

**EVALUATION OF DAMAGE-BASED MATERIAL MODELS FOR CARBON
FIBER-REINFORCED POLYMER COMPOSITE LAMINATE UNDER
MIXED-MODE BENDING**

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MODE BENDING

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A project report submitted in partial fulfilment of the
requirements for the award of the degree of
Master of Engineering (Mechanical)

Faculty of Mechanical Engineering
Universiti Teknologi Malaysia

JUNE 2014

To my beloved father, mother, brother and sister, and my lovely friend, Hengameh,
who offered me
unconditional love and support throughout the course of this thesis...

*Seek the science that unties for you this knot.
Seek it as long as there's life in you still to be sought.
Leave that nothing that looks like it's something;
Seek that something that looks like it's nothing.*

Rumi, Persian Poet

13th-Century

ACKNOWLEDGEMENT

First and foremost, my utmost gratitude to *Prof. Dr. Mohd Nasir Tamin*, my supervisor whose sincerity and encouragement, I will never forget. Prof. Nasir. I would also like to appreciate *Dr. Wong King Jye* who helped me a lot during this research and spending a lot of his precious time and energy for me. All computational solid mechanics laboratory (CSM Lab) members are also acknowledged, which create friendly atmosphere in laboratory for cooperation. I would also like to convey my special appreciation to my beloved Mother, Father and my lovely family Saeed, Mahboobeh and Hengameh for all the mental supports motivation to help me get through the challenges I faced throughout this master project. Last but not least, my appreciation is also extended to all my friends who involved whether directly or indirectly in helping me to complete this project especially Ali Farokhi Nejad.

ABSTRACT

Composite structures present high strength, low weight and design flexibility in terms of fiber orientation and number of plies and used vastly in advanced and modern applications. Among them, carbon fiber-reinforced polymer composites (CFRP) are used widely in aeronautic and automotive industries in which components are subjected to different loading types and this will necessity of investigation on fracture analysis using damage mechanics concepts. The reliability of structures made of composites, depends on continual process of damage initiation and propagation. In the current research, a specific CFRP composite is being tested and finite element simulated under monotonic loading which creates Mode I, Mode II and Mixed Mode (I&II) of fracture. The specimen is designed and fabricated by Institute of Automotive and Transport Engineering (ISAT) and the damage development can be tracked easily on the localized interface. The Double Cantilever Beam (DCB), End Notched Flexure (ENF) and Mixed Mode Flexure (MMF) experiments and FE simulation have been used simultaneously to investigate the damage under Mode I, Mode II and Mixed Mode (I &II) of fracture loading condition. Although, Damage model used is cohesive zone model (CZM) which is developed and validated before. Results showed that the CZM-based FE model is correlated well with experimental results and based on the experimental-computational approach, CZM parameters can be obtained and damage model will be characterized so that finite element method can be validated and stress and deformation analyses using FE results are feasible.

ABSTRAK

Bahan struktur bahan rencam mempunyai sifat kekuatan yang tinggi, ringan dan fleksibel dalam rekabentuk terutamanya pada orientasi gentian dan jumlah lapisan dan ianya banyak digunakan dalam aplikasi moden. Diantara jenis bahan rencam, polimer bertetulangkan gentian karbon banyak digunakan secara meluas terutama dalam industri penerbangan dan automotif di mana komponen tersebut dikenakan beban tekanan yang berbeza dan keperluan untuk menganalisis keretakan dengan menggunakan konsep *Damage Mechanics*. Kebolehpercayaan sesuatu struktur yang diperbuat daripada bahan rencam bergantung kepada proses yang berterusan terutamanya di permulaan kerosakan dan perebakannya. Dalam kajian ini, spesifik bahan rencam polimer bertetulangkan gentian karbon (CFRP) telah diuji dan kaedah simulasi kaedah unsur terhingga yang dikenakan beban tanjakan pada keadaan Mod I, Mod II dan Mod Campuran (I&II). Spesimen direka dan dibuat di Institut Kejuruteraan Automatif dan Pengangkutan, justeru perkembangan tahap kerosakan dapat dikesan dengan mudah pada antarafasa setempat. Eksperimen dan simulasi kaedah unsur terhingga pada Rasuk Berganda Julur (DCB), Lenturan Takuk Akhir (ENF) dan Lenturan Campuran Mod (MMF) telah digunakan pada masa yang sama untuk mengkaji kerosakan dalam Mod I, Mod II dan Mod Campuran (I dan II) pada keadaan beban patah. Model kerosakan yang digunakan adalah *cohesive zone model* (CZM) yang mana telah dibangunkan dan disahkan sebelum ini. Keputusan daripada simulasi model *CZM-FE* adalah berkait rapat dengan keputusan eksperimen dengan menggunakan kaedah eksperimen-perkomputeraan. Parameter CZM juga boleh diperolehi daripada *Damage Model* supaya kaedah unsur terhingga dapat disahkan. Keputusan analisis tegasan dan perubahan bentuk berdasarkan keputusan daripada kaedah unsur terhingga adalah boleh diguna pakai.

