

OPTIMIZATION OF CONSTRUCTAL DESIGN OF A MICROCHANNEL
HEAT SINK

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ABSTRACT

Past studies have shown that a good microchannel design depends on its lower thermal resistance. However, latest theory based on entropy generation minimization (EGM) stated that lower entropy generation rate must also be considered for an optimized microchannel design as is discussed in basic thermodynamics. The present study applies the entropy generation minimization (EGM) method on Li and Peterson's optimized parallel flow rectangular microchannel. Thermal resistance model, total pressure drop model, and entropy generation rate model are derived to analyze the effects of channel aspect ratio, channel width to channel pitch ratio, heat flux, and pumping power on thermal performance of the microchannel heat sink. The thermal resistance agreed with that of Li and Peterson's of channel aspect ratio of 6 and a channel number of 120, but the entropy generation rate was not minimized. For the same entropy generation rate value with the same design, an optimum channel number is found to be 60. Only by increasing the channel aspect ratio above 6 gives a minimized entropy generation rate. Therefore, for Li and Peterson's optimized microchannel design of channel aspect ratio below 6, a minimum entropy generation rate does not exist. Variation of heat flux between 100 and 1000 W/m² showed no effect on thermal resistance. Lower thermal resistance is obtained with higher overall pumping power at the expense of increasing entropy generation rate. Thus for a parallel microchannel heat sink to be superior in reducing the thermal resistance and in increasing thermodynamics performance, entropy minimization should be taken into account.

ABSTRAK

Kajian-kajian terdahulu menunjukkan bahawa penenggelam haba saluran mikro yang baik berdasarkan kepada rintangan haba yang rendah. Walaubagaimanapun, teori terbaru berkaitan penghasilan entropi minimum (EGM) menyatakan bahawa kadar penghasilan entropi perlu di ambil kira untuk rekabentuk penenggelam haba saluran mikro yang optimum seperti yang telah dibincangkan dalam asas termodinamik. Kajian ini menggunakan kaedah EGM ke atas penenggelam haba segiempat tepat aliran mikro selari yang optimum Li dan Peterson. Model rintangan haba, model susutan jumlah tekanan, dan model kadar penghasilan entropi telah diterbitkan bagi mengkaji kesan nisbah bidang saluran, nisbah lebar saluran kepada lebar picnya, fluks haba dan kuasa pam ke atas prestasi sinki haba saluran mikro. Rintangan haba yang diperolehi adalah hampir sama dengan kajian Li dan Peterson untuk nisbah bidang saluran adalah enam dan bilangan saluran adalah 120. Tetapi, kadar penghasilan entropi adalah tidak minimum. Bagi nilai kadar penghasilan entropi untuk rekabentuk yang sama, bilangan saluran yang optimum ialah 60. Kadar penghasilan entropi adalah minimum, hanya jika nisbah bidang saluran melebihi enam. Oleh itu, rekabentuk optimum penenggelam haba aliran mikro Li dan Peterson dengan nisbah bidang saluran kurang daripada enam, kadar penghasilan entropi minimum tidak dapat diperolehi. Nilai fluks haba di antara 100 hingga 1000 W/m² tidak memberi kesan kepada rintangan haba. Rintangan haba yang rendah diperolehi pada kuasa pam yang tinggi, tetapi kadar penghasilan entropinya adalah juga tinggi. Kesimpulannya, untuk sesebuah penenggelam haba aliran mikro selari menjadi berkesan bagi mengurangkan rintangan haba dan menaikkan prestasi termodinamiknya, kadar penghasilan entropinya perlu di ambil kira.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	DECLARATION	i
	DEDICATION	ii
	ACKNOWLEDGEMENT	iii
	ABSTRACT	iv
	ABSTRAK	v
	TABLE OF CONTENTS	vi
	LIST OF TABLES	ix
	LIST OF FIGURES	xi
	LIST OF SYMBOLS	xvi
	LIST OF APPENDICES	xx
1	INTRODUCTION	1
	1.1 Background	1
	1.2 Overview of Microchannel	3
	1.3 Entropy Generation Minimization (EGM)	7
	1.4 Constructal Theory	8
	1.5 Literature Review	9
	1.6 Study Scopes and Objectives	14

2	PROBLEM DEFINITION	16
	2.1 Problem Domain	16
	2.2 Problem Assumptions	20
	2.3 Modeling and Governing Equations	22
	2.3.1 First Law of Thermodynamics	22
	2.3.2 Second Law of Thermodynamics	27
3	METHODOLOGY	33
	3.1 Introduction	33
	3.2 Thermal Resistance Model	34
	3.3 Entropy Generation Model	42
	3.4 Total Pressure Drop Model	45
	3.4.1 Derivation of Mean Velocity, U_m	46
	3.4.2 Derivation of Reynolds Number, Re_{D_h}	50
4	RESULTS AND DISCUSSIONS	55
	4.1 Effect of Channel Aspect Ratio on Thermal Resistance	55
	4.2 Effect of Channel Width to Channel Pitch Ratio on Thermal Resistance	63
	4.3 Effect of Channel Aspect Ratio on Entropy Generation Rate	77
	4.4 Effect of Channel Width to Channel Pitch Ratio on Entropy Generation Rate	82
5	CONCLUSIONS AND RECOMMENDATIONS	88
	5.1 Conclusions	88
	5.2 Recommendations	94

