NANO-PHYSICS OF TRANSIENT PHENOMENON IN SEMICONDUCTING DEVICES AND CIRCUITS

ISMAIL SAAD¹*, MICHAEL L. P. TAN², RAZALI ISMAIL³ & VIJAY K. ARORA⁴

Abstract. As devices are scaled down to nanoscale, the high-field and quantum effects are becoming important in characterization and performance evaluation of semiconducting devices and circuits. By using the theory developed by Arora [1], transient phenomena in semiconducting devices and circuits is elaborated. It is shown that in the high-electric field, the current is limited by the saturation velocity that is ballistic, independent of scattering interactions. The enhanced scattering in quantum wells reduces the mobility of a given device, but does not change the nature of saturation velocity. The saturation velocity is comparable to thermal velocity for non-degenerate semiconductors and Fermi velocity for the degenerate semiconductors. The emission of an optical phonon may further lower this velocity. Similarly, transit-time delay and RC switching delays are enhanced over and above what is expected for the application of Ohm’s law. The effect of current and voltage division laws is also elaborated.

Keywords: Nanoscale device; quantum effect; fermi velocity; ballistic transistor; saturation velocity; high-field effects


Kata kunci: Peranti skala-nano; kesan kuantum; halaju Fermi; transistor balistik; halaju tepu; kesan medan tinggi

1.0 INTRODUCTION

The transient phenomenon in electrical circuits and devices defines the speed of a circuit or resistor. When a stimulus to a device or a system is turned on, the response of
the system is not instantaneous. The circuit response is decided by the parasitic elements that impeded the response. There are two major factors that limit the speed of a given circuit. One is the transit time delay, \( \tau_t = \frac{L}{\nu_d} \) the time a carrier (electron or hole) takes in transiting the device. Here, \( L \) is the length of the device and \( \nu_d \) is the drift velocity.

The second factor that gains predominance is the delay due to \( RC \) time constant \( \tau_{RC} \). As a device size is scaled down where \( \tau_t < \tau_{RC} \) due to the enhanced resistance as Ohm’s law breaks down. In the twentieth century, the circuit design was based on the application of Ohm’s law that is based on linear velocity response to the applied electric field \( E, \nu_d = \mu_o E \), where \( \mu_o \) is the low-field ohmic mobility. In macro-scale devices (\( L = 1 \text{ cm} \)) typical of a carbon resistor found in the physics laboratory, a logic voltage of \( V = 5 \text{ V} \) will give an electric field \( E = 5 \text{ V/cm} \) low-enough to retain the validity of Ohm’s law. The pioneering work of Arora [1] indicated that the linear velocity response to the electric field breaks down when the electric field exceeds the critical value \( E_{co} = \frac{V_t}{\theta} \) for the onset of nonlinear behavior. \( V_t = k_B T / q \) is the thermal voltage at temperature \( T \) with value 25.9 mV at room temperature. With a typical mean free path of \( \ell_s = 100 \text{ nm} \), this critical electric field for onset of nonlinear behavior is \( E_{co} = 2.59 \text{ kV/cm} \). With modern devices going below 100 nm, a logic voltage of 5 V will produce electric field exceeding 500 kV/cm. This electric field is well above the critical value in triggering the nonohmic behavior resulting in velocity saturation that is ballistic independent of the scattering-limited ohmic mobility.

In Section II, the transient velocity response to the electric field is discussed and a velocity-field characteristic is presented. Its effect on current-voltage characteristics is delineated. Section III considers the transient response to the applied impulse in an \( RC \) circuit. Section IV discusses the impact of the breakdown of Ohm’s law on current and voltage division.

