SEQUENCES AND CUBES OF
FINITE VERTICES OF FUZZY
TOPOGRAPHIC TOPOLOGICAL MAPPING

AZRUL AZIM MOHD YUNUS

A thesis submitted in fulfilment of the requirements for the award of the degree of Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

SEPTEMBER 2015
To my beloved parents Mohd Yunus Razali and Ozairah Zakaria who are a great source of joy and inspiration to me
ACKNOWLEDGEMENT

First of all, my heart and soul say thanks to ALLAH (SWT), the Lord Almighty, for His generosity to give me the health, strength and ability to complete this thesis.

I wish to express my deepest gratitude to my supervisor, Prof. Dr. Tahir Ahmad, who has suggested the research topic and directed the research. I thank him for his enduring patience.

I further wish to express my indebtedness to Universiti Sains Islam Malaysia for the financial support during the tenure of this research. This work was also supported in part by the Malaysian Ministry of Higher Education (MOHE) through the Research Management Centre (RMC), Universiti Teknologi Malaysia (GUP Q.J130000.3026.00M30).

Finally, I would like to convey my sincerest gratitude to my family members, especially my father Mohd Yunus bin Razali and my mother Ozairah bt. Zakaria for their support, and my relatives, friends and colleagues.
ABSTRACT

Fuzzy Topographic Topological Mapping (FTTM) was first developed by Fuzzy Research Group (FRG) of UTM. FTTM is a novel method for solving neuromagnetic inverse problems to determine the current source, i.e. epileptic foci in epilepsy disorder patient. FTTM consists of four components which are connected by three algorithms. FTTM is specially designed to have equivalent topological structures between its components. In addition, FTTM was generalized as a set of vertices which led to infinitely many forms of FTTM. This includes the possibilities of finite vertices of FTTM. In this research, the structure for finite vertices of FTTM, namely FK where K represents the number of vertices is established. Firstly, the sequences of FK, given by FK_n are constructed as sequences of polygons. In this process, geometrical and algebraic structures for some FK_n are obtained and proven in this thesis. Some patterns on FK_n are observed and defined recursively. Several new features for sequences of FK_n are introduced, such as sequence of vertices, sequence of faces, and sequence of cubes. Consequently, some theorems are proven in order to describe patterns for the sequence of cubes for FK_n. Interestingly, the cube of FK_n appears to be an example of generalized Fibonacci sequence, namely the k-Fibonacci sequence. Furthermore, the number of new elements produced from the combination of sequences of FK_n can be expressed as a combination of cubes of FK_n.
ABSTRAK

# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DECLARATION</td>
<td></td>
<td>ii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td></td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td></td>
<td>iv</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td></td>
<td>v</td>
</tr>
<tr>
<td>ABSTRAK</td>
<td></td>
<td>vi</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td></td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td></td>
<td>xii</td>
</tr>
<tr>
<td>LIST OF ABBREVIATIONS</td>
<td></td>
<td>xiv</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td></td>
<td>xv</td>
</tr>
<tr>
<td>LIST OF APPENDICES</td>
<td></td>
<td>xvi</td>
</tr>
<tr>
<td>1 INTRODUCTION</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Background and Motivation</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1.2 Research Background</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>1.3 Problem Statement</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>1.4 Research Objectives</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>1.5 Scope of the Study</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>1.6 Significance of Findings</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>1.7 Research Methodology</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>1.7.1 Constructive</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>1.7.2 Difference Equation</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>1.7.3 Mathematical Induction</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>1.8 Thesis Organization</td>
<td></td>
<td>8</td>
</tr>
</tbody>
</table>
2 LITERATURE REVIEW 11
2.1 Introduction 11
2.2 Fuzzy Topographic Topological Mapping 11
   2.2.1 FTTM Version 1 12
   2.2.2 FTTM Version 2 12
2.3 Different Version of FTTM 13
2.4 Homeomorphisms of FTTM Version 1 and FTTM Version 2 14
2.5 Generalized FT TM 14
2.6 Illustration of Sequence of FT TM 16
2.7 Pascal’s Triangle 22
2.8 FT TM as Pascal’s Triangle 24
2.9 Fibonacci Sequence and Fibonacci Cubes 25
2.10 $k$-Fibonacci Sequences 26
2.11 Conclusion 28

