MAGNETOHYDRODYNAMICS OF BLASIUS VISCOELASTIC FLUID WITH VISCOUS DISSIPATION AND SUCTION/INJECTION EFFECTS

NURATIKAH HANI BINTI ABD RANI

A thesis submitted of the

requirements for the award of the degree of

Master of Science (Engineering Mathematics)

Faculty of Science

Universiti Teknologi Malaysia

JANUARY 2013

To my beloved father and mother

Thank you for your support.

ACKNOWLEDGEMENT

Alhamdulillah, I am grateful to Allah S.W.T for His grace and mercy for giving me time, strength and patience in completing this study. Without His willingness, I would not be able to complete this study.

I would like to take this opportunity to express my gratitude to my supervisor, Dr. Sharidan Shafie for all the guidance, knowledge and advice throughout the completion of my study. All his efforts are very much appreciated. I also would like to extend my sincere gratitude to Abdul Rahman Mohd Kasim who kindly offered precious and helpful comments in the preparation of this study.

My appreciation would also go to my beloved family who either directly or indirectly involved in bringing this successful. Their views and opinions are helpful indeed.

Last but not least, I would like to express my heartiest appreciation to all those involved in helping me to complete this study. Unfortunately it is impossible to list all of them in this limited space. Thank you very much.

ABSTRACT

Due to the existence of viscoelastic fluid in technological applications, the research in the viscoelastic fluids has increase rapidly. In this study, the magnetohydrodynamics flow for the Blasius viscoelastic fluid along with the effects of viscous dissipation and suction or injection is considered. Since the equation of motion in viscoelastic fluid is one order higher than the Navier-Stokes or boundary layer equations, an extra boundary condition is imposed by augmenting the boundary condition at infinity. The governing equations are transformed into a non dimensional boundary layer equation by using non dimensional variables. The equations are solved numerically by using Keller-box method. Numerical results consist of the velocity and temperature profiles are presented graphically for assorted values of magnetic parameter, M, viscoelastic parameter, *K*, suction or injection parameter, f_w, Prandtl number, Pr and the ratio moving parameter, λ. It is found that, as the values of all parameters increased, the velocity profiles are also increased but opposite situation occurred in temperature profiles.

ABSTRAK

Oleh kerana wujudnya bendalir likat kenyal di dalam beberapa kegunaan teknologi, penyelidikan berkenaan bendalir likat kenyal telah meningkat dengan mendadak. Dalam kajian ini, aliran hidrodinamik magnet untuk bendalir likat kenyal Blasius berserta dengan kesan pelepasan likat dan sedutan atau suntikan dipertimbangkan. Disebabkan oleh persamaan gerakan dalam bendalir likat kenyal lebih tinggi satu peringkat berbanding dengan persamaan Navier-Stokes atau persamaan lapisan sempadan, maka syarat sempadan tambahan diperlukan dengan menambah syarat sempadan di infiniti. Persamaan menakluk diubah ke bentuk persamaan lapisan sempadan tak bermatra, dengan menggunakan pemboleh ubah tak bermatra. Persamaan diselesaikan secara berangka dengan menggunakan kaedah Kotak-Keller. Keputusan berangka yang terdiri daripada profil halaju dan suhu dipersembahkan secara grafik bagi pelbagai nilai parameter magnetik, M, parameter bendalir likat kenyal, *K*, parameter sedutan atau suntikan, f_w, nombor Prandtl, $\Pr$, dan parameter nisbah gerakan, λ. Didapati bahawa apabila semua nilai parameter meningkat, profil halaju juga meningkat namun keadaan sebaliknya berlaku bagi profil suhu.

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A_1	-	kinematics tensor
A_2	-	kinematics tensor
B_0	-	magnetic field strength
$\vec{C}$	-	momentum of the mass
C_f	-	skin friction coefficient
E_t	-	total energy
Ec	-	Eckert number
$\vec{F}$	-	sum of all forces acting
f	-	dimensionless stream function
f_w	-	dimensionless injection or suction parameter
I	-	identify matrix
K	-	viscoelastic parameter
k_0	-	vortex viscosity
M	-	magnetic parameter
Nu_x	-	local Nusselt number
p	-	pressure scalar
Pr	-	Prandtl number
Q	-	heat flow
Re	-	Reynolds number
T	-	temperature
$\underline{u}$	-	velocity vector

U_w	-	constant velocity
U_∞	-	dimensional constant
u	-	velocity component in x-direction
v	-	velocity component in y-direction
$\underline{v}$	-	fluid velocity
W	-	work done
x	-	coordinate in direction of surface motion
y	-	coordinate in direction normal to surface motion

