CHARACTERISTICS OF SOLITARY WAVE IN FIBER BRAGG GRATING

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CHARACTERISTICS OF SOLITARY WAVE IN FIBER BRAGG GRATING

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A thesis submitted in fulfillment of the requirements for the award of the degree of Master of Science (Physics)

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To all the beloved person in life especially
Mama, Papa, abg boby, abg wawan, saidah.

The difference between a successful person and others is
not a lack of strength, not a lack of knowledge,
but rather a lack of will.
When the world says ‘give up’,
hope whispers ‘try it one more time’

To my dearest friends:
Wondergirl UTM, Wondergirl Trans, Malayaparkers,
There are no limits to our possibilities.
Thank you for always supporting me.

To teachers:
Dr Saktioto and Prof Jalil,
I am indebted to my father for living,
but to my teacher for living well.
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ABSTRACT

In this study, nonlinear parameters are characterized in terms of the potential energy. This study investigates the photon energy of soliton propagating in a fiber Bragg grating (FBG). The formation of potential well of bright soliton in FBG is performed under Bragg resonance condition. In the presence of Kerr- nonlinearity, using the coupled-mode theory, the nonlinear coupled mode equation is defined. The existence of bright soliton is simulated in the FBG. This is done by applying the equation of bright soliton to the potential well. The results are obtained using MatLab software version R2010a. The nonlinear parameter in this study is initially set to $\alpha = 1.0$, $\beta = 0.7$ and $\gamma = 0.1$. The simulation of potential energy distribution throughout the grating is examined by varying the value of nonlinear parameters of $\alpha$. The changes of nonlinearity parameter depends on the motion of photon in the potential well. This influences the existence of bright Bragg soliton in FBG. The couple mode theory and Stokes parameter provide important information on the total energy and energy differences between the forward and backward propagating modes. The propagation of pump signal and probe signal is investigated in order to monitor photon trapping with soliton. By applying Split Step Method on nonlinear Schrodinger equation (NLSE), the input and output pulse of the wave are obtained for different Fast Fourier Transform (FFT), window size and step size. The associated Legendre equation is applied to the probe equation and solving soliton interaction equation. From the photon trapping profile, it is shown that photon can be trapped in certain time by soliton.
ABSTRAK

TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DECLARATION</td>
<td></td>
<td>ii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td></td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td></td>
<td>iv</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td></td>
<td>v</td>
</tr>
<tr>
<td>ABSTRAK</td>
<td></td>
<td>vi</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td></td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td></td>
<td>xi</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td></td>
<td>xiii</td>
</tr>
<tr>
<td>LIST OF APPENDICES</td>
<td></td>
<td>xvii</td>
</tr>
</tbody>
</table>

1 INTRODUCTION

1.1 Introduction 1
1.2 Background of the Study 2
1.3 Problem Statement 3
1.4 Aims and Objectives 4
# LITERATURE REVIEW

2.1 Introduction

2.2 Fiber Bragg Grating

2.3 Kerr Effect, Self Phase Modulation and Cross Phase Modulation

2.4 Solitary Waves

2.5 Bright and Dark Soliton

2.6 Photon Trapping and Soliton Interaction (Collision)

2.7 Nonlinear Schrödinger Equation

2.8 Split Step Method and Associated Legendre Equation

# RESEARCH METHODOLOGY

3.1 Introduction

3.2 The Modelling of Solitary Waves

3.3 The Modelling of Photon Trapping with Soliton

3.4 Split-Step Fourier Method

3.5 Flow Chart of Research Methodology
# RESULTS AND DISCUSSIONS

4.1 Introduction 33  
4.2 Findings of Potential Well of Bright Solitary Waves 33  
4.3 Findings of Photon Trapping with Soliton 37  
4.4 Findings of NLSE Pump Equation Using Split Step Method 38

# CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction 44  
5.2 Conclusions 44  
5.3 Recommendations 45

# REFERENCES

5.1 Introduction 44  
5.2 Conclusions 44  
5.3 Recommendations 45

# APPENDICES

6 REFERENCES 46  
7 APPENDICES 50  
8 PUBLISHED PAPER 62
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE NO.</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Table of Few Legendre Polynomial</td>
<td>20</td>
</tr>
<tr>
<td>2.2</td>
<td>Table of Few Powers in Terms of Legendre Polynomial</td>
<td>20</td>
</tr>
<tr>
<td>2.3</td>
<td>Table of Few Associated Legendre Functions</td>
<td>21</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE NO.</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Flow Chart of Methodology</td>
<td>5</td>
</tr>
<tr>
<td>2.1</td>
<td>Basic Diagram of Fiber Bragg Grating</td>
<td>8</td>
</tr>
<tr>
<td>2.2</td>
<td>Soliton Illustration from Balancing of Kerr Nonlinearity and GVD</td>
<td>11</td>
</tr>
<tr>
<td>2.3</td>
<td>Illustration of Self-Focusing</td>
<td>11</td>
</tr>
<tr>
<td>2.4</td>
<td>Graph of Dark Soliton</td>
<td>14</td>
</tr>
<tr>
<td>2.5</td>
<td>Graph of Bright Soliton</td>
<td>15</td>
</tr>
<tr>
<td>2.6</td>
<td>Potential Well and Soliton Illustration</td>
<td>15</td>
</tr>
<tr>
<td>2.7</td>
<td>Soliton Collision</td>
<td>17</td>
</tr>
<tr>
<td>3.1</td>
<td>Design of Two Pump Soliton</td>
<td>25</td>
</tr>
<tr>
<td>3.2</td>
<td>Flow Chart of Methodology</td>
<td>28</td>
</tr>
<tr>
<td>3.3</td>
<td>Flow Chart of Potential Well of Solitary Waves</td>
<td>29</td>
</tr>
<tr>
<td>3.4</td>
<td>Flow Chart of Photon Trapping With Soliton</td>
<td>30</td>
</tr>
<tr>
<td>3.5</td>
<td>Flow Chart of Nonlinear Schrodinger Equation solution using Split Step Method</td>
<td>31</td>
</tr>
<tr>
<td>3.6</td>
<td>Flow Chart of Associated Legendre Equation</td>
<td>32</td>
</tr>
<tr>
<td>4.1(a)</td>
<td>Well Potential of Bright Solitary Waves for</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 1.0, \beta = 0.7$ and $\gamma = 0.1$</td>
<td></td>
</tr>
<tr>
<td>4.1(b)</td>
<td>Well Potential of Bright Solitary Waves for</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>$\alpha = -1.0, \beta = 0.7$ and $\gamma = 0.1$</td>
<td></td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.2</td>
<td>Graph of Photon Trapping</td>
<td>37</td>
</tr>
<tr>
<td>4.3</td>
<td>Graph of Input and Output of Pump Equation for nt = 100 and T_{max} = 5</td>
<td>38</td>
</tr>
<tr>
<td>4.4</td>
<td>Graph of Input and Output of Pump Equation for nt = 200 and T_{max} = 10</td>
<td>39</td>
</tr>
<tr>
<td>4.5</td>
<td>Graph of Associated Legendre Equation</td>
<td>41</td>
</tr>
<tr>
<td>4.6</td>
<td>Graph of Relationship Between kRt and tanh kRt</td>
<td>42</td>
</tr>
<tr>
<td>4.7</td>
<td>Graph of Relationship Between Nonlinear Parameter and Degree of Eigenfunction</td>
<td>43</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

\( \lambda_B \) - Bragg wavelength
\( \Lambda \) - Spatial period (or pitch) of the periodic variation
\( N_{\text{eff}} \) - Effective index for light propagating in a single mode fiber
\( A(z) \) - Forward propagating modes
\( B(z) \) - Backward propagating modes
\( \psi(x,y) \) - Transverse modal field distribution
\( \omega \) - Frequency
\( \beta \) - Propagation constant of the mode
\( n_2^2(x,y,z) \) - refractive index variation along the fiber
\( K \) - spatial frequency of the grating
\( \Delta n^2 \) - index modulation of the grating
\( \Gamma \) - Coupling coefficient
\( r \) - Radius of the core of FBG
\( a \) - Radius of the cladding of FBG
\( l \) - Length of the grating
\( R \) - Reflectivity of the grating
\( n_2 \) - Kerr coefficient
\( \delta n_g(z) \) - Periodic index variation inside the grating
\( n_2 I \) - Nonlinear index change