3 SOME SEQUENCES OF FINITE VERTICES OF FT TM 29
3.1 Introduction 29
3.2 Sequence of $F^2_n$ 29
3.3 Sequence of $F^3_n$ 34
3.4 Sequence of $F^4_n$ 40
3.5 Sequence of $F^5_n$ 49
3.6 Some Definitions on Cube of $FK_n$ 58
3.7 Conclusion 60

4 SOME PROPERTIES OF SEQUENCES OF FINITE VERTICES OF FT TM 61
4.1 Introduction 61
4.2 Sequence of Edges and Faces For $FK_n$ 61
4.3 Conclusion 76
5 CUBES OF FINITE VERTICES OF FTTM IN RELATION TO $k$-FIBONACCI SEQUENCE 77

5.1 Introduction 77

5.2 Generalization of Sequence of Cubes Via $k$-Fibonacci Numbers 78

5.3 Conclusion 94

6 THE RELATION BETWEEN SEQUENCE OF $FK_n$ AND CUBE OF $FK_n$ 95

6.1 Introduction 95

6.2 Elements of $FK_n$ 95

6.2.1 Sequence of $F2$ 95

6.2.2 Sequence of $F3$ 97

6.2.3 Sequence of $F4$ 100

6.3 Relation Between Sequence of $FK_n$ and Cube of $FK_n$ 100

6.4 Coefficients for $FK_n$ 101

6.5 Conclusion 106

7 CONCLUSION 107

7.1 Conclusion 107

7.2 Recommendations 109

REFERENCES 110

APPENDIX A 115
# List of Tables

<table>
<thead>
<tr>
<th>TABLE NO.</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Vertices, edges, faces and cubes for sequences of FTTM for $n = 1, 2, 3, \ldots, 10$</td>
<td>22</td>
</tr>
<tr>
<td>3.1</td>
<td>Schematic diagram of $F2_n$ for $n = 1, 2, 3, \ldots, 4$</td>
<td>31</td>
</tr>
<tr>
<td>3.2</td>
<td>The number $eF2_n$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>32</td>
</tr>
<tr>
<td>3.3</td>
<td>The number of $F2_{2/n}$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>33</td>
</tr>
<tr>
<td>3.4</td>
<td>The number of vertices, edges, and squares for sequences of $F2_n$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>34</td>
</tr>
<tr>
<td>3.5</td>
<td>Schematic diagram of $F3_n$ for $n = 1, 2, 3, 4.$</td>
<td>36</td>
</tr>
<tr>
<td>3.6</td>
<td>The number of $eF3_n$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>37</td>
</tr>
<tr>
<td>3.7</td>
<td>The Number of $fF3_n$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>38</td>
</tr>
<tr>
<td>3.8</td>
<td>The number of $F3_{2/n}$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>39</td>
</tr>
<tr>
<td>3.9</td>
<td>The number of $F3_{3/n}$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>40</td>
</tr>
<tr>
<td>3.10</td>
<td>The number of vertices, edges, faces and triangles for sequences of $F3_n$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>41</td>
</tr>
<tr>
<td>3.11</td>
<td>Schematic diagram of $F4_n$ for $n = 1, 2, 3, 4.$</td>
<td>42</td>
</tr>
<tr>
<td>3.12</td>
<td>The Number of $eF4_n$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>44</td>
</tr>
<tr>
<td>3.13</td>
<td>The Number of $fF4_n$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>45</td>
</tr>
<tr>
<td>3.14</td>
<td>The number of $F4_{2/n}$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>46</td>
</tr>
<tr>
<td>3.15</td>
<td>The number of $F4_{3/n}$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>47</td>
</tr>
<tr>
<td>3.16</td>
<td>The number of $F4_{4/n}$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>47</td>
</tr>
<tr>
<td>3.17</td>
<td>The number of vertices, edges, faces and cubes for sequences of $F4_n$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>48</td>
</tr>
<tr>
<td>3.18</td>
<td>Schematic diagrams of $F5_n$ for $n = 1, 2, 3, 4.$</td>
<td>51</td>
</tr>
<tr>
<td>3.19</td>
<td>The number of $eF5_n$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>52</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>---------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.20</td>
<td>The Number of $f F5_n$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>53</td>
</tr>
<tr>
<td>3.21</td>
<td>The number of $F5_{2/n}$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>54</td>
</tr>
<tr>
<td>3.22</td>
<td>The number of $F5_{3/n}$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>55</td>
</tr>
<tr>
<td>3.23</td>