2.0 TRANSIENT VELOCITY RESPONSE

The drift velocity \( \nu_d(t) \) as a function of time when an electric field is switched on at \( t = 0 \) by applying a voltage source \( V \) to a sheet resistor (Figure 1) is given by [2]

\[
\nu_d(t) = \mu_o E \left(1 - e^{-t/\tau_c}\right)
\]

where \( \tau_c \) is the mean collision time for a carriers. The steady state response \( \nu_d = \mu_o E \) is arrived when \( t >> \tau_c \). However, the presence of the high electric field triggers an otherwise unexpected phenomenon of a quantum emission giving an inelastic scattering length \( \ell_q = E_q/qE \) which may become comparable to the tradition mean free path \( \ell_s \).

The transient time \( t \) is then limited by the time required to emit the quantum \( \tau_q = \ell_q / \nu_i \), where \( \nu_i \) is the intrinsic velocity that depends on the temperature and degeneracy of the sample [3]. The steady state velocity is then limited by

\[
\nu_d = \mu_o E \left(1 - e^{-\tau_0/\tau_c}\right) = \mu_o E \left(1 - e^{-\ell_q/\nu_i}\right)
\]
This equation by itself is enough to explain the deviation from the linear relationship and velocity eventually saturates to the value

$$v_{\text{sat}} = \frac{E_Q}{mV_i}$$  \hspace{1cm} (3)

Here $E_Q = \hbar \omega_o (N_o + 1)$ is the expected average of the quantum of energy $\hbar \omega_o$ and $(N_o + 1)$ is the probability of quantum emission with $N_o = 1 / \left( e^{\hbar \omega_o / k_BT} - 1 \right)$ the Bose-Einstein probability distribution.

The velocity response of Equation (2) does not take into account the electron distribution that is given by Arora’s distribution function [1]. When this distribution is included, the velocity response to the electric field is given by:

$$v_d = v_i \tanh \left( \frac{E_i / E_c}{E_i / \ell} \right), \quad E_c = E \left( \ell, \ell' \right) = V_i / \left[ \ell (1 - e^{-E_Q / qE_i}) \right]$$  \hspace{1cm} (4)

When this velocity response is taken into account the I-V characteristics are given by [3]

$$I = I_{\text{sat}} \tanh \left( V / V_c \right), \quad I_{\text{sat}} = n_i q v_{\text{sat}} W, \quad V = EL, \quad V_c = E_c L$$  \hspace{1cm} (5)

Here $n_i$ is the sheet concentration of the carriers per unit area for a resistor of length $L$ and width $W$. In the low-field and low-voltage limit ($V \ll V_c$), Equation (5) reverts to well-known Ohm’s law:

$$I = \frac{V}{R_o}, \quad R_o = \frac{1}{n_i q \mu_o} \frac{L}{W}$$  \hspace{1cm} (6)

However, in the other extreme, $V \gg V_c$, the current saturates to $I_{\text{sat}}$ with $v_{\text{sat}} = v_i \tanh \left( E_Q / k_BT \right)$.

Figure 2 is a direct proof of breakdown of Ohm’s law that is consistent with the experimental results of Greenberg and de Alamo [4] who observed a saturation current of 565 mA/mm width of the resistive channel of InGaAs sheet resistor. Note that for mesoscale resistor of 80 $\mu$m the I-V characteristics appear linear as the critical voltage

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**Figure 1**  A prototype quantum-well resistor with confinement length $d$ in nanometer regime with standing electron waves with digital energies
for the onset of nonlinear behavior is much larger. Hence the approach towards saturation is much slower. On the other hand, for 5-µm resistor the approach towards saturation is much faster. Two features are noticeable from Figure 2. Firstly, the ohmic resistance (the inverse slope of I-V curves) is directly proportional to the length of the resistor. As length increases, so does the resistance. Secondly, the saturation current is independent of the length of the resistor; it depends only on the width of the resistive channel. Although not explicitly shown, if width is varied while length is kept constant, the resistance will decrease with the width of the channel, but the saturation current is directly proportional to the width of the channel. Higher conductance (low resistance) does not necessarily lead to the higher saturation current or saturation velocity. Moreover, the saturation current is ballistic in the sense that it is not limited by the scattering processes that limit the ohmic resistance. These results are successfully applied to I-V characteristics of a nanoscale MOSFET in [5].

