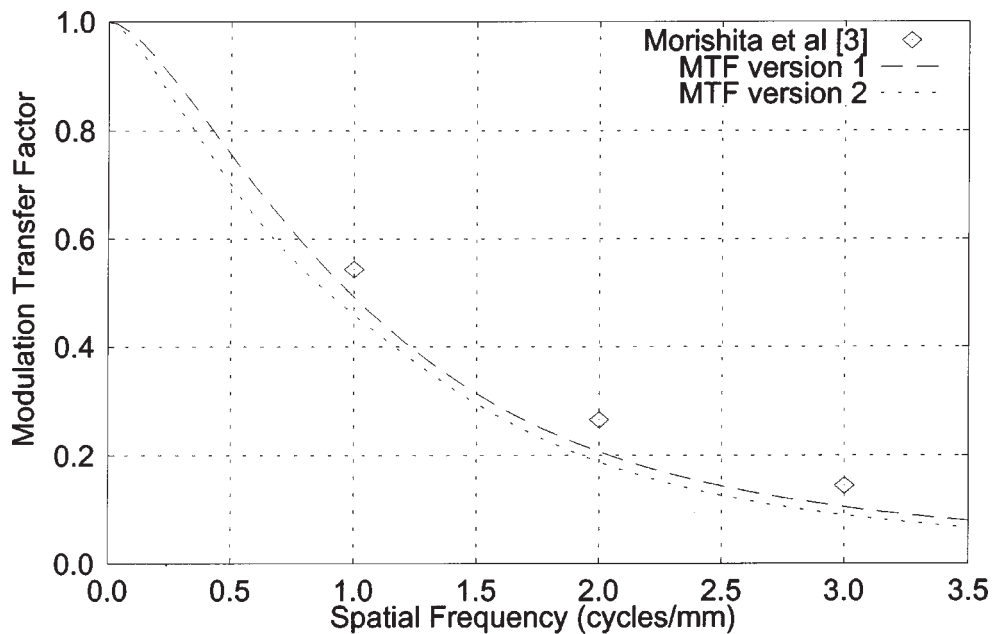


Fig. 5 – A comparison of the MTF obtained from the first and second versions of the calculation. The MTF from the second version was smaller than the first version by an average value of 0.02 units. Results of Morishita et al. [3], obtained from four institutions by the SWRF method are also shown in the Figure.

Appendix

The MTF calculator uses an interactive curve fitting software (Gnufit version 1.2, by C. Grammes, available from WWW site <http://src.doc.ic.ac.uk/packages/gnuplot>, file to download: gft12dos.zip. Newer version might also be used, gp371cyg.zip). The calculator was implemented as follow:

1. Prepare a normalized SWRF data in the form of a computer ASCII file (call it swrf.dat) consisting 2 columns, the first column (x column) consists of the spatial frequency values, the second column (y column) consists of the SWRF values.

2. Load the Gnufit program.

3. Define the following functions and variables:

```
f(x)=(c*exp(-pi**2*x**2/d)+a/(1.+4.*pi**2*x**2/b**2))/(c+a)
g(x)=e*((c*exp(-pi**2*x**2/d)+a/(1.+4.*pi**2*x**2/b**2))/(c+a))
m5(x)=f(x)+f(3.*x)/3.-f(5.*x)/5.+f(7.*x)/7.+f(11.*x)/11.
m9(x)=m5(x)-f(13.*x)/13.-f(15.*x)/15.-f(17.*x)/17.+f(19.*x)/19.
m12(x)=m9(x)+f(21.*x)/21.+f(23.*x)/23.-f(29.*x)/29.
mtf12(x)=m12(x)*pi/4.
a = 0.34
b = 4.42
c = 1.54
d = 260.0
e = 1.0
```

In these functions x stands for the spatial frequency in cycles/mm. Thus $f(x)$ represents $f(u)$ in Equation (2), or $r(u)$ in Equation (1). Furthermore, $mtf12(x)$ represents $R(u)$ to the 12th term. The initial guesses of the fit parameters were taken from Boone and Seibert [5]. Other values might work as well.

4. Fit the SWRF data:

```
fit f(x) 'swrf.dat' via a, b, c, d
```

If the operation fails to converge, redefine one or any number of the variables to new values, and repeat 4. The values of the variables can be displayed by the command `show variables`. For the second version of the calculation use

```
fit g(x) 'swrf.dat' via a, b, c, d, e
```

5. Plot the SWRF data and its fit:

```
plot 'swrf.dat', f(x)
```

If you are satisfied with the fit, do 6. Otherwise redefine one or any number of the variables to new values, and repeat 4. For the second version of the calculation the command is

```
plot 'swrf.dat', g(x)
```

6. Plot the MTF:

```
set xrange [0:3.5]
plot mtf12(x)
```

Numerical value of the MTF at any spatial frequency can be obtained by the print command. To get the MTF at 2 cycles/mm, type:

```
print mtf12(2)
```

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