

CHARACTERISTICS OF SOLITARY WAVE IN FIBER BRAGG GRATING

MARDIANA SHAHADATUL AINI BINTI ZAINUDIN

UNIVERSITI TEKNOLOGI MALAYSIA

CHARACTERISTICS OF SOLITARY WAVE IN FIBER BRAGG GRATING

MARDIANA SHAHADATUL AINI BINTI ZAINUDIN

A thesis submitted in fulfillment of the
requirements for the award of the degree of
Master of Science (Physics)

Faculty of Science
Universiti Teknologi Malaysia

JULY 2012

To all the beloved person in life especially
Mama, Papa, abg boby, abg wawan, saidah.

The difference between a successful person and others is
not a lack of strength, not a lack of knowledge,
but rather a lack of will.

When the world says 'give up',
hope whispers 'try it one more time'

To my dearest friends:
Wondergirl UTM, Wondergirl Trans, Malayaparkers,
There are no limits to our possibilities.
Thank you for always supporting me.

To teachers:
Dr Saktioto and Prof Jalil,
I am indebted to my father for living,
but to my teacher for living well.

ACKNOWLEDGEMENTS

First and foremost, I would like to express my deepest gratitude to Allah S.W.T for giving the strength to complete my research successfully.

Secondly, I would like to convey my deepest appreciation to my supervisors, Dr. Saktioto and Prof. Dr. Jalil Ali for all their guidance and support throughout the duration of this research and thesis writing. I am greatly indebted to them for the knowledge imparted and the precious time they allocated to guide me. Dr. Saktioto provided the overall framework of this studies. I would like to extend my sincere appreciation to my family especially mom and dad for their tender support, morally and financially. I would also like to thanks to members of the Institute of Advanced Photonics and Sciences (APSI) for their assistance. They had provided me with ample information, co-operation and help during the process of conducting my research.

ABSTRACT

In this study, nonlinear parameters are characterized in terms of the potential energy. This study investigates the photon energy of soliton propagating in a fiber Bragg grating (FBG). The formation of potential well of bright soliton in FBG is performed under Bragg resonance condition. In the presence of Kerr- nonlinearity, using the coupled-mode theory, the nonlinear coupled mode equation is defined. The existence of bright soliton is simulated in the FBG. This is done by applying the equation of bright soliton to the potential well. The results are obtained using MatLab software version R2010a. The nonlinear parameter in this study is initially set to $\alpha = 1.0$, $\beta = 0.7$ and $\gamma = 0.1$. The simulation of potential energy distribution throughout the grating is examined by varying the value of nonlinear parameters of α . The changes of nonlinearity parameter depends on the motion of photon in the potential well. This influences the existence of bright Bragg soliton in FBG. The couple mode theory and Stokes parameter provide important information on the total energy and energy differences between the forward and backward propagating modes. The propagation of pump signal and probe signal is investigated in order to monitor photon trapping with soliton. By applying Split Step Method on nonlinear Schrodinger equation (NLSE), the input and output pulse of the wave are obtained for different Fast Fourier Transform (FFT), window size and step size. The associated Legendre equation is applied to the probe equation and solving soliton interaction equation. From the photon trapping profile, it is shown that photon can be trapped in certain time by soliton.

ABSTRAK

Dalam kajian ini, ciri-ciri parameter tak linear bagi gelombang soliton dikaji dari segi tenaga keupayaannya. Kajian ini menyiasat tenaga foton dalam soliton di dalam gentian parutan Bragg (FBG). Pembentukan soliton cerah di dalam FBG dilakukan di bawah keadaan resonan Bragg. Dengan kehadiran kesan tak linear-Kerr, menggunakan teori mod pengganding, persamaan linear mod bersama ditakrifkan. Simulasi berjaya menunjukkan kewujudan soliton terang dalam FBG. Ini telah dilakukan dengan menggunakan persamaan gelombang terang bersendirian dan menghasilkan graf perigi keupayaan. Keputusan telah diperolehi dengan menggunakan perisian Matlab versi R2010a. Parameter tak linear dalam kajian ini pada mulanya ditetapkan $\alpha = 1.0$, $\beta = 0.7$ and $\gamma = 0.1$. Simulasi agihan tenaga keupayaan seluruh gentian parutan diperolehi dengan mengubah nilai parameter tak linear α . Perubahan parameter tak linear bergantung atas pergerakan foton dalam perigi keupayaan tenaga. Ini mempengaruhi kewujudan soliton terang Bragg di dalam FBG. Teori pasangan mod dan parameter Stokes menyediakan maklumat penting mengenai perbezaan tenaga dan jumlah tenaga antara mod pergerakan ke hadapan dan ke belakang. Kajian ini juga menyiasat perambatan isyarat pam dan isyarat probe bagi memantau pemerangkapan foton oleh soliton. Dengan menggunakan kaedah Langkah Split pada persamaan Schrodinger tak linear (NLSE), nadi input dan output gelombang telah diperolehi untuk transformasi Fourier cepat, tettingkap saiz dan saiz langkah yang berbeza. Persamaan Legendre bersekutu juga digunakan untuk menyelesaikan persamaan interaksi soliton. Graf pemerangkapan foton telah menunjukkan bahawa soliton boleh memerangkap foton dalam masa tertentu.