REFERENCES

APPENDICES

LIST OF TABLES

TABLE NO.	TITLE	PAGE
Table 1.1	Mean free path calculations for gases at atmospheric pressure (Kandlikar, 2003)	6
Table 1.2	Knudsen number ranges for various types of flow (Kandlikar, 2003)	7
Table 2.1	Microchannel heat sink dimension	19
Table 2.2	The cooling liquid thermal properties	22
Table 4.1	Calculated data for, α_c of range 2 to 6 at $q=500 \text{ W/m}^2$ with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \text{ }\mu\text{m}$)	56
Table 4.2b	Variation of α_c and β with, N and W_{pitch} under channel height constraint, $H_c=180 \text{ }\mu\text{m}$	65
Table 4.3a	Variation of W_{pitch} and α_c with N at constant β under channel height constraint, $H_c=360 \text{ }\mu\text{m}$	70

TABLE NO.	TITLE	PAGE
Table 4.3b	Variation of W_{pitch} and α_c with N at constant β under channel height constraint, $H_c=180 \mu\text{m}$	71

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
Figure 1.1	Liquid cooling using microchannels heat sink (Joshi, 2002)	4
Figure 1.2	Rectangular, trapezoidal and double-trapezoidal KOH-etched microchannels (Morini, 2004)	5
Figure 2.1	Technology selection guide of heat exchanger in term of thermal efficiency and power dissipation (www.aavidthermalloy.com)	17
Figure 2.2	Structure of a rectangle microchannel heat sink and the unit of cell (Li and Peterson, 2005)	18
Figure 2.3	The entrance length of the microchannel with the variation of channel aspect ratio, α_c with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$)	20
Figure 2.4	A cycle composed of a reversible and irreversible process	28

FIGURE NO.	TITLE	PAGE
Figure 2.5	The entropy of a control volume changes as a result of mass flow as well as heat flow	31
Figure 3.1	A model of microchannel convective heat transfer	42
Figure 4.1	Effect of, α_c of range 2 to 6 with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on thermal resistance	56
Figure 4.2	Effect of α_c of range 2 to 6 with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on total pressure drop	57
Figure 4.3	Effect of heat flux at α_c of range 2 to 6 with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on thermal resistance	58
Figure 4.4	Effect of pumping power α_c of range 2 to 6 with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on Reynolds number	59
Figure 4.5	Effect of pumping power at α_c of range 2 to 6 with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on thermal resistance	60
Figure 4.6	Effect of α_c of range 2 to 20 with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on thermal resistance	61

FIGURE NO.	TITLE	PAGE
Figure 4.7	Effect of α_c of range 2 to 20 with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on total pressure drop	62
Figure 4.8a	Effect of β on thermal resistance under channel height constraint, $H_c=360 \mu\text{m}$	66
Figure 4.8b	Effect of β on thermal resistance under channel height constraint, $H_c=180 \mu\text{m}$	67
Figure 4.9	Effect of β on fin efficiency, η_{fin} for $N=40$ under channel height constraint, $H_c=360 \mu\text{m}$	67
Figure 4.10a	Effect of β on total pressure drop under channel height constraint, $H_c=360 \mu\text{m}$	72
Figure 4.10b	Effect of β on total pressure drop under channel height constraint, $H_c=180 \mu\text{m}$	72
Figure 4.11a	Effect of, N on thermal resistance under channel height constraint, $H_c=360 \mu\text{m}$	73
Figure 4.11b	Effect of, N on thermal resistance under channel height constraint, $H_c=180 \mu\text{m}$	74

FIGURE NO.	TITLE	PAGE
Figure 4.12a	Effect of N on total pressure drop under channel height constraint, $H_c=360 \mu\text{m}$	75
Figure 4.12b	Effect of N on total pressure drop under channel height constraint, $H_c=180 \mu\text{m}$	76
Figure 4.13	Effect of α_c of range 2 to 6 at $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on entropy generation, S_{gen}	78
Figure 4.14	Effect of α_c of range 2 to 20 at $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on entropy generation, S_{gen}	79
Figure 4.15	Effect of heat flux for α_c of range 2 to 6 with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on entropy generation, S_{gen}	80
Figure 4.16	Effect of pumping power on for α_c of range 2 to 6 with $\beta=0.6$ and $N=100$ ($W_{pitch}=100 \mu\text{m}$) on entropy generation, S_{gen}	81
Figure 4.17a	Effect of β on entropy generation, S_{gen} under channel height constraint, $H_c=360 \mu\text{m}$	83

FIGURE NO.	TITLE	PAGE
Figure 4.17b	Effect of β on entropy generation, S_{gen} under channel height constraint, $H_c=180 \mu\text{m}$	83
Figure 4.18a	Effect of N on entropy generation, S_{gen} under channel height constraint, $H_c=360 \mu\text{m}$	85
Figure 4.18b	Effect of, N on entropy generation, S_{gen} generation, S_{gen} under channel height constraint, $H_c=180 \mu\text{m}$	85

