ROOT FINDING OF A SYSTEM OF NONLINEAR EQUATIONS USING THE COMBINATION OF NEWTON, CONJUGATE GRADIENT AND QUADRATURE METHODS

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A dissertation submitted in fulfilment of the
requirement for the award of the degree of
Master of Science (Mathematics)

Fakulty of Science
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JANUARY 2013

This dissertation is dedicated to my parents,

Zulkefli bin Idris & Nor Zihan binti Jusoh

for their love, endless support and encouragement.

Also, this dissertation is dedicated to my beloved siblings,

Nor Zulfa Irwani binti Zulkefli,

Nor Atirah Izzah binti Zulkefli,

Zul Hasanuddin bin Zulkefli,

Zul Faziruddin bin Zulkefli,

Zul Syamiluddin bin Zulkefli

for a great source of motivation and inspiration.

ACKNOWLEDGEMENTS

In the name of ALLAH, this dissertation is finally managed to complete. It is an honor for me to send my fullest gratitude to all these people that have involved directly or indirectly in completing this dissertation.

I would like to thank my supervisor, Dr Yeak Su Hoe because without his valuable guidance and support throughout the duration of this dissertation, I could never complete this dissertation as it is now. His assistance and support throughout the duration of this dissertation really motivate me greatly.

I would also like to thank my beloved family. They will always be there to listen and comfort me when I get stuck with obstacles and challenges. They have always been willing to provide their concern and mentally support continuously when I needed.

Last but not least, I would like to take this opportunity to thank all of my friends and those who give me a big hand during this dissertation. Thank you for helping me without any complaint.

ABSTRACT

A system of nonlinear equations is a well-known problem in many fields of science and engineering. There may have no solution or multiple solutions for the systems. This dissertation is aimed to solve a system of nonlinear equations using combination of Newton, conjugate gradient and quadrature methods. This new method is able to improve the approximated solution of a system of nonlinear equations. C++ compiler is used to achieve the objectives of this dissertation. Besides that, conjugate gradient method is used to find sufficiently accurate starting approximations for the Newton-based techniques. Three examples of system of nonlinear problems have been presented in this dissertation. With the help of C++ compiler, the numerical technique has been reviewed. On the other hand, this dissertation is intended to analyze the efficiency and effectiveness of these methods through numerical simulations. Numerical results show that these methods surpassed the other methods in both efficiency and effectiveness.

ABSTRAK

Sistem persamaan tak linear merupakan masalah yang terkenal dalam pelbagai bidang sains dan kejuruteraan. Ianya berkemungkinan tiada penyelesaian atau mempunyai pelbagai penyelesaian terhadap sistem. Disertasi ini bertujuan untuk menyelesaikan sistem persamaan tak linear menggunakan gabungan dari kaedah Newton, 'Conjugate Gradient' dan 'Quadrature'. Kaedah baru ini dapat meningkatkan penghampiran penyelesaian sistem persamaan tak linear. Perisian C++ akan digunakan untuk mencapai objektif-objektif disertasi ini. Selain itu, 'Conjugate Gradient Method' digunakan untuk mencari ketepatan penghampiran dengan menggunakan permulaan dari teknik-teknik berasaskan Newton. Tiga contoh masalah sistem tak linear dibentangkan dalam disertasi ini. Dengan bantuan perisian C++, teknik berangka telah dikaji semula. Selain itu, disertasi ini bertujuan untuk menganalisis kecekapan dan keberkesanan kaedah ini melalui simulasi berangka. Keputusan berangka menunjukkan bahawa kaedah ini mengatasi kaedah lain dalam kecekapan dan keberkesanan.

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INTRODUCTION

1.1 Research Background

Numerical methods is a popular subject and the basis of all branches of science and technology. In the past, solving problems numerically often meant a great deal of programming and numerical problems. The solution of systems of nonlinear equations is a well-known problem in many fields of science and engineering. Several types of numerical methods exist, each with their advantages and disadvantages.

