ENHANCING HIGHER ORDER THINKING SKILLS THROUGH MATHEMATICAL THINKING IN AN OUTSIDE CLASSROOM LEARNING ENVIRONMENT: A THEORETICAL FRAMEWORK

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Abstract

Emphasis on enhancing students' higher order thinking skills (HOTS) has been one of the objectives of Malaysian education system. The success of HOTS depends upon an individual's ability to create complex ideas, reorganize and embellish knowledge in the context of thinking situation. Generating HOTS in learning Mathematics starts from the process itself involving various processes of mathematical thinking. However, the inculcation of HOTS using mathematical thinking in normal Malaysian *classroom setting is rather limited and often inadequate. Furthermore, it is much less practised in an outside classroom environment. Therefore, learning activities which can promote the inculcation of mathematical HOTS should be developed and implemented in the process of teaching and learning of mathematics. This paper reports an attempt to design and develop a framework aimed at promoting mathematical HOTS among Malaysian secondary schools. The framework uses appropriate questions and prompts to support each of the four Mason's mathematical thinking processes practised in an outside classroom environment.*

1.0 Introduction

Promoting higher order thinking skills (HOTS) recently become serious agenda in Malaysia education system. Students' thinking skills automatically comes from the learning process itself. Even though numbers of literatures which support this goal seem to be growing, to systematically apply thinking strategies in learning Mathematics has yet to be scrutinized. Thus, a proper theoretical framework is needed and it is common in educational research where, for every learning process created, it must in line and backed with specific educational theories as those theories should be able to describe how the learning process takes place. We will first look into how HOTS is supported by the mathematical thinking and outside classroom learning environment, respectively as to achieve the target of this research study that is enhancing HOTS in learning Mathematics.

2.0 Higher Order Thinking Skills in Mathematics

HOTS is at highest level in cognitive hierarchy. It has been defined by researchers with different definition. HOTS does not involves with algorithm process, it is complex and variety of solution (Resnick; 1987), ability to think critically, logically and creatively (King, Ludwika and Rohani, 1998) and involves with analyzing, evaluating and creating processes (Anderson and Krathwohl, 2001; Madhuri, Kantamreddi, and Prakash, 2012;

Ramirez and Ganaden, 2008; Zohar and Dori, 2003).

In Malaysia, Bloom's Taxonomy be as a guidance for teachers in teaching and learning especially in assessment. Teachers need to create the items based on six cognitive levels namely knowledge, understand, application, analysis, synthesis and evaluation (Bloom, 1956). In 2001, Anderson & Krathwohl have revised the taxonomy to remembering, understanding, applying, analyzing, evaluating and creating. Analyzing, evaluating and creating known as higher order thinking.

The inculcation of HOTS in schools has been implementing in various aspects such as pedagogy, teaching and learning method as well as assessment. Pedagogy itself includes a sort of methods in promoting HOTS (Goethals, 2013). Moreover, problem solving activities in groups may enhancing HOTS (Aizikovitsh., 2012; Barak and Dori, 2009). The activities should be related with real life problems in order to achieve effective learning process (Zohar and Dori, 2003). This can gives the chance among students to argue, question, critic and build new concepts through self-exploration.

In learning Mathematics, HOTS synonym with mathematical thinking since it requires conjecturing, reasoning and proving, abstraction, generalization and specialization (Burton, 1984; Mason, Burton and Stacey (2010); Schoenfeld, 1992, 1994).

3.0 Mathematical Thinking

Numerous definition of mathematical thinking and it is depends on aim and how it is used. Schoenfeld (1992, 1994) proposed five cognitive levels in mathematical thinking and problem solving namely the knowledge base, problem solving strategies, monitoring and control, beliefs and affects and practices. He stressed that through problem solving activities, mathematical thinking can be inculcated directly. When a student be able to solve the problems, he also achieved good thinking skills.

However, according to Tall (2004) mathematical thinking involves with conceptual-embodied, proceptual-symbolic and axiomatic-formal known as Three World of Mathematics. Although, this theory focus on algebra and calculus in higher education level besides less axiomatic-formal syllabus in secondary education while Schoenfeld's framework more focus on overall problem solving process including beliefs and affects.

Mason *et al*., (2010) defined mathematical thinking as a dynamic process which, by enabling us to increase the complexity of ideas we can handle and expands our understanding. In order to achieve mathematical thinking, Mason *et al.* (2010) have been proposed four processes namely specializing, generalizing, conjecturing and convincing. These processes guide students in how they solve questions, tasks or problems given through questioning strategies by teachers. This atmosphere will provokes students to keep thinking and it will leads them to HOTS. Furthermore, Mason's mathematical thinking lead to a deeper understanding of ourselves as well as more critical assessment of what we hear and see. This will reflects to mathematical concepts they have learnt.