LIST OF SYMBOLS

A_c	-	cross-section area of a single fin [m ²]
A_{eff}	-	total effective heat transfer are [m ²]
c_p	-	specific heat [J/kg.°C]
D_h	-	hydraulic diameter [m]
E	-	energy [J]
\dot{E}	-	energy rate [J/s]
ΔE	-	energy change [J]
f	-	friction factor
H_c	-	channel height [m]
Kn	-	Knudsen number
h	-	specific enthalpy of the fluid [J/kg]
h_{av}	-	average heat transfer coefficient [W/m ² .°C]
k	-	thermal conductivity of solid [W/m.°C]
k_{eq}	-	ratio of thermal conductivity of fluid to solid $\equiv k_f/k$
k_f	-	thermal conductivity of fluid [W/m.°C]
L_e	-	entrance length [m]
L_x	-	total length of microchannel heat sink [m]
L_y	-	total height of microchannel heat sink [m]
L_z	-	total width of microchannel heat sink [m]
m	-	fin parameter [m ⁻¹]
\dot{m}	-	mass flow rate [kg/s]

N	-	total number of microchannels
Nu_{Dh}	-	Nusselt number based on hydraulic diameter $\equiv D_h h_a / k_f$
P	-	pressure [Pa]
\bar{P}	-	pumping power [W]
ΔP	-	total pressure drop [Pa]
Pe_{Dh}	-	Peclet number based on hydraulic diameter $\equiv D_h U_m / \alpha$
Pr	-	Prandlt number
Q	-	heat transfer [J]
\dot{Q}	-	rate of heat transfer [W]
\dot{Q}_b	-	heat transfer rate from the base [W]
\dot{Q}_{fin}	-	heat transfer rate from the fin [W]
q	-	heat flux [W/m^2]
R	-	gas constant [$J/kg \cdot ^\circ C$]
R_{th}	-	thermal resistance [$W/^\circ C$]
Re_{Dh}	-	Reynolds number based on hydraulic diameter $\equiv D_h U_m / \nu$
S_{gen}	-	total entropy generation rate [$W/^\circ C$]
S	-	entropy [$J/^\circ C$]
\dot{S}	-	rate of entropy [$J/^\circ C \cdot s$]
ΔS	-	entropy change [$J/^\circ C$]
s	-	specific entropy of fluid [$J/kg \cdot ^\circ C$]
T	-	temperature [$^\circ C$]
T	-	thickness [m]
t	-	time [s]
U_m	-	mean velocity in channels [m/s]
U	-	internal energy [J]
ΔU	-	change of internal energy [J]
u	-	specific internal energy [J/kg]
\dot{V}	-	total volume flow rate [m^3/s]

ν	-	specific volume of fluid [m^3/kg]
W	-	work [J]
\dot{W}	-	rate of work [J/s]
W_c	-	channel width [m]
W_{pitch}	-	width of the pitch [m]
W_w	-	fin thickness [m]

Greek Symbols

ΔP	-	pressure drop across microchannel [Pa]
α	-	thermal diffusivity [m^2/S]
α_c	-	channel aspect ratio $\equiv H_c/W_c$
α_{hs}	-	heat sink aspect ratio $\equiv L_x/W_c$
β	-	fin spacing ratio $\equiv W_c/W_w$
γ	-	ratio of specific heat $\equiv C_p=C_v$
μ	-	absolute viscosity of fluid [kg/m.s]
η_{fin}	-	fin efficiency
Θ	-	energy associated with internal, potential, kinetic and pressure energy [J]
Θ_b	-	base stream temperature, $T_b - T_a$ [$^{\circ}\text{C}$]
ν	-	kinematic viscosity of fluid [m^2/s]
ρ	-	fluid density [kg/m^3]
λ	-	mean free path for gases

Subscripts

a	-	ambient
av	-	average

b	-	base
conv	-	convective
CV	-	control volume
eq	-	equivalent
f	-	fluid
fin	-	single fin
gen	-	generation
hs	-	heat sink
in	-	entrance
int rev	-	internally reversible
m	-	mean value
sys	-	system
out	-	exit
1	-	initial
2	-	final

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
Appendix A-1	Friction factor from experimental results and theoretical predictions for (a) laminar, and (b) turbulent flow (Garimella, 2005)	100
Appendix A-2	Heat transfer from experimental results and theoretical predictions for (a) laminar, and (b) turbulent flow (Garimella, 2005)	101
Appendix A-3	Experimental results on the Poiseuille number for laminar flows in microchannel : Liquid (Morini, 2004)	102
Appendix A-4	Experimental results on the Poiseuille Number for laminar flows in Microchannel: Gases (Morini, 2004)	103

APPENDIX	TITLE	PAGE
Appendix A-5	Experimental results on the laminar-to-turbulent flow transition in microchannel quoted in the open literature (Morini, 2004)	104
Appendix A-6	Experimental results on the Nusselt number for single-phase internal flows through microchannel (Morini, 2004)	105
Appendix A-7	Comparison between the experimental correlations for the Nusselt number in microchannels : (a) Gas flow (b) Liquid flow (Morini, 2004)	106
Appendix A-8	Summary of correlations for laminar Convective heat transfer inside microchannels (Morini, 2004)	107
Appendix A-9	Values of C and $C_{f,t}$ as a function of the microchannel cross-section geometry (Morini, 2004)	108
Appendix A-10	(a) The optimize first element (b) The optimized second construct (c) Second construct containing an unspecified number of first elements (d) The optimize third construct (Bejan, 1997)	109