<td>The number of $F5_{4/n}$ For $n = 1, 2, 3, \ldots, 10$</td>
<td>55</td>
</tr>
<tr>
<td>3.24</td>
<td>The number of $F5_{5/n}$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>56</td>
</tr>
<tr>
<td>3.25</td>
<td>The number of vertices, edges, faces and cubes for sequences of $F5_n$ for $n = 1, 2, \ldots, 10$</td>
<td>57</td>
</tr>
<tr>
<td>5.1</td>
<td>Sequence of $FK_{3/n}$ for $n = 1, 2, 3, \ldots, 10$</td>
<td>78</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Human brain</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Neuromagnetic field</td>
<td>2</td>
</tr>
<tr>
<td>1.3</td>
<td>MEG Systems</td>
<td>3</td>
</tr>
<tr>
<td>1.4</td>
<td>FTTM version 1</td>
<td>4</td>
</tr>
<tr>
<td>1.5</td>
<td>FTTM version 2</td>
<td>4</td>
</tr>
<tr>
<td>1.6</td>
<td>Equivalence of Structure of <em>FTTM</em> Componentwise</td>
<td>5</td>
</tr>
<tr>
<td>1.7</td>
<td>Sequences of FTTM&lt;sub&gt;n&lt;/sub&gt;</td>
<td>7</td>
</tr>
<tr>
<td>1.8</td>
<td>Research Framework</td>
<td>10</td>
</tr>
<tr>
<td>2.1</td>
<td>FTTM version 1</td>
<td>12</td>
</tr>
<tr>
<td>2.2</td>
<td>FTTM version 2</td>
<td>13</td>
</tr>
<tr>
<td>2.3</td>
<td>EEG projection</td>
<td>14</td>
</tr>
<tr>
<td>2.4</td>
<td>Equivalence of FTTM Version 2</td>
<td>14</td>
</tr>
<tr>
<td>2.5</td>
<td>Generalized FTTM</td>
<td>15</td>
</tr>
<tr>
<td>2.6</td>
<td>Sequence of FTTM&lt;sub&gt;n&lt;/sub&gt;</td>
<td>16</td>
</tr>
<tr>
<td>2.7</td>
<td>Examples of sequence of FTTM&lt;sub&gt;n&lt;/sub&gt;</td>
<td>18</td>
</tr>
<tr>
<td>2.8</td>
<td>k-FTTM</td>
<td>19</td>
</tr>
<tr>
<td>2.9</td>
<td>Reproduction of Pascal’s Triangle from Chu Shih Chieh</td>
<td>21</td>
</tr>
<tr>
<td>2.10</td>
<td>Pascal’s Triangle based on binomial coefficients</td>
<td>23</td>
</tr>
<tr>
<td>2.11</td>
<td>Modern day interpretation of Pascal’s triangle</td>
<td>23</td>
</tr>
<tr>
<td>2.12</td>
<td>FTTM in Pascal’s Triangle</td>
<td>24</td>
</tr>
<tr>
<td>2.13</td>
<td>Example of Fibonacci cubes</td>
<td>26</td>
</tr>
<tr>
<td>3.1</td>
<td>Sequence of F&lt;sup&gt;2&lt;/sup&gt;&lt;sub&gt;n&lt;/sub&gt;</td>
<td>30</td>
</tr>
</tbody>
</table>
3.2 Sequence of $F3_n$ 35
3.3 $k - F3_n$ 39
3.4 Sequence of $F4_n$ 43
3.5 Sequence of $F5_n$ 49
3.6 $k - F5$ 57
3.7 Number of cubes for $FK_{k/n}$ 58
5.1 Relations Between FITM, Pascal triangle, and $k$-Fibonacci Sequence 79
5.2 Some examples of cubes $FK_n$ 80
6.1 Element of $F2_1$ 96
6.2 Elements of $F2_2$ 96
6.3 Elements of $F2_3$ 96
6.4 Elements of $F2_4$ 97
6.5 New elements produced from the combination of two $F3$ in $F3_2$ 98
6.6 New elements produced from the combination of two $F3$ in $F3_3$ that starts with $A_1$ 98
6.7 New elements produced from the combination of two $F3$ in $F3_3$ that starts with $A_2$ 99
6.8 New elements produced from the combination of two $F3$ in $F3_3$ that starts with $A_3$ 99
6.9 New elements produced from combination of three $F3$ in $F3_3$ 100
7.1 Geometry of $FK_n$ for $k=2, 3, 4$, and 5 108
7.2 Some Examples of Polygonal Numbers 109
# LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>UTM</td>
<td>Universiti Teknologi Malaysia</td>
</tr>
<tr>
<td>FTTM</td>
<td>Fuzzy Topographic Topological Mapping</td>
</tr>
<tr>
<td>FRG</td>
<td>Fuzzy Research Group</td>
</tr>
<tr>
<td>MEG</td>
<td>Magnetoencephalography</td>
</tr>
<tr>
<td>EEG</td>
<td>Electroencephalogram</td>
</tr>
<tr>
<td>BM</td>
<td>Base Magnetic Plane</td>
</tr>
<tr>
<td>FM</td>
<td>Fuzzy Magnetic Field</td>
</tr>
<tr>
<td>MC</td>
<td>Magnetic Contour Plane</td>
</tr>
<tr>
<td>TM</td>
<td>Topographic Magnetic Field</td>
</tr>
<tr>
<td>BMI</td>
<td>Base Magnetic Image Field</td>
</tr>
<tr>
<td>FMI</td>
<td>Fuzzy Magnetic Image Field</td>
</tr>
<tr>
<td>MI</td>
<td>Magnetic Image Field</td>
</tr>
<tr>
<td>TMI</td>
<td>Topographic Magnetic Image Field</td>
</tr>
<tr>
<td>FK</td>
<td>Finite vertices of FTTM</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