In Malaysia, mathematical thinking be one of the aims in curriculum of Mathematics. It is refer to learning mathematics effectively through problem solving and decision making (MOE, 2003b). Mathematical thinking was taught as relate to higher order thinking, critical and analytical thinking as well as problem solving. Even though, teachers are not familiar with mathematical thinking, they still have an idea that mathematical thinking is somehow related with higher order thinking. This because the word 'mathematical thinking' was not stated explicitly in the Malaysian mathematics syllabus (Lim and Hwa, 2006). Furthermore, inculcation of HOTS using mathematical thinking in Malaysian classroom is rather limited due to no clear understanding about mathematical thinking, examination oriented culture, 'finish syllabus syndrome' and lack of appropriate instrument (Lim and Hwa, 2006). Therefore, there is a need to have much more empirical study focusing on mathematical thinking in Malaysian classroom.

This study is focus on learning activities development which using mathematical thinking strategy in enhancing analysing, evaluating and creating skills which known as HOTS (Anderson and Krathwohl, 2001). Therefore, Mason's mathematical thinking seems as the suitable framework in developing tasks and learning activities.

4.0 The Potential of Using Mason's Mathematical Thinking in an Outside Classroom Learning Environment

Roselainy (2009) used the ideas of mathematical thinking as proposed by Mason, Burton and Stacey (1982). In presenting those ideas, Mason's mathematical thinking focused on four processes: specializing, generalizing, conjecturing and convincing. Specializing requires students to turning the questions or problems into familiar situations or else close to their understanding in order to create feeling of confidence and ease in otherwise unfamiliar situations. Generalization is the ability to recognize those patterns and making an attempt in expressing it mathematically. It starts when the students sense an underlying pattern, even if they cannot articulate it while conjecturing involves with giving statement which appears reasonable. In learning mathematics, students are encourage to justify their answers and solutions. This can be injected and assessed through questions and prompts. Finally, when the students be able to convince themselves, a friend and an enemy shows that they have completely achieve mathematical thinking process.

Although mathematical thinking studies seems growing, most of them were implementing in the classroom setting (Kashefi, Zaleha and Yudariah, 2009; Roselainy, 2009; Yudariah and Roselainy, 2004). In addition, Khasefi, Zaleha and Yudariah (2012) stated that poor prior knowledge and poor mastery basic mathematics' concept were the reason behind student difficulties in problem's solving. He suggested it was necessary to use new strategies and tools when teaching students with a wide variance in their preparation and abilities.

In conjunction to inculcate mathematical HOTS and application of problem solving strategies as well as the deeper understanding of mathematics concept, learning in an outside classroom environment brings an opportunity for students to see mathematics as cross-curricular (Ofsted, 2008). It gives greater curiosity leading to more effective exploration and creative ideas driving investigations. Students not only experience mathematics in concrete and novel settings, but can be liberated from the sometimes restrictive expectations of the classroom setting.

From this mathematics learning experiences, it enhances the process of thinking through the tasks or problems given. Discussion among the students while solving the problems or activities require the mathematical reasoning and creative thinking (Milrad, 2010). It is supported by Sollervall, Otero, Milrad, Johansson, and Vogel (2012), where outside classroom learning may enhance the student's mathematical HOTS. According to Jordet (2010), students engage in practical outside classroom activities, they learn by doing and dealing with a concrete 'real-life' context. This differs from more abstract classroom situation. He proposed in his model of characteristics of school based outdoor learning, implications for pedagogy involve with problem solving, explorative, practical, constructive, creative and playful. The potential for learning outside the classroom is seen to enhance mathematics learning process more effectively (Fägerstam and Samuelsson, 2012). Learning through the outside classroom environment may give the opportunity for student to enhance the process of mathematical thinking when they are free to solve the tasks and problems given.

In addition, Mason *et al.*, (1982) stressed that suitable learning atmosphere which encourages students in promoting their mathematical thinking and freethinking classroom context is necessary. This context are suitability for questioning, convenience for expressing thoughts and assurance for the challenge. Thus, a sufficient questions and prompts are needed to support mathematical thinking in a systematic and organized manner.

5.0 The Needs of Questions and Prompts

As we all know, thinking happens so fast in one's mind and often we found ourselves come out with the idea or suggestion without even notice how we actually arrived to that point. Thus, through series of questioning oneself on related aspects of experiences which aims to create confusion and conflict. By doing so, we will then have clear understanding on how one can reaches to a conclusion and able to gain some rationale justification along with it, thus the same strategy is applicable to be used in the future encounter of related questions or problem.