APPENDIX	TITLE	PAGE
Appendix A-11	Summary of the optimized geometries and fluid flow resistance of the first nine constructs (Bejan, 1997)	111
Appendix A-12	Color displays of in flowing (blue) and out flowing (red) streams in the (a) first construct (b) second construct and (c) third construct (Bejan, 1997)	112
Appendix B-1	MATLAB 7 Code	113

CHAPTER 1

INTRODUCTION

1.1 Background

Thermal resistance is one of the most important parameter on designing heat transfer system. Thermal resistance of a system is defined as the ability of the system to resist the flow of heat. In heat transfer augmentation system, the better the system will be the lesser the thermal resistance. Minimization of thermal resistance means heat can easily flow from the heat generated body to its surrounding air.

Tuckerman and Pease (Tuckerman and Pease, 1981) designed and tested a very compact, water-cooled integral heat sink of microscopic channel size for silicon integrated circuits. In the study, high-aspect ratio channels offered a good thermal performance as it exhibited significant drop in thermal resistance. Since that, numerous studies have been done by Knight et al. (Knight et al, 1992), Fedorov and Viskanta (Fedorov and Viskanta, 2000), Qu and Mudawar (Qu and Mudawar, 2002), Li and Peterson (Li and Peterson, 2005) and many other researchers on microchannel research. Most of the studies defined the high thermal performance according to the lowest thermal resistance exhibited by the microchannel heat sink.

Microchannel as flow passages having hydraulic diameters of 10 to 200 micrometers (Kandlikar et. al, 2003) analogous to the alveolar duct of human lung system size is claimed to be one of the most effective devices in high heat-flux heat transfer application. Microchannel research study has generated increasing interest among microelectronics manufacturers, engineers and researchers on developing next-generation power electronics for cooling high heat fluxes up to 100 Watts/cm² at high temperatures in compact, low volume and lightweight packages. The research interest is due to microchannel heat sink wide application as efficient compact heat exchanger in electronics cooling, automotive industry; for example hybrid electric and fuel cell vehicles development, and also in military, biological and other commercial applications where aggressive cooling techniques are required. In addition, new microfabrication techniques like etching, vapor deposition, diffusion bonding, extruded aluminum multichannel tubes (Garimella, 2005) and micromachining, microstamping, hybridization, and system-on-chip integration (Kandlikar et. al, 2003) have also increased application and research study on microchannel heat sink.

Besides all rewarding effects of microchannels application, high friction factors and pressure gradient due to the high available surface area for a given volume seem to be some of its drawbacks (Kandlikar, 2003). Moreover, challenges like rarefaction effects in gas flows, electric double layer, entrance region and developing flows, experimental errors issues as well as inaccurate prediction in heat transfer and fluid flow behavior from the studies conducted need to be clearly addressed.

Thermal performance analysis and optimization have been the subject of interest for most microchannel researchers. The thermal performance analysis and optimization were generally done to identify the best microchannel design that exhibit low thermal resistance and thus, good thermal performance. However, past studies did not consider the simultaneous effect of heat transfer and fluid friction on evaluating overall thermal performance of microchannel. Moreover, Bejan (Bejan, 2000) stated that the most common method of thermal performance analysis and optimization is only by assumption on the system geometry, architecture and components.

The assumption principle did not actually maximize the thermodynamics performance of the system or minimize its reversibility. Bejan pioneered the method of deduction of thermal engineering system geometry, architecture and components based on maximization of thermodynamics performance. The entropy generation minimization (EGM) method is used in his study (Bejan, 2001). EGM method considered simultaneously the effect of heat transfer and fluid friction on thermal engineering design. Imperfection of the thermodynamics system is optimally distributed and leads to the final system configuration.

The present study will utilize the EGM principle on Li and Peterson's (Li and Peterson, 2005) microchannel design. An analytical study which employs EGM method formulation developed by Khan et al. (Khan et al., 2006) is used for comparison with thermal performance.

1.2 Overview on Microchannels

Microchannel heat exchangers use channel width of 10 to 1,000 μm . By constraining the flow passage to such narrow channels, thermal diffusion lengths are short, and the characteristic heat transfer coefficients are very high. Since the effect of heat transfer is dominance, relatively short flow passages are required to minimize thermal resistance of the system. Furthermore, with many flow passages in parallel in a small device, the pressure drop can be small as well. These are important factors in the design of microchannel heat sink. Joshi (Joshi, 2002) illustrated a schematic diagram of a liquid cooling microchannel heat sink as shown in *Figure 1.1*.