\( \bar{n} \) - Average refractive index of the medium

\( E_{f,b}(z,t) \) - Forward and backward propagating waves

\( \kappa \) - Coupling between the forward and backward propagating waves in the FBG

\( k_i \) - Incident wavevector

\( k_f \) - Wavevector of the scattered radiation

\( n_{eff} \) - Effective refractive index of the fiber core at free space center wavelength

\( \Delta n \) - Amplitude of the induced refractive index perturbation formed in the core of the fiber

\( z \) - Distance along the fiber in longitudinal axis

\( \lambda \) - Wavelength

\( \lambda_{uv} \) - UV wavelength

\( N \) - Number of grating

\( \bar{P}_{unperturbed} \) - Unperturbed polarization

\( \bar{P}_{grating} \) - Perturbed polarization

\( \mu \) - transverse mode number

\( \hat{e}_z \) - unit vector along the propagation direction \( z \)

\( \delta_{\mu\nu} \) - Kronecker's delta

\( \bar{E} \) - Electric field vectors

\( \bar{H} \) - magnetic field vectors
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{D}$</td>
<td>Displacement vectors</td>
</tr>
<tr>
<td>$\vec{B}$</td>
<td>Flux density</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of light</td>
</tr>
<tr>
<td>$\vec{E}(z,t)$</td>
<td>Electric field</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Central frequency</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Wavenumber</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Total power inside the grating</td>
</tr>
<tr>
<td>$e_f$</td>
<td>Forward propagating modes</td>
</tr>
<tr>
<td>$e_b$</td>
<td>Backward propagating modes</td>
</tr>
<tr>
<td>$\Gamma_s$</td>
<td>Self Phase Modulation</td>
</tr>
<tr>
<td>$\Gamma_x$</td>
<td>Cross-phase modulation effects</td>
</tr>
<tr>
<td>$C$</td>
<td>Constant of integration</td>
</tr>
<tr>
<td>$\hat{\delta}$</td>
<td>Detuning parameter</td>
</tr>
<tr>
<td>$V(A_0)$</td>
<td>Potential energy distribution in a FBG structures while the light propagating through the grating structures</td>
</tr>
</tbody>
</table>
## LIST OF APPENDICES

<table>
<thead>
<tr>
<th>APPENDIX</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Matlab Coding for Well Potential of Solitary Waves</td>
<td>50</td>
</tr>
<tr>
<td>B</td>
<td>Matlab Coding for Photon Trapping</td>
<td>51</td>
</tr>
<tr>
<td>C</td>
<td>MatLab Coding of Solving NLSE Using Split Step Method</td>
<td>52</td>
</tr>
<tr>
<td>D</td>
<td>MatLab Coding of Legendre Equation</td>
<td>55</td>
</tr>
<tr>
<td>E</td>
<td>Value of P3</td>
<td>57</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Introduction

Rapid development and investigation about gap soliton in Fiber Bragg Grating (FBG) have brought wide applications and devices in the optical fiber communication field. FBG that based on the Kerr-non-linearity is the outstanding device for studying nonlinear phenomena (Senthilnathan and Porsezian, 2003). Recently, there are many researches in designing nonlinear fiber Bragg gratings with a low reflectivity, that allows us to obtain compressed pulses with a very low wing intensity (Amir Rosenthal and Moshe Horowitz, 2006). Shapira and Horowitz (2009) have developed a model to study nonlinear pulse propagation in a FBG written. The first experimental observation of nonlinear propagation effects in FBG, resulting in nonlinear optical pulse compression and soliton propagation (Benjamin and Slusher, 1996).

In this thesis, the organised study of solitary waves in potential well that trapped photon is introduced. This research is studied numerically. Mathematical modelling is defined and developed through the first principle of optical soliton derivation. Simulated result obtained is able to characterize the soliton waves in
FBG. Further details about FBG and soliton history, development, theory, simulation are expounded in this thesis.