3.0 TRANSIENT RESPONSE OF AN RC CIRCUIT

In digital signal processing, the transit time delay ($\tau_t$) and RC time constants ($\tau_{RC}$) compete in limiting the speed of a signal. Considerable progress has been made in reducing the transit-time delay due to scaling down of the size of the devices that is now in nano-regime. Efforts are underway to utilize low-resistivity materials and low-$k$ dielectrics to shorten the RC time delay. However, there are intrinsic factors that enhance RC timing delay due to resistance blow-up when the step voltage $V$ exceeds the critical voltage $V_c$ for the onset of nonohmic behavior.
In the presence of a digital signal applied to an RC circuit of Figure 3, the turn-on transients enhance the switching time delay of the circuit. This enhancement of the switching delay over and above what is predicted by the Ohm’s law \((\tau_o = RC)\) is given by [6]

\[
\frac{\tau_{RC}}{\tau_o} = \ln \left[ \frac{\sinh \left( \frac{V}{V_c} \right)}{\sinh \left( \frac{V}{eV_c} \right)} \right]
\]

(7)

The transit time delay \(\tau_t\) is given by:

\[
\tau_t = \frac{L}{v_{sat} \tanh \left( \frac{V}{V_c} \right)}
\]

(8)

**Figure 3** A prototype RC circuit with resistor a few nanometer in length

**Figure 4** The normalized RC time delay and transit time delay as a function of normalized applied voltage
The comparison of the two time delay given in Figure 4 indicates that the transit time delay dominates in the ohmic regime \((V << V_c)\). However, there is a considerable enhancement in the \(RC\) time constants when \(V >> V_c\). In this regime, the transit time delay is negligible and does not limit the circuit behavior. Therefore, an optimized channel length can be designed to make the delay as small as possible.

### 4.0 VOLTAGE AND CURRENT DIVISION

It is important to illustrate the effect on current and voltage division in a circuit with two resistors of different lengths connected together. This effect is extensively discussed in [3]. Consider two resistors of same \(W/L\) ratio (i.e. the same ohmic resistance) connected in series with stimulating source of voltage \(V\). The predictions of voltage division based on Ohm’s law tell us that each resistor will have a voltage drop of \(V/2\). However, when deviations from Ohm’s law are considered, we find that the voltage across smaller length resistor gets a larger share of the voltage \(V\). This is important information to study the effect of parasitic regions at the end of a conducting channel whose resistance are extracted under ohmic conditions. However, when the device or circuit is excited by a voltage source with voltage larger than the critical voltage \(V_c\) of either resistor dramatic changes in the division of the voltage are expected. Similarly, when two parallel conducting channels with different lengths exist, the smaller length resistor will become more resistive and more current will flow through the larger length resistor even if both resistors have the same ohmic value. This resistance blow up effect transforms both the steady-state and transient behavior of the nano-scale circuits.

Other set of effects are the appearance of quantum waves on the nanoscale dimensions. Depending upon how many of the three Cartesian directions are in the quantum domain, the standing waves in the quantum-confined directions can transform the behavior of the traveling quantum waves in the quasi-free (classical) directions. These are some interesting works left for future studies.

### 5.0 CONCLUSION

The transient response to the stimulation applied to an \(RC\) circuit is substantially different from what has been studied in the past using Ohm’s law. The Ohmic resistance depends on \(W/L\) ratio while saturation current depends only on the width of the channel. A smaller length channel becomes more resistive. Normally, the operational parameters (resistance, transit time, \(RC\) time constant, etc.) are extracted under ohmic conditions. However, their values may change when a device is operating in the nonohmic regime. Therefore, these results are useful in correct interpretation and performance evaluation of miniaturized devices.
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