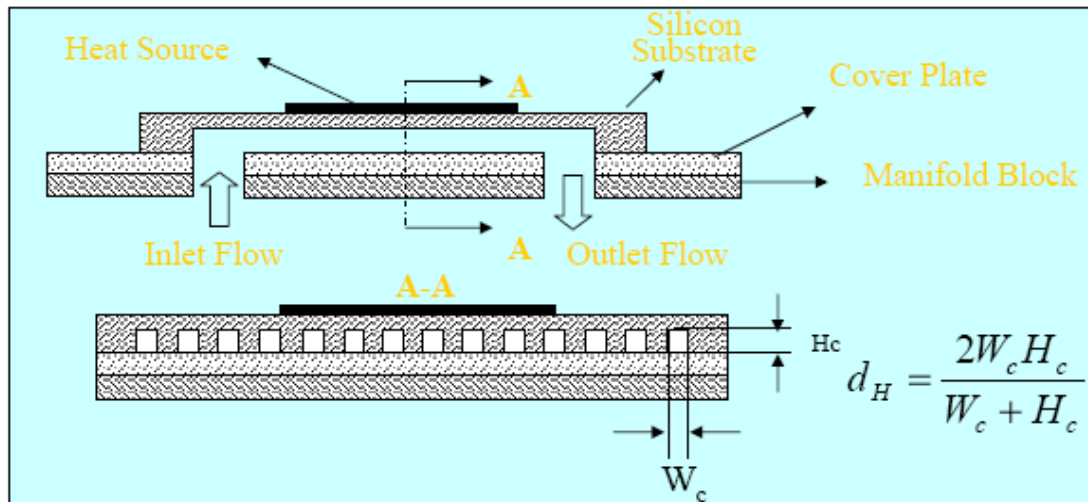


Figure 1.1 - Liquid cooling using microchannels heat sink (Joshi, 2002)

All devices with characteristic dimensions of flow passage between 1 μm and 1 mm are called micro-devices (Morini, 2004). However, Kandlikar (Kandlikar, 2003) classified channels between the ranges of 10 μm to 200 μm as microchannel. Generally, the micro-systems can be subdivided into three categories (Morini, 2004):

- MEMS: Micro-Electro-Mechanical Systems (for instance, air bag acceleration sensors, HD reader, etc.).
- MOEMS: Micro-Opto-Electro-mechanical Systems (for instance, micro endoscope, etc.).
- MFD: Micro-flow devices (for instance, micro heat exchangers, micro-pumps, etc.).

The geometry of MFD microchannels depends on the technology used to build the MFD. The four process technologies that can be used to fabricate MFD are (Morini, 2004):

- Micromechanical machining (such as diamond machining, laser processes, focused ion beam, microdrilling);
- X-ray micromachining (such as LIGA Litographie-Galvanoformung-Abformung);

- Photolithographic-based processes (such as Silicon chemical etching);
- Surface and surface-proximity-micromachining (epimicromachining) processing techniques.

Numerous experimental studies have been conducted on microchannels with trapezoidal, rectangular, double-trapezoidal (hexagonal), and also circular cross-section. However, the results obtained showed inconsistency in term of the fluid flow and heat transfer behavior. *Figure 1.2* shows geometry that can be fabricated using Alkali hydroxide + Water-etched (KOH-etched).

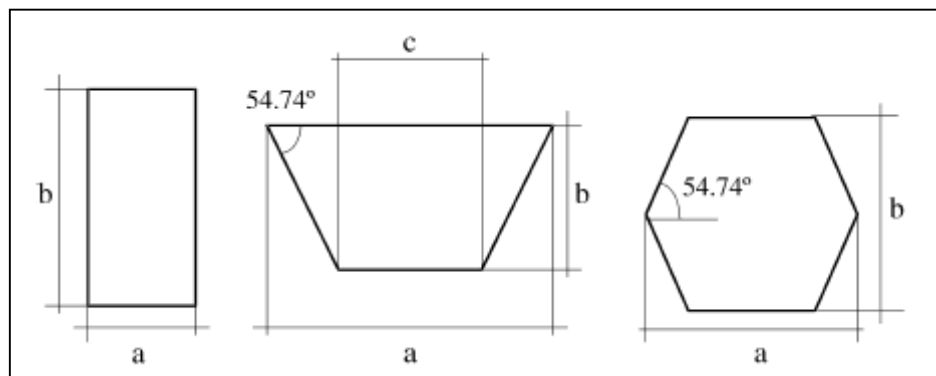


Figure 1.2 - Rectangular, trapezoidal, and double-trapezoidal KOH-etched microchannels (Morini, 2004)

The flow regime through a MFD depends strongly on the method used in order to induce the motion of the fluid. The fluid flow in a MFD can be obtained in two different ways (Morini, 2004):

- By applying an external pressure gradient (*pressure driven motion*). In this case a Poiseuille flow profile is generated along the channel. In fact, because the Reynolds number associated with the flow is in general small (due to the small hydraulic diameter of the channel), the flow is usually laminar and the velocity varies across the entire cross-sectional area of the channel.

- By applying an external electric field (*electrokinetically driven flow*). In this case the fluid velocity only varies within the so-called Debye screening layer near the channel walls.

Some of the important terminologies in microchannel study are:

1. Knudsen Number

Knudsen number is a measure of the departure from the continuum,

$$Kn = \frac{\lambda}{D_h}$$

where,

$$D_h = \text{hydraulic diameter of the flow channel, } \frac{2W_c H_c}{W_c + H_c}$$

$$\lambda = \text{mean free path for the gases calculated from, } \lambda = \frac{\mu\sqrt{\pi}}{\rho\sqrt{2RT}}$$

with R is a gas constant in (J/kg.K), μ is the dynamic viscosity in N/ms and ρ is the density in kg/m³ and T is the absolute temperature in Kelvin

Mean free paths for some common gases and the Knudsen number for various types of flow are presented in *Table 1.1* and *Table 1.2* respectively.