TABLE OF CONTENTS

CHAPTER	TITLE	PAGE
	DECLARATION	ii
	DEDICATION	iii
	ACKNOWLEDGEMENT	iv
	ABSTRACT	v
	ABSTRAK	vi
	TABLE OF CONTENTS	vii
	LIST OF TABLES	x
	LIST OF FIGURES	xi
	LIST OF SYMBOLS	xiii
	LIST OF APPENDICES	xvii
1	INTRODUCTION	
	1.1 Introduction	1
	1.2 Background of the Study	2
	1.3 Problem Statement	3
	1.4 Aims and Objectives	4

	1.5	Scope of the Study	4
	1.6	Research Methodology	4
	1.7	Significance of the Study	6
	1.8	Organization of the Study	6
2		LITERATURE REVIEW	
	2.1	Introduction	7
	2.2	Fiber Bragg Grating	7
	2.3	Kerr Effect, Self Phase Modulation and Cross Phase Modulation	9
	2.4	Solitary Waves	12
	2.5	Bright and Dark Soliton	13
	2.6	Photon Trapping and Soliton Interaction (Collision)	16
	2.7	Nonlinear Schrödinger Equation	17
	2.8	Split Step Method and Associated Legendre Equation	18
3		RESEARCH METHODOLOGY	
	3.1	Introduction	22
	3.2	The Modelling of Solitary Waves	23
	3.3	The Modelling of Photon Trapping with Soliton	25
	3.4	Split-Step Fourier Method	26
	3.5	Flow Chart of Research Methodology	28

4	RESULTS AND DISCUSSIONS		
	4.1	Introduction	33
	4.2	Findings of Potential Well of Bright Solitary Waves	33
	4.3	Findings of Photon Trapping with Soliton	37
	4.4	Findings of NLSE Pump Equation Using Split Step Method	38
5	CONCLUSIONS AND RECOMMENDATIONS		
	5.1	Introduction	44
	5.2	Conclusions	44
	5.3	Recommendations	45
6	REFERENCES		46
7	APPENDICES		50
8	PUBLISHED PAPER		62

LIST OF TABLES

TABLE NO.	TITLE	PAGE
2.1	Table of Few Legendre Polynomial	20
2.2	Table of Few Powers in Terms of Legendre Polynomial	20
2.3	Table of Few Associated Legendre Functions	21

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
1.1	Flow Chart of Methodology	5
2.1	Basic Diagram of Fiber Bragg Grating	8
2.2	Soliton Illustration from Balancing of Kerr Nonlinearity and GVD	11
2.3	Illustration of Self-Focusing	11
2.4	Graph of Dark Soliton	14
2.5	Graph of Bright Soliton	15
2.6	Potential Well and Soliton Illustration	15
2.7	Soliton Collision	17
3.1	Design of Two Pump Soliton	25
3.2	Flow Chart of Methodology	28
3.3	Flow Chart of Potential Well of Solitary Waves	29
3.4	Flow Chart of Photon Trapping With Soliton	30
3.5	Flow Chart of Nonlinear Schrodinger Equation solution using Split Step Method	31
3.6	Flow Chart of Associated Legendre Equation	32
4.1(a)	Well Potential of Bright Solitary Waves for $\alpha = 1.0, \beta = 0.7$ and $\gamma = 0.1$	34
(b)	Well Potential of Bright Solitary Waves for $\alpha = -1.0, \beta = 0.7$ and $\gamma = 0.1$	35

4.2	Graph of Photon Trapping	37
4.3	Graph of Input and Output of Pump Equation for $nt = 100$ and $T_{\max} = 5$	38
4.4	Graph of Input and Output of Pump Equation for $nt = 200$ and $T_{\max} = 10$	39
4.5	Graph of Associated Legendre Equation	41
4.6	Graph of Relationship Between kRt and $\tanh kRt$	42
4.7	Graph of Relationship Between Nonlinear Parameter and Degree of Eigenfunction	43

LIST OF SYMBOLS

λ_B	-	Bragg wavelength
Λ	-	Spatial period (or pitch) of the periodic variation
N_{eff}	-	Effective index for light propagating in a single mode fiber
$A(z)$	-	Forward propagating modes
$B(z)$	-	Backward propagating modes
$\psi(x, y)$	-	Transverse modal field distribution
ω	-	Frequency
β	-	Propagation constant of the mode
$n_g^2(x, y, z)$	-	refractive index variation along the fiber
K	-	spatial frequency of the grating
Δn^2	-	index modulation of the grating
Γ	-	Coupling coefficient
r	-	Radius of the core of FBG
a	-	Radius of the cladding of FBG
l	-	Length of the grating
R	-	Reflectivity of the grating
n_2	-	Kerr coefficient
$\delta n_g(z)$	-	Periodic index variation inside the grating