Greek symbols

α	-	thermal diffusivity
α_1	-	viscoelasticity of fluid
α_2	-	cross-viscosity of fluid
σ	-	stress tensor
σ_0	-	electrical conductivity
η	-	dimensionless similarity variable
θ	-	dimensionless temperature
λ	-	ratio moving parameter
μ	-	viscosity coefficient
υ	-	kinematic viscosity
ψ	-	stream function
ρ	-	fluid density

Subscripts

w	-	condition at the surface
∞	-	condition at ambient medium

Superscripts

' - differentiation with respect to η

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

There are two types of fluid which are Newtonian fluid and non-Newtonian fluid. Newtonian fluid is a type of fluid where the relation between shear stress and shear rate is proportionally linear. On the other hand, non-Newtonian fluid is a type of fluid where the relation between shear stress and shear rate is not a linear function of the spatial variation of velocity at a given temperature and pressure. Non-Newtonian fluids fail to obey Newton's law of friction since non Newtonian fluids form very wide class of different materials, whose only common features are fluidity. It is now generally recognized that non-Newtonian fluids are more appropriate than Newtonian fluids in industrial and technological applications and numerous models were suggested for non-Newtonian fluids with their constitutive equations varying greatly in complexity (Othman, 2010).

Viscoelastic fluid is a fluid that returns to its original shape either fully or partially after the applied stress is released. It is also the subject that explains both the elastic and viscous behaviour of materials. Viscosity describes how powerfully fluids resist rapid changes in shape. For example, honey. Since it tends to deform slowly, thus, it is very viscous. When honey is poured from a jar, it moves very slowly. This is because the internal stresses between molecules increase with relative velocity between molecules. As the movement of the molecules increase, the resistance to that movement become greater. Viscous materials display a time-dependent reaction to deformation.

Elastic deformation can be observed in the way a rubber band acts. Besides rubber bands, elastic deformation can also be observed in springs. The harder a spring is compressed, the more force is needed to hold it there. Springs are called linear because if doubling the amount of compression it will double the amount of force. Though it may not be obviously to the eye, when forces are applied to metals, they get linearly compressed or stretched out. Elastic materials quickly return to their original sizes when all forces are removed. Fundamentally, when changing shape, all materials show some elastic and viscous effects.

Magnetohydrodynamics (MHD) is the study of the interaction of electrically conducting fluids and electromagnetic forces. The word magnetohydrodynamics is from the words magneto which means magnetic field, hydro means liquid, and dynamics means movement. Synonyms of MHD that are less commonly used are the terms hydromagnetics and magnetofluiddynamics MHD problems arise in a broad range of situations. The description of MHD flows connects both the equations of fluid dynamics which is the Navier-Stokes equations and the equations of electrodynamics which is Maxwell's equations. These equations are mutually coupled through the Lorentz force and Ohm's law for moving electrical conductors and these differential equations have to be solved simultaneously, either analytically or numerically.

Since vicoelastic fluid grabs our attention, thus in this study, problem in viscoelastic fluid will be solved. To make it more interesting, MHD flow will be considered in this study.

1.2 Statement of Problem

Due to the existence of viscoelastic fluid in many of technological applications over the past decades, the interest in the viscoelastic fluids has increase largely. For that reason, this study will investigate the problem on the viscoelastic model concentrating on classical Blasius problem over flat plate. Besides, the study also will explore how the boundary layer flow and heat transfer nature execute with viscous dissipation included into the energy equation. Further, the effects of viscoelastic parameter and the ration of moving parameter on the skin friction and heat transfer coefficient with the presence of magnetohydrodynamics (MHD) flow will be studied completely.

1.3 Objectives of the Study

The objectives of this study are:

1. To carry out the mathematical formulation of governing equations of the boundary layer flow of a viscoelastic fluid with the effects of magnetohydrodynamics (MHD) flow.
2. To transform the governing equations into non-dimension equations.
3. To observe the effects of magnetohydrodynamics (MHD) flow to the fluid flow characteristic with the present of viscous dissipation and suction or injection effects.

1.4 Scope of the Study

This study will consider two-dimensional viscoelastic fluid model in Cartesian coordinate and the fluid is assumed to be incompressible. This study will consider the MHD boundary layer flow with suction or injection, and viscous dissipation.

1.5 Methodology of the Study

1.5.1 Mathematical Modelling – Problem Formulation

The governing boundary layer flow equations will be acquired for the new model outlined in the objectives.

1.5.2 Mathematical Analysis – Nonsimilar Transformation

The governing equations will be transformed into a set of coupled nonlinear differential equations

1.5.3 Numerical Computation – Keller's Box Method

The differential equations will be solved numerically by using an implicit finite difference scheme that is the Keller Box method. This method had been described in the books by Cebeci and Bradshaw (1984), and Cebeci (2002). There are four steps involve in this method.