*Table 1.1 – Mean free path calculations for gases at atmospheric pressure
(Kandlikar, 2003)*

Gas	T (K)	R (J/kg.K)	ρ (kg/m³)	μ (kg/m.s)	λ (μm)
Air	300	287.0	1.1614	1.846×10^{-5}	0.068
Helium	300	2077.03	0.1625	1.99×10^{-5}	0.194
Hydrogen	300	4124.18	0.08078	8.96×10^{-5}	0.125
Nitrogen	300	296.8	1.1233	1.782×10^{-5}	0.066

Table 1.2 – Knudsen numbers ranges for various types of flow (Kandlikar, 2003)

Range of Knudsen Numbers	Type of Flow
$0.001 > Kn$	Continuum Flow: no rarefaction effects
$0.1 > Kn > 0.001$	Slip Flow: rarefaction effects that can be modeled with a modified continuum theory accounting for wall slip
$10 > Kn > 0.1$	Transition Flow: a type of flow and free molecular flow that is analyzed statistically; i.e with Boltzman equation
$Kn > 10$	Free Molecular Flow: motion of individual molecules must be modeled and treated statistically

1.3 Entropy Generation Minimization (EGM)

Bejan first introduced optimization of heat sink using EGM in 1996 (Bejan, 1996). EGM method evaluates the relevant system parameters that simultaneously relate thermal performance to the viscous effects of the cooling system. The entropy generation associated with thermal resistance and fluid friction effects provide direct assessment of lost potential for work. In a heat sink system the lost potential for work means the lost ability of the system to transfer heat to the surrounding cooling medium. Optimization using entropy generation minimization method will eventually produce the optimum dimensions and performance within the given set of constraint.

1.4 Constructal Theory

The main discovery of constructal theory is purely geometric: *any finite-size portion of this composite can have its shape optimized such that its overall resistance to flow is minimal* (Bejan, 1997). Bejan (Bejan, 1997) stated that by optimizing volume shape at every length scale the optimal-access solution for the total volume can be obtained. The sequence of this optimized structure will begin with the smallest building block (elemental system), and proceeds toward larger building block assemblies and constructs).

Constructal theory is an emerging new theory. The study on constructal theory was pioneered by Bejan. He predicted shape flow phenomena existing in nature using constructal theory. Proportionality between width and depth in rivers of all sizes, the nearly round cross-sections of all blood vessels and bronchial passages, the dendritic shape of the snowflake and the pattern formed by cracks in a solid that shrinks upon cooling or drying (e.g., mud cracks) are some of the natural structure that have been observed. Bejan (Bejan, 1998) stated that natural structure is often exhibited by a flow that connects a point to a finite-size volume (an infinity of points). A tree-shaped, loopless network is the most visible part of the natural structure. A tree network is not only exist in living systems but also to nonliving systems. The examples of tree network for living and nonliving system are lungs, botanical trees, vascularized tissues, river basins and deltas, lightning, turbulent jets, neural dendrites, dendritic crystals, street patterns -urban growth and other patterns of transportation and telecommunication.

Currently, tree and other natural structures (animate and inanimate) are nondeterministic (Bejan, 1997). Nondeterministic is analogous to fractal geometry, where any tree network can be generated by repeating a suitably designed algorithm and interrupting it at a small and finite scale. Fractal geometry is descriptive, not predictive as the algorithm and the smallest scale have to be postulated. This doctrine was challenged by Bejan (Bejan, 1997 and 1999), which showed that all the geometric details of the tree and the rest of the volume-to-point flow path can be predicted in purely deterministic way.

Bejan claimed that every portion of the given volume can have its shape optimized, such that its resistance to flow is minimal. The requirement of constructal is the time arrow of its construction, starting from the smallest building block which is an elemental system, and proceeds toward larger building blocks which is assemblies or constructs. To differentiate between determinism theory and fractal thinking, the geometric optimization approach developed by Bejan (Bejan, 1996 and 1997) was named constructal theory. The constructal theory basic construction and general idea are illustrated in *Appendix A-10* to *A-12*.

Constructal theory has been applied to the construction of fluid flow and thermodynamics system. Constructal theory together with entropy generation minimization can determine the optimal configuration of the thermodynamics system. Constructing the heat sink design based on entropy generation minimization is expected to simultaneously reduce the thermal resistance and pressure drop for the system.

1.5 Literature Review

Experimental work by Tuckerman and Pease (Tuckerman and Pease, 1981) revealed that the very compact water-cooled integral heat sink for silicon integrated circuits has exhibited maximum power dissipation capability up to 790 W/cm^2 . The experiment was conducted using rectangular channel of width in the range of 50 to 60 μm . Since that pioneering work, various analytical, numerical and analytical studies have been conducted to further evaluate and investigate the performance as well as heat transfer and fluid flow through a microchannel.

Garimella (Garimella, 2005) did a review on previous experimental works for a single-phase flow. Results of the study are shown in *Appendix A-1* and *A-2*. *Appendix A-1* shows relationship between friction factor and Reynolds number and *Appendix A-2* illustrates relationship between Nusselt number and Reynolds number.