The system of nonlinear equations

$$\begin{aligned} &f_1(x_1, x_2, \dots, x_n) = 0, \\ &f_2(x_1, x_2, \dots, x_n) = 0, \\ &\vdots \\ &f_n(x_1, x_2, \dots, x_n) = 0, \quad \text{where } n = 0, 1, 2, \dots \end{aligned} \tag{1.1}$$

where each function, f_n maps a vector $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ of the n-dimensional space $\Re^n$ into the real line $\Re$. The system of nonlinear equations in n unknowns can also be represented by defining a function F mapping $\Re^n$ into $\Re^n$ as

$$\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_n(\mathbf{x}))^T = 0. \quad (1.2)$$

One of the methods for solving a system of nonlinear equations is Newton's method. There may be no solutions or multiple solutions. There is in general no algorithm to find a solution in finite time, and we must accept approximate solutions as a substitute (Kahaner, Moler and Nash, 1989). The method for solving the problem will be iterative. However, it is much harder to guarantee the convergence of a system of equations or to guarantee the existence of a solution (Kahaner, Moler and Nash, 1989).

In recent years, several iterative methods have been developed to solve the nonlinear system of equations $\mathbf{F}(\mathbf{x}) = 0$ by using essentially Taylor's polynomial, quadrature and other techniques (Noor and Waseem, 2009). It is already known that quadrature formula and nonlinear equations are connected. Therefore, in this research we will use these quadrature formula to obtain methods for solving nonlinear equations.

Besides that, the general conjugate gradient method was proposed as a technique for solving nonlinear system, real valued, linear system and n dimensional problems. Following that, the general conjugate gradient method was developed to solve unconstrained optimization problems (Fletcher, 1989). The preconditioned conjugate gradient method has been accepted as a powerful tool for solving the linear systems of equations. Applications of the nonlinear algorithm are mainly confined to the diagonally scaled conjugate gradient.

In this research, we analyze the combination of Newton, conjugate gradient and quadrature methods for solving a system of nonlinear equations. A program was developed to solve this problem with C++ compiler.

1.2 Problem Statements

Newton's method is one of the numerical algorithms that can be used to find the root of a system of nonlinear equations. However, the method does not converge or fail if the Jacobian matrix is near to zero or singular. Usually Newton's method is expected to converge with initial values that is close to the solution. Besides that, Newton's method requires computation of $f'(x)$. This may be time-consuming, difficult or impossible, especially if evaluating $f(x)$ requires the evaluation of an integral. Hence, the use of quadrature that will evaluate the integral while conjugate gradient can avoid the singularity. Therefore, combination of Newton, conjugate gradient and quadrature methods are designed to improve the effectiveness of the existing Newton's method. In this research, the combination of Newton and conjugate gradient methods are called preconditioned conjugate gradient method.

1.3 Research Objectives

The main objectives of this research are:

(i) To study the technique of Newton's method, conjugate gradient method and quadrature method.

(ii) To improve the effectiveness of the existing Newton's method.

(iii) To solve a system of nonlinear equations using the combination of Newton, conjugate gradient and quadrature methods with the help of C++ compiler.

1.4 Scope of the Research

This research focuses on solving a system of nonlinear equations. We will only concentrate on Newton's method, conjugate gradient method and quadrature method. The problem of a system of nonlinear equations will be solved using the combinations of Newton, conjugate gradient and quadrature methods. First, we will combine the methods of Newton and conjugate gradient where it is called preconditioned conjugate gradient method. Then, we will combine this method with quadrature method.

The above method is calculated using C++ compiler. At the end, a system of nonlinear equations will be solved by programming using C++ compiler in many fields of science and engineering.

1.5 Significance of the Research

From this research, the combinations of Newton, conjugate gradient and quadrature methods will be able to improve the effectiveness of approximated

solution of a system of nonlinear equations. Besides, the problem will be solved more accurately and faster. On the other hand, the research can be used as a reference for the future studies in many fields of science and engineering.

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