1. Reduce the differential equations to a first-order system;
2. Write the differential equations by using central differences;
3. The resulting equations are linearized by using the Newton's method and write them in matrix-vector form;
4. The linear system are solved by using block-tridiagonal elimination method

In this study, the Keller Box method is not mention in detail since this method had been studied by Lok *et al.* (2008) and Anwar *et al.* (2008).The numerical algorithms is developed by using FORTRAN and to plot the associated graph, Matlab® is used.

1.6 Significant of the Study

Nowadays, non-Newtonian fluids have achieved significant important as in stretching a sheet in a viscoelastic fluid, the power required is less than when it is placed in a Newtonian fluid. Besides, the heat transfer for a viscoeleastic fluid is found to be less than that of a Newtonian fluid. For mathematicians, this study might increase the usage of the application of differential equations in viscoelastic fluid and the mathematicians might also gain more knowledge about the characteristic and the properties of the viscoelastic fluid.

Rheologist is a person who studies the deformation and flow of matter, particularly non-Newtonian flow of liquids and plastic flow of solids. Since the effects of MHD to the fluid flow characteristic is studied in this study, thus, both the mathematicians and rheologists will gain new knowledge based on the results

In addition, there are many applications of viscoelastic fluids in some industrial-manufacturing processes. For example, in a food manufacture. In this manufacture, the degree of fluidity, consistency, and other mechanical properties are essential in understanding how stable the food will remain, how long it can be stored, and in determining food quality.

Another example is in petroleum drilling or mud drilling. Everyone that takes part in the drilling operations acknowledges that muds behave with non-Newtonian fluid flow properties since their viscosity is not only affected by pressure and temperature but is also related to the velocity at which the mud flows through the

REFERENCES

Abel. S., Veena P. H., Rajagopal K., and Pravind V. K. Non-Newtonian Magnetohydrodynamic Flow Over a Stretching Surface with Heat and Mass Transfer. *International Journal of Non-Linear Mechanics.* 2004. 39: 1067-1078.

Abel S. M., Siddheshwar P. G., Mahantesh M., and Nandeppanavar. Heat Transfer in a Viscoelastic Boundary Layer Flow Over a Stretching Sheet with Viscous Dissipation and Non-Uniform Heat Source. *International Journal of Heat and Mass Transfer.* 2007. 50: 960–966.

Anderson H. L. MHD Flow of a Viscoelastic Fluid Past a Stretching Surface. *Acta Mechanica.* 1992. 95: 227-230.

Anwar I., Amin S. and Pop I. Mixed Convection Boundary Layer Flow of a Viscoelastic Fluid Over a Horizontal Circular Cylinder. *International Journal of Non-Linear Mechanics,* 2008. 43: 814-821.

Ariel P. D., Hayat T., and Asghar S. The Flow of an Elastico-Viscous Fluid Past a Stretching Sheet with Partial Slip. *Acta Mechanica.* 2006. 187: 29-35.

Cebeci T. and Bradshaw P. *Physical and Computational Aspects of Convective Heat Transfer*, Springer, New York. 1984.

Cebeci T. *Convective Heat Transfer.* California: Horizon Publishing Incorporated. 2002.

Chen C. H. Effect of Viscous Dissipation on Heat Transfer in a Non-Newtonian Liquid Film over an Unsteady Stretching Sheet. *Journal of Non-Newtonian Fluid Mechanics.* 2006. 135: 128-135.

Cortell R. A Note on Flow and Heat Transfer of a Viscoelastic Fluid over a Stretching Sheet. *International Journal Non-Linear Mechanics*, 2006. 41: 78-85.

Cortell R. Flow and heat transfer in a moving fluid over a moving flat surface. *Theory Computer Fluid Dynamics*, 2007. 21: 435-446.

Cortell R. Numerical Comparisons of Blasius and Sakiadis Flows. *MATEMATIKA*. 2010. 26 (2): 187-196.

Dandapat B. S., Gupta A. S. Flow and Heat Transfer in a Viscoelastic Fluid over a Stretching Sheet. *International Journal Non-Linear Mechanics*. 1989. 24: 215-219

Datti P. S., Prasad K. V., Abel M. S., and Joshi A. MHD Visco-Elastic Fluid Flow Over a Non-Isothermal Stretching Sheet. *International Journal of Engineering Science*. 2004. 42: 935-946.

Dunn J. E. and Fosdick. R. L. *Thermodynamics, stability, and boundedness of fluids of complexity 2 and fluids of second grade*. Berlin/Heidelberg: Springer. 1974.

Fang T. Further Study on a Moving-Wall Boundary-layer Problem with Mass Transfer. *Acta Mechanica*, 2003. 163: 183–188.