The work done by different researchers exhibited little agreement for the fluid flow and the heat transfer trend. However, the experimental results show significant difference from the theoretical predictions which is based on macroscale behavior. Furthermore, lower Reynolds number for transition of laminar to turbulent regimes from the study conducted exhibits clear distinction between macro and micro-scales channels (Garimella, 2005).

Another important bibliographical review from open literature on experimental works of convective heat transfer through microchannels was done by Morini (Morini, 2004). The review was done based on experimental work accomplished by Peng and Peterson (Peng and Peterson, 1996), Mala and Li (Mala and Li, 1999), Harms et al. (Harms et al., 1998), Qu and Mudawar (Qu and Mudawar, 2002), Wu and Cheng (Wu and Cheng, 2003) and others. Morini has presented the analysis by comparing the friction factor, laminar-to-turbulent transition, and also on the Nusselt number (*Appendix A-1 to A-6*). Morini made a similar conclusion as Garimella by stating that the friction factor and the Nusselt number in the microchannels are not in good agreement with the conventional theory and also deviates from one another. The predicted reasons of disagreement between the results are caused by rarefaction and compressibility effects, viscous dissipation effects, electro-osmotic effects (EDL), property variation effects, channel surface conditions (relative roughness) and experimental uncertainties (Morini, 2004). Morini also suggested that further systematic studies need to be conducted in order to clearly explain the transport mechanisms through microchannel.

Besides various experimental works done, analytical studies were also conducted to investigate fluid flow and heat transfer behavior of microchannel. Initially, classical fin analysis was adopted in the analytical analysis of modeling overall heat transfer and optimizing the geometry of microchannels. However, assumptions like the fin efficiency, fixed heat transfer coefficient, constant temperature difference between the solid and the bulk fluid, and negligible axial heat conduction lead to inaccurate result (Li and Peterson, 2005). On the contrary, simple fin analysis by Qu and Mudawar (Qu and Mudawar, 2002) that employed heat transfer correlations which takes into account thermal entrance effect exhibits more accurate results than assuming fully developed correlations. Kim and Kim (Kim and

Kim, 2005) conducted a model based on an averaging approach for a uniform wall temperature. In this model, the microchannel heat sink is modeled as a fluid-saturated porous medium, which is frequently termed the porous-medium approach. Mathematically, this is equivalent to averaging the velocity and temperature distributions in the direction perpendicular to the flow direction.

Continuum model derived from Navier-Stokes equation consisting conservation of mass, momentum, and energy equations has been applied numerically to illustrate the transport mechanism of a microchannel heat sink. Even though there were analyses on two-dimensional (2-D) numerical model conducted in the past, employing a three-dimensional numerical model is the best solution to study microchannel heat transfer and fluid flow. Fedorov and Viskanta (Fedorov and Viskanta, 2000) developed a three dimensional (3-D) model to investigate the conjugate heat transfer in a microchannel heat sink. The incompressible laminar Navier Stokes equations of motion were employed as the governing conservation equations and further solved using finite-volume method. The theoretical model developed is validated by comparing with the available experiment data of thermal resistance and the friction coefficient over a wide range of Reynolds numbers. The analysis provides a unique fundamental insight into the complex heat flow pattern established in the channel due to combined convection and conduction effects in the three-dimensional setting.

Another investigation on a 3-D numerical analysis was conducted by Qu and Mudawar (Qu and Mudawar, 2002). The fluid flow and heat transfer occurring in microchannel with rectangular cross-sections was modeled based on finite different method and SIMPLE algorithm to solve the governing equations. The model was validated by comparing the predictions with analytical solutions and available experimental data. A few interesting conclusions were made from the study. It was found that the temperature rise along the flow direction in the solid and fluid regions can be approximated as linear. The highest temperature is exhibited at the heated base surface of the heat sink immediately above the channel outlet. It was also found that, the heat flux and Nusselt number have much higher values near the channel inlet and vary around the channel periphery. It is concluded that Reynolds number affects the length of the flow developing region and for a relatively high Reynolds

number of 1400, fully developed flow may not be achieved inside the heat sink. Increasing the thermal conductivity of the solid substrate reduces the temperature at the heated base surface of the heat sink, especially near the channel outlet (Qu and Mudawar, 2002).

One of the most recent 3-D numerical model was developed by Li et al. (Li et al., 2004). They conducted a detailed numerical simulation of the heat transfer occurring in silicon-based microchannel heat sinks using a simplified 3-D conjugate heat transfer model with a 2-D and 3-D heat transfer. The microchannel heat sink evaluated in the investigation, consists of a 10-mm long and 10-mm wide silicon substrate with rectangular microchannel having bottom width ranging from 20 μm to 220 μm , aspect ratio ranging from 2 to 7, and a pitch ranging from 70 μm to 250 μm (the number of channels ranges from 40 to 140). Li and Peterson (Li and Peterson, 2005) conducted the same numerical model as Li et. al (Li et. al, 2004) in order to investigate the most optimized geometry of a microchannel with liquid flow. It was concluded that the optimal number of channels was found to be 120 per centimeter (or $W_{pitch} \approx 80\mu\text{m}$) with an aspect ratio, as large as possible (i.e, very deep grooves). The results obtained from this study will be the basis on comparing the present study analysis.