$\cong$ - Homeomorphism
$
\leq$
 - Less than or equal
$
\geq$
 - Greater than or equal
$
\implies$
 - Implies
$
\in$
 - Element of
$vFTTM$
 - Vertices of FTTM
$eFTTM$
 - Edges of FTTM
$fFTTM$
 - Faces of FTTM
$FTTM_{2/n}$
 - Cubes with combination of two number of FTTM in FTTM$_n$
$FTTM_{3/n}$
 - Cubes with combination of three number of FTTM in FTTM$_n$
$FTTM_{4/n}$
 - Cubes with combination of four number of FTTM in FTTM$_n$
$FTTM_{k/n}$
 - Cubes with combination of $k$ number of FTTM in FTTM$_n$
$FK_n$
 - Sequence of finite vertices of FTTM
$vFK_n$
 - Vertices of finite vertices of FTTM
$eFK_n$
 - Edges of finite vertices of FTTM
$fFK_n$
 - Faces of finite vertices of FTTM
$FK_{r/n}$
 - Cubes with combination of $r$ number of $FK$ in $FK_n$
# LIST OF APPENDICES

<table>
<thead>
<tr>
<th>APPENDIX</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Papers published during the author’s candidature</td>
<td>116</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

The human brain (see Figure 1.1) is the most important structure in our body. It is also the most complexly organized structure known to exist [1]. There are four lobes in both halves of the cortex: frontal, parietal, temporal and occipital.

The outermost layer of the brain is called the cerebral cortex. The cerebral cortex has a total surface area of about $2500cm^2$, folded in a complicated way, so that it fits into the cranial cavity formed by the skull of the brain. There are at least $10^{10}$ neurons in the cerebral cortex. These neurons are the active units in a vast signal-handling network [1]. When information is being processed, small currents flow in the neural system, producing a weak magnetic field (see Figure 1.2).

![Human brain](http://www.newscientist.com)

Figure 1.1: Human brain (source from http://www.newscientist.com)
Different parts of brain produce different patterns of magnetic fields [2]. The small area of brain tissues that triggers epileptic seizures is called epileptic foci. It is very important to accurately locate the epileptic foci in the cortical region for a successful surgery [1]. Both invasive and noninvasive methods of locating the epileptic foci have been used in the past, but only the invasive pathway has yielded necessary results for surgical removal.

