ANALYTICAL SOLUTION OF CONTAMINANT TRANSPORT IN STREAMS

NURUL SYAZWANI BINTI MOHD AFANDI

This project is submitted in part fulfillment of the requirements for the award of the degree of Master of Science (Engineering Mathematics).

Faculty of Science
Universiti Teknologi Malaysia

JANUARY 2012
For dearest,

abah and ummi

&

siblings

aya, mat capiq, siti, idah, sabila
ACKNOWLEDGEMENT

First of all, thanks to Almighty Allah s.w.t for give me blessing and opportunity to undertake this study. I sincerely have to thank all those who have supported and helped me writing this report. A special thanks and a deepest appreciation to my supervisor, PM.Dr. Zainal Abd Aziz for his valuable direction and assistance in making this project a success.

A warmest gratitude to my family especially my parents, for their encouragement and advice.

I also would like to thank my friends for being such a great friends and support me during the elaboration of this project.
ABSTRACT

The behavior of contaminant transport in stream is greatly influenced by surface and groundwater interaction. The improvement of the prediction of downstream water quality can be implemented if the more attention gives to this problem and model these processes accurately. In this study, an analytical solution is presented for solute transport in streams including the effect of transient storage. Transient storage model is introduced to represent the movement of solute from main stream into stagnant zone and back to the main stream. The governing equation of the transient storage model discretized by analytical solution and transformed using laplace transformation. Then the results of the concentration profile obtained graphically. The results show that the concentration profile of the main stream approaches classical advection dispersion equation whereas the storage zone exchanged coefficient, $\alpha=0$ which show that there is no mass exchange of solute between the main channel and the storage zone.
ABSTRAK

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LIST OF SYMBOLS

\begin{align*}
A & \quad \text{- Main channel cross sectional area} \\
A_s & \quad \text{- Storage zone cross-sectional area} \\
C(x, t) & \quad \text{- Main Channel solute concentration} \\
C_{in} & \quad \text{- Upstream boundary condition} \\
C_L & \quad \text{- Lateral inflow solute concentration} \\
C_s & \quad \text{- Storage zone solute concentration} \\
D & \quad \text{- Dispersion coefficient in the channel} \\
H(t - t_1) & \quad \text{- Heaviside function} \\
p & \quad \text{- Laplace parameter} \\
Q & \quad \text{- Volumetric flow rate} \\
q_{Lin} & \quad \text{- Lateral inflow rate} \\
q_{Out} & \quad \text{- Lateral outflow rate} \\
\nu & \quad \text{- Velocity in the channel} \\
\alpha & \quad \text{- Storage exchange coefficient} \\
\mathcal{L} & \quad \text{- Laplace Transform}
\end{align*}
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CHAPTER 1

INTRODUCTION

1.1 Background of The Study

The surface water quality in a region is largely determined both by the natural process and the anthropogenic influences urban, industrial and agricultural activities, and increasing exploitation of water resources. Water quality monitoring has one of the highest priorities in environmental protection policy. While, surface/ground water interactions influence downstream water quality significantly since the concentration distribution both in a stream and in ground water changes due to this exchange and biogeochemical reactions occur between the minerals in the subsurface and the minerals in the streams. These interaction involves the phenomenon of transient storage. Transient storage occurs in streams with zones of water that are immobile relative to the water in the main channel. Immobile zones may include pools, eddies, and water isolated behind rocks, vegetation, or other irregular bottom relief, as well as interstitial water within gravel beds and banks. Transient storage is controlled by the prevailing physical conditions of the stream such as discharge, channel structure, and bed composition. As a solute pulse moves along such a stream, solute mass enters storage zones, and the concentration of the solute in the active channel is attenuated. After the main body of the pulse passes, transient storage zones become a source of solutes to the stream. This transfer of solute mass back to the active
channel causes a gradual tailing of the concentration profile. The mass transfer into and out of a storage zone was postulated to depend on the difference in concentration between the stream and the storage zone. Generally, in transient storage models, streams are divided into two distinct zones. The first zone represents the main flow region that includes the process of advection, dispersion and lateral flow. The second one represents the storage zone area, which is stagnant relative to the main channel flow of a stream. Both zones are linked by the mass exchange process between them.

![Illustration Model of Main Channel and Storage Zone](image)

**Figure 1.1** Illustration Model of Main Channel and Storage Zone.

This model includes the effect of both surface storage, in which water is stationary relative to the main channel and the hyporheic zone, to which water moves from the main channel, flows through and returns to the main channel. Hyporheic zone which defined as a porous area which connects stream water and subsurface water that results in exchange of water between two different media. It become an important habitat for numerous aquatic organisms. It contain a wide variety of subterranean fauna and zoobenthos, either at various stages of their lives or throughout their life histories.
1.2 Statement of the Problem

It is a well-known fact that the quality of the stream water is getting worst day by day. The main problem caused by the bad quality of stream is that it kills life that inhabits water-based ecosystems. This problem occurs caused by solute transport in stream. Existing of the storage zone in stream channel caused the interaction which bring solute transport flowing in a stream. The transient storage zone actually represent the effect of the bank storage. The bank storage is the process of temporary storage of stream water in streambanks due to rapid rise which may be caused by storm. Based on this problem, the effect of the interaction between surface and ground water which lead to the quality of concentration of the stream water is considered in this study.

1.3 Objective of the study

The objectives of this study are:

1. To develop analytical solution for the transient storage model using Laplace transform.
2. To investigate the effect of the surface/ground water interaction on stream solute transport.
1.4 Scope of the study

The scope of the study is by doing derivation on governing equation of transient storage models using analytical solutions and transforms the equation using Laplace transformation for the case of continuous and finite injection of a tracer into a stream. The surface/ground water interaction on stream solute transport using tracer test is investigated.

1.5 Significance of The Study

The significance of the study is to help in guiding engineering and management decision concerned with the appropriate utilization of the river. This can also prevent the additional cost for water treatment which can lead the economy stability.

1.6 Outline of The Study

In this study, there are five chapters including the introduction and conclusion. First chapter described briefly about the background of the study, statement of the problem, objectives, scope and significance of the study.

Literature review of this study about the transient storage model will be conducted in Chapter 2. This chapter also contains the Fick’s Law, conceptual of the transient storage model, advection diffusion equation, history of water quality model and mass transfer.
Chapter 3 discuss about derivation of the governing equation of the transient storage model and transform into the Laplace transformation. Chapter 4 will describe the result of the study and followed by Chapter 5 which will state about the conclusion and recommendation of the study.
REFERENCES