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LIST OF SYMBOLS

| | | |
|-------------------------|---|--|
| a | - | Crack length |
| A | - | Area |
| b | - | Width |
| D | - | Damage variable |
| E | - | Young`s modulus of elasticity |
| G | - | Shear modulus |
| G_c | - | Critical energy release rate |
| L | - | length |
| P | - | Load |
| S | - | Maximum interfacial strength in shear mode |
| t | - | Thickness |
| β | - | Mode mixity |
| δ | - | Deflection , separation |
| δ_0 | - | Relative displacement at damage onset |
| δ_f | - | Relative displacement at fracture |
| δ_m | - | Total mixed-mode relative displacement |
| δ_{shear} | - | Tangential displacement |
| Φ | - | Diameter |
| η | - | Power at B-K criterion |
| ϑ | - | Poisson ratio |
| τ | - | Shear stress, traction force |

CHAPTER 1.0

INTRUDUCTION

1.1 Introduction

In this chapter, the background of the study and some discussions on key issues relating to damage mechanics of advanced composite structures, under Mixed-mode bending loading condition will be presented and briefly described. Moreover, the finite element method as a key numerical procedure to study the behavior of material during evolution of damage under tension and shear stress will be shortly discussed. Subsequently, the objectives of the study will be either presented or followed by a discussion on the scope and significance of the study.

The current research will focus on modeling and finite element (FE) simulation of a specific carbon fiber reinforced polymer (CFRP) with specific number of plies and fiber orientation with pre-existing crack and in three point flexural Mixed-mode loading condition. The FE simulation should then validated with real-world conditions, therefore a systematic experimental procedure will be

conducted and the results of both numerical modeling and simulation and also, experimental data will be compared with each other to investigate on accuracy and reliability of numerical method.

1.2 Background of the Study

Carbon-fiber-reinforced polymer, carbon-fiber-reinforced plastic or carbon fiber reinforced thermoplastic (CFRP, CRP, CFRTTP or often simply carbon fiber, or even carbon), is an extremely strong and light fiber-reinforced polymer which contains carbon fibers. It has many applications in aerospace and automotive fields, such as Formula One racing. The compound is also used in sailboats, rowing shells, modern bicycles, and motorcycles because of its high strength-to-weight ratio and very good rigidity. Improved manufacturing techniques are reducing the costs and time to manufacture, making it increasingly common in small consumer goods as well, such as fishing rods, hockey sticks, paintball equipment, archery equipment, tent poles, racquet frames, stringed instrument bodies, drum shells, golf clubs, helmets used as a paragliding accessory and pool/billiards/snooker cues (Kolor et al.).

Unlike metals, composite materials are inhomogeneous (on a gross scale) and anisotropic. They accumulate damage in a general rather than a localized fashion, and failure does not always occur by the propagation of a single macroscopic crack. The micro-structural mechanisms of damage accumulation, including fiber breakage and matrix cracking, debonding, transverse-ply cracking, and delamination, occur sometimes independently and sometimes interactively, and the predominance of one or the other may be strongly affected by both materials variables and testing conditions.