1.2 Background of Study

Fiber Bragg Grating (FBG) is formed by the periodic variations of the refractive index in the fiber core. It is a type of distributed Bragg reflector constructed in a short segment of optical fiber that reflects particular wavelengths of light and transmits all others. FBG are very attractive components because as well as being passive, linear, and compact, they possess strong dispersion in both reflection and transmission (Natalia et. al, 1997). A fiber Bragg grating can therefore be used as an inline optical filter to block certain wavelengths, or as a wavelength-specific reflector. Most FBGs are used in single-mode fibers, and in that case the physical modeling is often relatively simple. The primary application of fiber Bragg gratings is in optical communications systems. They are used as flexible and low cost in-line components to manipulate any part of the optical transmission and reflection spectrum.

One of the important criteria in an effective communication is propagation of light which is not affected by the waveguide until the end of transmission. A wave having those behaviours is called a soliton. It is a nonlinear wave which has the following two properties:

i. A localized wave propagates without change of its properties.

ii. Localized waves are stable against mutual collisions and retain their identities.
Optical gap solitons refer to nonlinear waves propagating in optical fibers whose linear refractive index has a periodic variation. Gap soliton was obtained when the spectrum of bragg grating solitons lies within the photonic band gap (Haryana Mohd Hairi et al, 2009). From the experiment, it is stated that solitons can be produced by the symmetric input pulse (Yuri S. Kivshar, 1990). As stated by F. Kh. Abdullaev and J. Garnier, many physical systems are described by nonintegrable systems which have stationary or moving localized solutions. Collisions between these solitary waves are usually inelastic, which is a clear indication that they are not true solitons (F. Kh. Abdullaev and J. Garnier). When photons are trapped by solitons, soliton collisions can be used to implement several operations that may prove useful in quantum communication and computing (Ken Steiglitz and Darren Rand, 2009).

1.3 Problem Statement

Based on the wave propagation in FBG and the theory of soliton, lead to some problems such as:

i. How potential well in solitary waves can exist?

ii. What is the impact of increasing nonlinearity in FBG towards the potential well plot?

iii. How is the energy transferred along the propagating modes for potential well?
1.4 Aims and Objectives

This research aims to optimize and characterised the nonlinear parameters in terms of the potential energy. A mathematical model on solitary wave will be developed to show the existence of bright solitary wave. The equations will be derived based on the coupled mode theory. A MatLab coding will be developed to solve these equations. This model will also be used to monitor energy of soliton along the propagation of solitary wave in FBG. The motion of a particle moving in FBG represents the pulse propagation in the grating structure of fiber optics. In order to describe the photon motion, the function of potential energy is depicted via modelling and the simulation.

1.5 Scope of the study

This study is developed from a model of optical soliton in FBG using the coupled-mode theory including the Kerr nonlinearity, group velocity dispersion (GVD) and self phase modulation (SPM). This study also included the applications of Split Step Fourier Method (SSM) in order to solve Nonlinear Schrodinger Equation numerically. On the other hand, Associated Legendre equation also applied to the soliton equation.

1.6 Research Methodology

This study covers two main areas, namely, modelling and simulation on the existence of optical soliton in grating structure in FBG. Figure 1.1 shows the flow and steps undertaken to conduct this research.
To find basic potential well equation for well potential.

Applying stability condition-the system possesses a single well potential.

To model and run the code using Matlab by setting several parameters such as the value of $\alpha$, $\beta$, $\gamma$ & $A_0$.

The Modelling of Solitary Waves.

The Modelling of Photon Trapping With Soliton.

Figure 1.1: Flow Chart of Methodology
1.7 Significance of the Study

This research will contribute towards the research areas of nanophotonics and optical solitons. The more specific significance of this study are as follows:

i. Manage to control photon in certain gap of soliton.
ii. Able to trap soliton in propagation of FBG.

1.8 Organization of the Study

This report is organized as follows. Chapter 1 is the research framework. This chapter contains some discussion on the introduction to our study, a description to the problem, the objectives of the study, the scope of study, the significance of the study and finally the chapter organization. Chapter 2 will brief about the theory that connects to this work embracing past research that has been done related to the study. Chapter 3 will elaborate a complete account on the research methodology that is used in this study. It will cover simulations related to mathematical modelling of FBG. Chapter 4 is the report on the results and its analysis. Chapter 5 gives the conclusions followed by recommendations for future works.
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Abdullaev, F. Kh. and Garnier, J. “Bright Solitons in Bose-Einstein Condensates”.