Magnetoencephalography (MEG) is one of the noninvasive neuroimaging techniques used to identify epileptic foci (see Figure 1.3). This study was first conducted by the University of California [3]. MEG is the study of magnetic field generated by currents in the neurons [4]. MEG consists of the superconducting quantum interface device (SQUID) detectors coupled with flux transformers. The recorded magnetic fields help in determining where the electrical currents originate and the strength of currents. MEG is completely noninvasive and non-hazardous. The recorded magnetic field gives information in the process to determine location, direction and magnitude of a current source. Estimating the cerebral current sources underlying a measured distribution of the magnetic field is called the neuromagnetic inverse problem [1].

There is a method for solving this problem, called Bayesian, that needs a priori information (data based model), and it is time consuming [5]. By using Bayesian, forward calculation is used to calculate the magnetic field caused by the current dipole at every possible point. The best location of the current source is determined by minimizing the sum of the squares of the difference between the measured and the
calculated value similar to the least squares method. Then, a point with a minimum least square is the location of a current source. On the other hand, Fuzzy Topographic Topological Mapping (FTTM) is a model for the solving neuromagnetic inverse problem. It does not need priori information and is less time consuming [6].

1.2 Research Background

FTTM was first developed by Fuzzy Research Group (FRG) group in 1999 in order to determine the location of epileptic foci in epilepsy disorder patients [6]. The model consists of four components, which are magnetic contour plane (MC), base magnetic plane (BM), fuzzy magnetic plane (FM), topographic magnetic field (TM) and three mathematical algorithms (see Figure 1.5). In 2002, Zakaria has developed FTTM version 1 (see Figure 1.4) to present a 3-D view of an unbounded single current source [7], and later, Rahman developed FTTM version 2 (see Figure 1.5) to present a 3-D view of a bounded multi current source [8]. The structure of FTTM will be discussed in detail in Chapter 2.
1.3 Problem Statement

FTTM version 1 and FTTM version 2 (See Figure 1.6) are specially designed to have equivalent topological structures between its components. This was proven by Yun [9]. In other words, FTTM version 1 and FTTM version 2 are homeomorphic component-wise. Yun also noticed that if there are two elements of FTTM that are homeomorphic to each other component-wise, it would generate more homeomorphisms [10]. The number of generating new elements of FTTM is

\[
\begin{bmatrix}
\binom{2}{1} \times \binom{2}{1} \times \binom{2}{1} \times \binom{2}{1}
\end{bmatrix} - 2 = 14 \text{ elements.}
\] (1.1)
Consequently, Yun proposed a conjecture such that if there exist $n$ elements of FTTM, then the number of new elements are $n^4 - n$ [10]. This conjecture is proven by Jamaian [11]. Consequently, Jamaian proposed an open problem as follows:

given cube of two, three, and four FTTM given by FTTM$_{2/n}$, FTTM$_{3/n}$ and FTTM$_{4/n}$ . Thus for every nonzero of FTTM$_{2/n}$, FTTM$_{3/n}$ and FTTM$_{4/n}$ appears in the third, fourth, and fifth main diagonal of Pascals triangle respectively, therefore for every nonzero sequence of $FK_{2/n}$, $FK_{3/n}$, $FK_{4/n}$,.....$FK_{l/n}$ they also obey the third, fourth, fifth, until $(l+1)^{th}$ main diagonal of Pascals triangle with $K$ representing the number of components [12]. The number of new elements of $FK_n$ can be written as,

$$FK_n = C_1\binom{n}{2} + C_2\binom{n}{3} + C_3\binom{n}{4} + \cdots + C_p\binom{n}{k}$$  \hspace{1cm} (1.2)$$

where $n \geq k$ with $k$ the number of component and $C_1$, $C_2$, $C_3$,....., $C_p$ are the coefficient for each combination. Since FTTM exists in a sequence, therefore the need to analyse the sequence of finite vertices of Fuzzy Topographic Topological Mapping ($FK_n$) is paramount.
1.4 Research Objectives

The aims of this research are as follows:

(i) To develop sequences of finite vertices of Fuzzy Topographic Topological Mapping (FTTM).

(ii) To prove the theorem on sequences of $FK_n$ using difference equation.

(iii) To find the relation between sequences of cubes of $FK_n$ and $k$-Fibonacci sequences.