1.2.1 Composite Material

Composite materials (also called composition materials) are materials made from two or more constituent materials with significantly different physical or chemical properties, that when combined, produce a material with characteristics different from the individual components. Composite materials have advantageous over metals such as high strength, light weight, design flexibility, consolidation of parts etc. Advanced composite materials are finding increasing application in aerospace, automotive, marine and many other industries due to the advantages in performance, structural efficiency and cost they provide. Composite structures have different classifications, such as Particle-Reinforced, Fiber-Reinforced and Structural composites. From these categories; fiber-reinforced composites have wide range of application in modern and advanced structures. Carbon Fiber-Reinforced Polymer (CFRP) composites are commonly employed in modern structural application such as aircraft wing, rotor blades, and automobile chassis.

1.2.2 Finite Element Method

The finite element method (FEM) is a numerical method seeking an approximated solution of the distribution of field variables in the problem domain that is difficult to obtain analytically. It is done by dividing the problem domain into several elements. Known physical laws are then applied to each small element, each of which usually has a very simple geometry. A continuous function of an unknown field variable is approximated using piecewise linear functions in each sub-domain, called an element formed by nodes. The unknowns are then the discrete values of the field variable at the nodes. Next, proper principles are followed to establish equations for the elements, after which the elements are „tied“ to one another. This process

leads to a set of linear algebraic simultaneous equations for the entire system that can be solved easily to yield the required field variable (Quek and Liu, 2003).

1.2.3 Cohesive Zone Model

The cohesive zone model (CZM) is one of the most modern evolutions in the area of fracture mechanics in which fracture formation is regarded as a gradual phenomenon in which the separation of the surfaces involving in the crack takes place across an extended crack tip, or cohesive zone, and is resisted by cohesive tractions. This method has several advantages in comparison with the conventional methods in fracture mechanics. For illustration, It is able to adequately predict the behavior of uncracked structures, including those with blunt notches or Size of non-linear zone need not be negligible in comparison with other dimensions of the cracked geometry in CZM, while in other conventional methods, it is not so, and, Even for brittle materials, the presence of an initial crack is needed for LEFM to be applicable.

The cohesive zone model (CZM) is widely used in modeling fracture and other failure phenomena in different types of materials. Applications can be found in homogeneous as well as composite materials (Zreid et al., 2013).

1.3 Research Objectives

- To develop finite element (FE) model of carbon fiber-reinforced polymer (CFRP) composite laminates with initial defect under mixed-mode bending (MMB) load.
- To validate and examine available damage-based models for the mixed-mode bending.

1.4 Problem Statement

How an appropriate available damage based can be evaluated to respect the initial defect under Mixed-Mode bending for CFRP?

1.5 Research Scopes

- Review of Finite Element (FE) formation for solid (8-node) element, FE modeling of CFRP composite laminates, continuum damage models, cohesive behavior of interface.
- Review the current status FE simulation of CFRP composite laminates based on damage mechanics approach. This work that composite of Mode I and Mode II loading case was performed by a doctoral candidate at CSMLab.

- Develop FE model of the CFRP composite beam $[0]_8$ with initial interface crack for mixed mode bending test setup. Established characteristic load-deflection curve.
- Perform flexural test on the composite beam specimen using mixed-mode bending (MMB) test setup in accordance to the test standard.
- Validate the FE model with measured load-deflection data. Analysis internal states of displacement, strain and stress in the laminates with respect to damage initiation, propagation and localized fracture.

1.6 Research Questions

- Is it possible to simulate the real Carbon Fiber-Reinforced Polymer (CFRP) in virtual space for predicting the behavior of this material under complex loading?
- How an appropriate available damage based can be evaluated to respect the initial defect under Mixed-Mode bending for CFRP?

1.7 Structure of Research

In chapter 1, the background of the study, statement of the problem, objectives, research questions, scope of project, research question and research hypothesis are described.

In chapter 2, review of the literature related to damage mechanics of composite structures and also some review on kind of fracture modes in mechanics, especially mixed mode as well as introduction to cohesive zone model will be covered with more details.

In chapter 3, the evaluation of methodology will present. Moreover, the used material, experimental and numerical techniques will be covered.

In chapter 4, the preliminary results and dissection on the results will be discussed.

