VARIABLE SELECTION USING
LEAST ANGLE REGRESSION

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Special dedicated to

My beloved family and friends
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ABSTRACT

The least-angle regression (LARS) (Efron, Hastie, Johnstone, and Tibshirani, 2004) is a technique used with the absence of data that consist of many independent variables. Suppose we expect a response variable to be determined by a linear combination of a subset of potential covariates. Then the LARS algorithm provides a means of producing an estimate of which variables to include, as well as their coefficients. The MATLAB programming codes are developed in order to solve the algorithms systematically and effortlessly.
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CHAPTER 1

INTRODUCTION

1.1 Background of The Problem

In statistics, regression analysis includes any techniques for modeling and analyzing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables. More specifically, regression analysis helps us understand how much dependent variable changes with changes in each of the independent variable, while the other independent variables are held fixed.

Regression analysis is widely used for prediction and forecasting. Regression analysis is also used to understand which among the independent variables are related to the dependent variable, and to explore the forms of these relationships.

There are simple regression and multiple regression. Simple regression is a model with only one independent variable (X) while multiple regression is a model with more than one
independent variables which are $Y = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \ldots + \beta_pX_p$. In theory, the more of independent variables use the more accurate explanation on the dependent variable. But in most practical situations, however, only a relatively small number of independent variable is to be considered because it will help to reduce the cost and time.

For the variables selection, common methods are being used are stepwise, forward and backward selection method. Stepwise selection has been proposed as a technique that combines advantages of forward and backward selection. At any point in the search, a single predictor variable may be added or deleted based solely on the $t$-statistics of their estimated coefficients. Commonly, the starting subset is the empty set. Some of the problems with stepwise variable selection are it yields R-squared values that are badly biased to be high, the method yields confidence intervals for effects and predicted values that are falsely narrow (See Altman and Anderson, 1989, Statistics in Medicine) and it has severe problems in the presence of collinearity.

Forward selection, which involves starting with no variables in the model, trying out the variables one by one and including them if they are statistically significant while Backward elimination, which involves starting with all candidate variables and testing them one by one for statistical significance, deleting any that are not significant on the basis of an F-distribution, calculate the p-value associated with restoring the term into the model.

The purpose of model selection algorithms such as all subsets, Forward Selection and Backward Elimination is to choose a linear model on the basis of the same set data to which the
model will be applied. Typically we have available a large collection of possible covariates from which we hope to select a parsimonious set for the efficient prediction of a response variable. Least Angle Regression (LARS), a new model selection algorithm, is a useful and less greedy version of traditional forward selection methods.

1.2 Problem Statement

The main problem in the multiple regression model is to select the independent variable. The idea is to choose a simpler model where the \( X \) is selected from the \( p \) variables.

1.3 Objectives of the Study

The main objectives of this research are:

1. To apply forward selection and LARS method in variable selection for a regression model
2. To identify the similarities and the differences between forward selection and LARS method
1.4  Scope of the Study

The study and proposed efficient algorithms for the extensions of common methods for factor selection and show that these extensions (LARS) give superior performance to the traditional forward selection method in factor selection problems. We study the similarities and the differences between these methods. The body fat data are used to illustrate the methods. This data were used to produce predictive equations for lean body weight, a measure of health.

1.5  Significance of the Study

Variable selection is very important to ensure that the result from data analysis will be more accurate. Therefore this research will focus on the least angle regression (LARS) to select the variables.
REFERENCES