It is well known that the cross-sectional shape of a channel can have significant influence on the fluid flow and heat transfer characteristics in noncircular microchannel. Various experimental works using different cross-sectional shape, dimension and diameter are shown in *Appendix A-3, A-4* and *A-6*. The reviewed conducted by Morini (Morini, 2004) demonstrated the fluid flow and heat transfer characteristics for liquid and gases. It can be stated that no conclusion can be made as there is inconsistency of the results from one to another. In addition, the derived empirical correlations may differ from each other, with each only applicable to the geometry of a particular heat sink and fabrication process.

Bejan (Bejan, 1996 and 1997) did a fundamental study on constructal theory in thermodynamics and fluid flow system. On optimizing the fluid flow path between one point and a finite size volume, overall flow resistance must be minimized. The argument is valid for a fixed flow rate and duct volume. *Appendix A-10* illustrates the

optimization of first, second and third construct using constructal theory. Bejan featured two novel methodologies in constructal theory which are *optimization process have a definite direction: from small volume elements toward larger volume (assemblies) and the smallest scale of the network is finite and known (predictable)*. Summary of the optimized geometries and fluid flow resistances of the first nine constructs is shown in *Appendix A-11*. Bejan further reviewed the constructal theory from thermodynamic and geometric optimization to predicting shape in nature (Bejan, 1998). Bejan (Bejan, 1999) extended the constructal theory to systems that is cooled volumetrically by tree networks of channels with fluid flow. Parallel-plate channel and round tubes are fluid channel geometries that have been optimized.

Apart from the parallel tree network, Bejan et al. (Bejan et al., 2000) did a study on flow systems of T-, Y-, and cross (+) shape. The study managed to deduce every geometric detail on the way to get the optimized flow structure. Based on the examples, it was concluded that optimized geometry has the effect of “partitioning” optimally on the system features for instance the pressure drop, flow resistance and amount of thermal insulation. Even though Bejan work was particularly done for macroscopic scales geometry, however constructal theory is a universal law either for a macro and micro scale geometry. Muzychka (Muzychka, 2005) did a study on forced convection cooled microchannel heat sink and heat exchanger by utilizing constructal design theory. The study was on determining the circular and non-circular optimal duct geometry using simple approach on approximate solution. Arrays of square channels, equilateral or isosceles right triangles give the highest heat transfer per unit volume. On the other hand, the square and isosceles right triangle is the outstanding choice for the most efficient packing (Muzychka, 2005). The drawback of this study was the optimized number of channels was not considered.

Even though EGM method has pioneered by Bejan (Bejan, 1996), many researchers have put their effort on utilizing the principle to various heat sink application. Sahin (Sahin, 1997) applied EGM method for various macro scale duct geometries with constant heat flux and laminar flow. In the study, circular duct was found to be superior in performance if the frictional contribution of entropy generation becomes significant. Work by Ogiso (Ogiso, 2001) demonstrated the

assessment of overall cooling performance in thermal design of electronics based on entropy generation. The analytical, numerical and experimental data provide an agreement that the overall cooling performance is increased with decreasing entropy generation rate. Ratts and Atul (Ratts and Atul, 2004) compared optimal Reynolds number and minimum entropy generation for square, equilateral and rectangle with aspect ratio two and eight. Rectangle cross section with aspect ratio eight, exhibited the smallest optimal Reynolds number, entropy generation and flow length. Khan et al. (Khan et al., 2006) developed a formulation on optimization of microchannel heat sink using entropy generation minimization method. Air was used as the cooling fluid in the analysis. It was illustrated that volume mass flow rate and channel aspect ratio are strongly affected the entropy generation of three different Knudsen number.

1.6 Study Scopes and Objectives

The present study will use Li and Peterson's (Li and Peterson, 2005) geometry of parallel rectangular microchannel structure using Khan et al. (Khan et al., 2006) EGM formulation. The ultimate objective of this study is to determine optimized parallel rectangular microchannel flow geometry that simultaneously improved heat transfer and fluid flow performance. This analytical investigation will make use the minimization of entropy generation principle and fin analysis theory as in Khan et al. (Khan et al., 2006). The constant heat flux and pumping power will be set as the boundary conditions for the incompressible cooling fluid under laminar fully developed flow.

The study will further evaluate the heat transfer and fluid flow performance of Li and Peterson's (Li and Peterson, 2005) parallel rectangular microchannel in term of entropy generation rate, S_{gen} . The existence of optimized channels aspect ratio, fin spacing and number of channels will also be inspected. The findings will then be compared with Li and Peterson (Li and Peterson, 2005) numerical

investigation results. The relationship between thermal resistance and entropy generation number will also be discussed.

The scopes of this study can be summarized as:

1. To evaluate the heat transfer performance of Li and Peterson's (Li and Peterson, 2005) parallel rectangular microchannel in terms of thermal resistance, R_{th} .
2. To compare the effect of thermal resistance on microchannel configuration using analytical formulation modified by Khan et al. (Khan et al., 2006) and numerical analysis conducted by Li and Peterson (Li and Peterson, 2005).
3. To evaluate the heat transfer and fluid flow performance on Li and Peterson's (Li and Peterson, 2005) parallel rectangular microchannel in term of entropy generation rate, S_{gen} .
4. To identify the existence of optimized channel aspect ratio, ratio of channel side wall thickness to the channel width and number of channels using entropy generation minimization formulation.

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