Finally, in chapter 5 conclusion and summary of preliminary results will be presented briefly.

REFERENCES

- Alfano, G. (2006). On the influence of the shape of the interface law on the application of cohesive-zone models. *Composites Science and Technology*. 66 (6), 723-730.
- Andena, L., Rink, M. and Williams, J. (2006). Cohesive zone modelling of fracture in polybutene. *Engineering Fracture Mechanics*. 73 (16), 2476-2485.
- Barenblatt, G. I. (1962). The mathematical theory of equilibrium cracks in brittle fracture. *Advances in applied mechanics*. 7 (1), 55-129.
- Benzeggagh, M. and Kenane, M. (1996). Measurement of mixed-mode delamination fracture toughness of unidirectional glass/epoxy composites with mixed-mode bending apparatus. *Composites Science and Technology*. 56 (4), 439-449.
- Borst, R. d., Remmers, J. J. and Needleman, A. (2006). Mesh-independent discrete numerical representations of cohesive-zone models. *Engineering Fracture Mechanics*. 73 (2), 160-177.
- Camanho, P. P., Davila, C. and De Moura, M. (2003). Numerical simulation of mixed-mode progressive delamination in composite materials. *Journal of Composite Materials*. 37 (16), 1415-1438.

- Chaboche, J., Feyel, F. and Monerie, Y. (2001). Interface debonding models: a viscous regularization with a limited rate dependency. *International Journal of Solids and Structures*. 38 (18), 3127-3160.
- Chalivendra, V. B., Hong, S., Arias, I., Knap, J., Rosakis, A. and Ortiz, M. (2009). Experimental validation of large-scale simulations of dynamic fracture along weak planes. *International Journal of Impact Engineering*. 36 (7), 888-898.
- Chandra, N., Li, H., Shet, C. and Ghonem, H. (2002). Some issues in the application of cohesive zone models for metal–ceramic interfaces. *International Journal of Solids and Structures*. 39 (10), 2827-2855.
- Cuntze, R. and Freund, A. (2004). The predictive capability of failure mode concept-based strength criteria for multidirectional laminates. *Composites Science and Technology*. 64 (3), 343-377.
- Dávila, C. G. and Camanho, P. P. (2003). Analysis of the effects of residual strains and defects on skin/stiffener debonding using decohesion elements. Paper presented at the Proc. 44th AIAA/ASME/ASCE/AHS structures, structural dynamics, and materials conf. Norfolk, VA.
- Dugdale, D. (1960). Yielding of steel sheets containing slits. *Journal of the Mechanics and Physics of Solids*. 8 (2), 100-104.
- Echaabi, J. and Trochu, F. (1997). Failure mode dependent strength criteria for composite laminates. *Journal of reinforced plastics and composites*. 16 (10), 926-945.

- Fedele, R., Raka, B., Hild, F. and Roux, S. (2009). Identification of adhesive properties in GLARE assemblies using digital image correlation. *Journal of the Mechanics and Physics of Solids*. 57 (7), 1003-1016.
- Fuchs, P. and Major, Z. (2011). Experimental determination of cohesive zone models for epoxy composites. *Experimental Mechanics*. 51 (5), 779-786.
- Ghaboussi, J., Pecknold, D. A., Zhang, M. and Haj-Ali, R. M. (1998). Autoprogressive training of neural network constitutive models. *International Journal for Numerical Methods in Engineering*. 42 (1), 105-126.
- Greszczuk, L. (1974). Microbuckling of lamina-reinforced composites. *Composite Materials: Testing and Design*. 3.
- Harris, B. (2003). *Fatigue in composites: science and technology of the fatigue response of fibre-reinforced plastics*: Woodhead Publishing.
- Hart-Smith, L. (1998). Predictions of the original and truncated maximum-strain failure models for certain fibrous composite laminates. *Composites Science and Technology*. 58 (7), 1151-1178.
- Hong, S. and Kim, K.-S. (2003). Extraction of cohesive-zone laws from elastic far-fields of a cohesive crack tip: a field projection method. *Journal of the Mechanics and Physics of Solids*. 51 (7), 1267-1286.