Therefore, in proposing strategies to provoke students to become aware of mathematical thinking processes, Watson and Mason (1998) described a framework to generate and organized generic questions which can be asked about mathematical topics in various context. To think mathematically and then HOTS demand the use of appropriate strategies of questioning (Watson and Mason, 1998). They have been listed complete verb in solving mathematical problem: exemplifying, specializing, completing, deleting, correcting, comparing, sorting, organizing, changing, varying, reversing, altering, generalizing, conjecturing, explaining, justifying, verifying, convincing and refuting. These verbs gives better chance for teachers in organizing their questioning strategies. Each processes in mathematical thinking supported by questions and prompts. Thus, these complete verbs have been divided into six heading:

1)Exemplifying, Specializing
2)Completing, Deleting, Correcting
3)Comparing, Sorting, Organizing
4)Changing, Varying, Reversing, Altering
5)Generalizing, Conjecturing
6)Explaining, Justifying, Verifying, Convincing, Refuting

Conjunction to elements in HOTS which are focused in this study (analyzing, evaluating and creating), these six heading will helps in enhancing HOTS through the questions and prompts designed.

Apart from that, the learning activities using three phases in provoking mathematical thinking. They are Entry Phase, Attack Phase and Review Phase (Mason *et al.*, 2010). The learning activities designed based on solving tasks or problems and these phases may help students in completing the tasks along with the questions and prompts given. It was supported by Roselainy and her colleagues (Yudariah and Roselainy, 2004; Roselainy, Yudariah and Mason, 2005) which used mathematical themes through speacially designed questions and prompts. It was provided linkages between mathematical ideas, to expose the structures of the mathematics, and to support students' generic skills.

Thus, questions and prompts will support mathematical thinking in order to guide in designing and developing learning activities in this study.

6.0 Proposed Theoretical Framework to Enhance Higher Order Thinking Skills

With the significant potential can be obtained from the academic community and eventually led to the empowerment of students, mathematical thinking processes can bring more benefit in outside classroom learning environment. Thus, this paper aims to suggest a framework on how mathematical thinking (Mason *et al.*, 2010) supported by Questions and Prompts (Watson and Mason, 1998) in an outside classroom environment (Jordet, 2010) to enhance HOTS among Malaysian secondary schools. The following figure represents the theoretical framework as suggested by this research study.

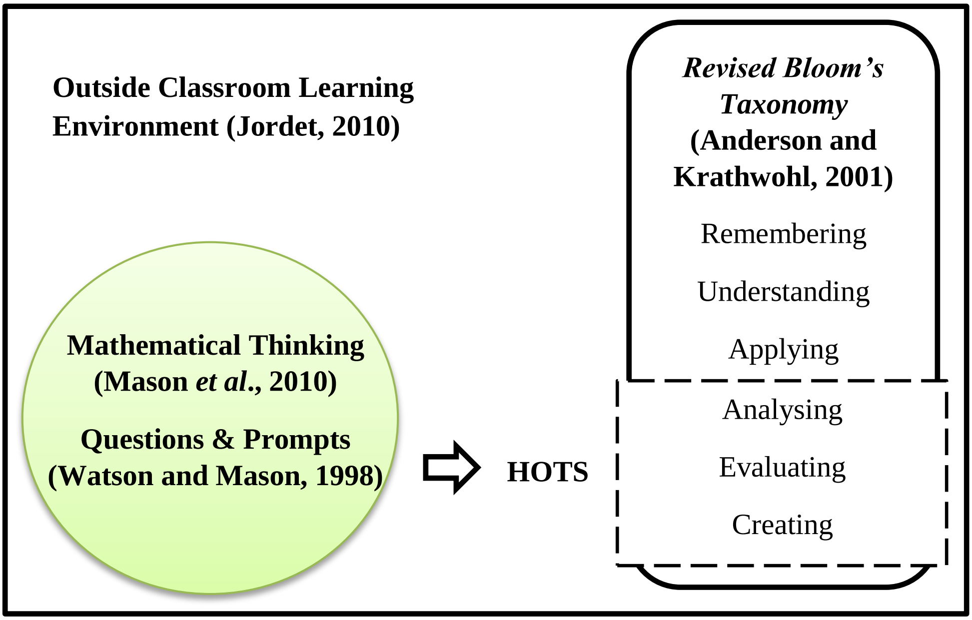

Figure 1 The Proposed Theoretical Framework

7.0 Conclusion

In summary, this article has described in detail regarding the theoretical underpinning of the selected pedagogical strategies and how they are blended together as a basis to carry out the research study. It is cited that "theory matters because without it education is just hit and miss…we risk misunderstanding not only the nature of our pedagogy but the epistemic foundations of our discipline" (Webb, 1996). In relation to this, it is hoped that if the proposed theories blended well on a paper, it will also work effectively in practice, where the HOTS using Mason's mathematical thinking in an outside classroom learning is expected to be achieved.

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