Fang T. Similarity Solutions for a Moving-Flat Plate Thermal Boundary Layer. *Acta Mechanica*, 2003. 163: 161–172.

Hsiao K. L. Conjugate Heat Transfer of Magnetic Mixed Convection with Radiative and Viscous Dissipation Effects for Second-Grade Viscoelastic Fluid Past a Stretching Sheet. *Applied Thermal Engineering*. 2007. 27: 1895-1903.

Ishak A., Nazar R. and Pop I. The Effects of Transpiration on the Flow and Heat Transfer over a Moving Permeable Surface in a Parallel Stream. *Chemical Engineering Journal,* 2009. 148: 63 – 67.

Ishak A., Nazar R., Bachok N., and Pop I. MHD Mixed Convection Flow Adjacent to a Vertical Plate with Prescribed Surface Temperature. *International Journal of Heat and Mass Transfer.* 2010 .53:4506–4510.

Jamaludin M. A. *Heat Transfer Over a Stretching Sheet with Viscous Dissipation and Internl Heat Generation.* Master Thesis. Universiti Teknologi Malaysia; 2008.

Kasim A. R. M., Othman Z. S., and Sharidan S. Generalized Blasius Problem for a Viscoelastic Fluid with Viscous Dissipation and Suction/Injection Effects. *Internationl Journal of Numerical Methods for Heat and Fluid Flow.* 2010.

Kasim A. R. M., Admon M. A., and Sharidan S. Free Convection Boundary Layer Flow of a Viscoelastic Fluid in the Presence of Heat Generation. *World Academy of Science, Engineering and Technology*. 2011. 51.

Kasim A. R. M, Mohammad N. F., Aurangzaib and Sharidan S. Natural Convection Boundary Layer Flow of a Viscoelastic Fluid on Solid Sphere with Newtonian Heating. *World Academy of Science, Engineering and Technology*. 2012. 64.

Klemp J. P. and Acrivos A. A Method for Integrating The Boundary-layer Equations Through a Region of Reverse Flow. *Journal Fluid Mechanics*, 1972. 53: 177-191.

Kumaran V., Banerjee A. K., Kumar V. A., and Pop I. Unsteady MHD Flow and Heat Trasfer with Viscous Dissipation Past a Stretching Sheet. *International Communications in Heat and Mass Transfer.* 2011. 38: 335-339.

Lok Y. Y., Pop I. and Ali J. Chamkha, Non-orthogonal Stagnation Point Flow of a Micropolar Fluid. *International Journal Engineering Science*, 2007. 45: 173-184.

Mushtaq M., Asghar S., and Hossain M. A. Mixed Convection Flow of a Second Grade Fluid Along a Vertical Stretching Flat Surface with Variable Surface Temperature. *Heat Mass Transfer*. 2007. 43: 1049-1061.

Othman Z. S. *Generalized Blasius Problem for a Viscoelastic Fluid with Viscous Dissipation and Suction/Injection Effects.* Master Thesis. Universiti Teknologi Malaysia; 2010.

Panton R. L. *Incompressible Flow*, 2nd Edition. Wiley, New York. 1996

Pinho F. T., and Coelho P. M. Fully-Developed Heat Transfer in Annuli for Viscoelastic Fluids with Viscous Dissipation. *Journal of Non-Newtonian Fluid Mechanics*. 2006. 138: 7-21.

Rajagopal K. R., Na T. Y., and Gupta S. A. Flow of a Fluid Over a Stretching Sheet. *Rheologica Acta*. 1984. 23: 213-215.

Rivlin R. S. and Ericksen J. L. Stress deformation relations for isotropic materials. *Journal Ration Mechanics Anal*, 1955. 4: 323–425.

Schlichting H. and Bussmann, K.: Exakte Lo¨ sungen fu¨ r die laminare Grenzschicht mit Absaugung und Ausblasen. *Schr. Deutsch. Akad. Luftfahrtforschung Ser.B*, 1943. 7(2).

Shit G. C., and Haldar R. Effects of Thermal Radiation on MHD Viscous Fluid Flow and Heat Transfer over Nonlinear Shrinking Porous Sheet. *Applied Mathematics and Mechanics.* 2011. 32(6): 677-688.

Vajravelu K. and Mohapatra R. N. On Fluid Dynamic Drag Reduction in Some Boundary Layer Flows. *Acta Mechanica*. 1990. 81: 59-68.

Vajravelu K. and Roper T. Flow in heat transfer in a second grade fluid over stretching sheet. *International Journal of Non-linear Mechanics*, 1999. 34:1031-1036.

Weyl H. On The Differential Equation of The Simplest Boundary-layer Problems. *Annals of Mathematics*, 1942. 43: 381–407.