- Huang, H., Springer, G. S. and Christensen, R. M. (2003). Predicting failure in composite laminates using dissipated energy. *Journal of Composite Materials*. 37 (23), 2073-2099.
- Jiang, L. (2010). A cohesive law for carbon nanotube/polymer interface accounting for chemical covalent bonds. *Mathematics and Mechanics of Solids*. 15 (7), 718-732.
- Kaw, A. K. (2005). *Mechanics of composite materials*: CRC press.
- Kim, C. H. and Yeh, H.-Y. (1994). Development of a new yielding criterion: the Yeh-Stratton criterion. *Engineering Fracture Mechanics*. 47 (4), 569-582.
- Kolloor, S. S. R., Redzuan, N. and Tamin, M. N. Shear-dominated Interlaminar Fracture Process in CFRP Composite Laminates.
- Kubair, D. V. and Geubelle, P. H. (2003). Comparative analysis of extrinsic and intrinsic cohesive models of dynamic fracture. *International Journal of Solids and Structures*. 40 (15), 3853-3868.
- Niu, M. C.-Y. (1992). *Composite airframe structures: practical design information and data*: Adaso Adastra Engineering Center.
- Orifici, A., Herszberg, I. and Thomson, R. (2008). Review of methodologies for composite material modelling incorporating failure. *Composite Structures*. 86 (1), 194-210.

- Perales, F., Dubois, F., Monerie, Y., Piar, B. and Stainier, L. (2010). Multi-body NSCD strategy as a multi-domain solver. Application to code coupling dedicated to the modeling of fracture of heterogeneous media. *European Journal of Computational Mechanics*. 19 389-417.
- Puck, A. and Schürmann, H. (1998). Failure analysis of FRP laminates by means of physically based phenomenological models. *Composites Science and Technology*. 58 (7), 1045-1067.
- Quek, S. and Liu, G. (2003). *Finite Element Method: A Practical Course: A Practical Course: Butterworth-Heinemann*.
- Reeder, J. (1992). *An Evaluation of Mixed-Mode Delamination Failure Criteria*.
- Rybicki, E. F. and Kanninen, M. (1977). A finite element calculation of stress intensity factors by a modified crack closure integral. *Engineering Fracture Mechanics*. 9 (4), 931-938.
- Shen, B. and Paulino, G. (2011). Direct extraction of cohesive fracture properties from digital image correlation: a hybrid inverse technique. *Experimental Mechanics*. 51 (2), 143-163.
- Tan, H., Liu, C., Huang, Y. and Geubelle, P. (2005). The cohesive law for the particle/matrix interfaces in high explosives. *Journal of the Mechanics and Physics of Solids*. 53 (8), 1892-1917.

- Tomar, V., Zhai, J. and Zhou, M. (2004). Bounds for element size in a variable stiffness cohesive finite element model. *International Journal for Numerical Methods in Engineering*. 61 (11), 1894-1920.
- Turon, A., Davila, C. G., Camanho, P. P. and Costa, J. (2007). An engineering solution for mesh size effects in the simulation of delamination using cohesive zone models. *Engineering Fracture Mechanics*. 74 (10), 1665-1682.
- Tvergaard, V. and Hutchinson, J. W. (1992). The relation between crack growth resistance and fracture process parameters in elastic-plastic solids. *Journal of the Mechanics and Physics of Solids*. 40 (6), 1377-1397.
- Valoroso, N. and Fedele, R. (2010). Characterization of a cohesive-zone model describing damage and de-cohesion at bonded interfaces. Sensitivity analysis and mode-I parameter identification. *International Journal of Solids and Structures*. 47 (13), 1666-1677.
- Zhu, Y., Liechti, K. M. and Ravi-Chandar, K. (2009). Direct extraction of rate-dependent traction–separation laws for polyurea/steel interfaces. *International Journal of Solids and Structures*. 46 (1), 31-51.
- Zreid, I., Fleischhauer, R. and Kaliske, M. (2013). A thermomechanically coupled viscoelastic cohesive zone model at large deformation. *International Journal of Solids and Structures*. 50 (25), 4279-4291.