(iv) To prove the conjecture proposed by Jamaian.

1.5 Scope of the Study

This research focuses on the goal to prove the conjecture proposed by Jamaian in [12] and the relation between generalized FTTM and $k$-Fibonacci sequence. This form of sequence for $FK_n$ was only limited to the form that were adopted by Jamaian in [12].

1.6 Significance of Findings

By proving the conjecture, other versions of FTTM can be introduced. In other words, a new version of FTTM besides FTTM version 1 and version 2 can be developed. The relation between sequences of FTTM, Pascal’s Triangle and also Fibonacci sequences are obtained.
1.7 Research Methodology

The research starts by studying different types of FTTM and the geometry for sequences of FTTM. There are three methods used in order to prove the theorem which are constructive, difference equation, and mathematical induction.

1.7.1 Constructive

According to Hein [13], a constructive proof is a method of proving that demonstrate the existence of a mathematical object with certain properties by creating or providing a method for creating such an object. In addition, the constructive method can be identified by certain keywords that appear in the statement such as there is, there are, there exist, for all, for each and for every [14]. Furthermore, this method never puts any condition that the statement of a problem should be identified first.

1.7.2 Difference Equation

A difference equation (also called a recurrence equation) is the discrete analog of a differential equation [13]. A difference equation involves an integer function \( f(n) \)
in a form, such as the following:

\[ f(n) - f(n + 1) = g(n). \] (1.3)

For non-homogenous equation

\[ U_{n+1} + aU_n = f(n) \] (1.4)

where \( a \) is constant, the solution \( U_n \) is given by

\[ U_n = \left( \begin{array}{c} \text{general solution of} \\ \text{associated homogeneous} \end{array} \right) + \left( \begin{array}{c} \text{one particular solution} \\ \text{of the non - homogeneous} \end{array} \right) \] (1.5)

1.7.3 Mathematical Induction

Mathematical induction is a way to prove statements for all positive integers [15]. There are two steps in mathematical induction: the basis and the inductive steps.

(i) The basis (base case): showing that the statement holds when \( n \) is equal to the lowest value that \( n \) is given in the question. Usually, \( n = 0 \) or \( n = 1 \).

(ii) The inductive step: showing that if the statement holds for some \( n \), then the statement also holds when \( n + 1 \) is substituted for \( n \).

1.8 Thesis Organization

In general, the thesis contains seven chapters. The first chapter serves as an introduction to the whole thesis. This chapter includes the background of the research, problem statements, objectives, scope and importance of the research.
Chapter 2 presents the literature review of this research. Various works by different researchers regarding FTTM are discussed in this chapter. Some definitions on k-Fibonacci sequence are also presented in this chapter.

Chapter 3 consists the geometrical features of $FK_n$. It consists of generalization of $FK_n$. The geometrical feature of $FK_n$ is discussed in this chapter. Several definitions on sequence of $FK_n$ are also provided.

Chapter 4 provides the theorems on sequences of $FK_n$. The proofs for the theorems are provided in this chapter.

In Chapter 5, the proofs of sequence of cubes $FK_n$ are provided. It reveals the relation between cube of $FK_n$, Pascal’s triangle, and k-Fibonacci sequence. This chapter covers the proof to the theorem and corollaries of cube $FK_n$.

Chapter 6 covers on the elements of $FK_n$ and the relation to cubes of $FK_n$. Finally, Chapter 7 consists of conclusions and recommendations for future work.

The framework of this research is summarized in Figure 1.8.
SEQUENCES AND CUBES OF FINITE VERTICES OF FUZZY TOPOGRAPHIC TOPOLOGICAL MAPPING

Introduction

CHAPTER 1
Introduction

CHAPTER 2
Literature Review

Theoretical Foundation

CHAPTER 3
Some Sequences of Finite Vertices FTTM

Implementation

CHAPTER 4
Some Properties of Sequences of Finite Vertices FTTM

CHAPTER 5
Cube of Finite Vertices of FTTM In Relation To $k$-Fibonacci Sequence

CHAPTER 6
Relation Between Sequence of $FK_n$ and Cube of $FK_n$

Conclusion

CHAPTER 7
Conclusion and Recommendation

Figure 1.8: Research Framework
REFERENCES