n_2I	-	Nonlinear index change
\bar{n}	-	Average refractive index of the medium
$E_{f,b}(z,t)$	-	Forward and backward propagating waves
κ	-	Coupling between the forward and backward propagating waves in the FBG
k_i	-	Incident wavevector
k_f	-	Wavevector of the scattered radiation
n_{eff}	-	Effective refractive index of the fiber core at free space center wavelength
Δn	-	Amplitude of the induced refractive index perturbation formed in the core of the fiber
z	-	Distance along the fiber in longitudinal axis
λ	-	Wavelength
λ_{uv}	-	UV wavelength
N	-	Number of grating
$\bar{P}_{unperturbed}$	-	Unperturbed polarization
$\bar{P}_{grating}$	-	Perturbed polarization
μ	-	transverse mode number
\hat{e}_z	-	unit vector along the propagation direction z
$\delta_{\mu\nu}$	-	Kronecker's delta
\vec{E}	-	Electric field vectors
\vec{H}	-	magnetic field vectors

\vec{D}	-	Displacement vectors
\vec{B}	-	Flux density
c	-	Speed of light
$\vec{E}(z,t)$	-	Electric field
ω_0	-	Central frequency
k_0	-	Wavenumber
P_0	-	Total power inside the grating
e_f	-	Forward propagating modes
e_b	-	Backward propagating modes
Γ_s	-	Self Phase Modulation
Γ_x	-	Cross-phase modulation effects
C	-	Constant of integration
$\hat{\delta}$	-	Detuning parameter
$V(A_0)$	-	Potential energy distribution in a FBG structures while the light propagating through the grating structures

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A	Matlab Coding for Well Potential of Solitary Waves	50
B	Matlab Coding for Photon Trapping	51
C	MatLab Coding of Solving NLSE Using Split Step Method	52
D	MatLab Coding of Legendre Equation	55
E	Value of P3	57

CHAPTER 1

INTRODUCTION

1.1 Introduction

Rapid development and investigation about gap soliton in Fiber Bragg Grating (FBG) have brought wide applications and devices in the optical fiber communication field. FBG that based on the Kerr-non-linearity is the outstanding device for studying nonlinear phenomena (Senthilnathan and Porsezian, 2003). Recently, there are many researches in designing nonlinear fiber Bragg gratings with a low reflectivity, that allows us to obtain compressed pulses with a very low wing intensity (Amir Rosenthal and Moshe Horowitz, 2006). Shapira and Horowitz (2009) have developed a model to study nonlinear pulse propagation in a FBG written. The first experimental observation of nonlinear propagation effects in FBG, resulting in nonlinear optical pulse compression and soliton propagation (Benjamin and Slusher, 1996).

In this thesis, the organised study of solitary waves in potential well that trapped photon is introduced. This research is studied numerically. Mathematical modelling is defined and developed through the first principle of optical soliton derivation. Simulated result obtained is able to characterize the soliton waves in

FBG. Further details about FBG and soliton history, development, theory, simulation are expounded in this thesis.

1.2 Background of Study

Fiber Bragg Grating (FBG) is formed by the periodic variations of the refractive index in the fiber core. It is a type of distributed Bragg reflector constructed in a short segment of optical fiber that reflects particular wavelengths of light and transmits all others. FBG are very attractive components because as well as being passive, linear, and compact, they possess strong dispersion in both reflection and transmission (Natalia *et. al*, 1997). A fiber Bragg grating can therefore be used as an inline optical filter to block certain wavelengths, or as a wavelength-specific reflector. Most FBGs are used in single-mode fibers, and in that case the physical modeling is often relatively simple. The primary application of fiber Bragg gratings is in optical communications systems. They are used as flexible and low cost in-line components to manipulate any part of the optical transmission and reflection spectrum.

One of the important criteria in an effective communication is propagation of light which is not affected by the waveguide until the end of transmission. A wave having those behaviours is called a soliton. It is a nonlinear wave which has the following two properties:

- i. A localized wave propagates without change of its properties.
- ii. Localized waves are stable against mutual collisions and retain their identities.

Optical gap solitons refer to nonlinear waves propagating in optical fibers whose linear refractive index has a periodic variation. Gap soliton was obtained when the spectrum of bragg grating solitons lies within the photonic band gap (Haryana Mohd Hairi *et. al*, 2009). From the experiment, it is stated that solitons can be produced by the symmetric input pulse (Yuri S. Kivshar, 1990). As stated by F. Kh. Abdullaev and J. Garnier, many physical systems are described by nonintegrable systems which have stationary or moving localized solutions. Collisions between these solitary waves are usually inelastic, which is a clear indication that they are not true solitons (F. Kh. Abdullaev and J. Garnier). When photons are trapped by solitons, soliton collisions can be used to implement several operations that may prove useful in quantum communication and computing (Ken Steiglitz and Darren Rand, 2009).

1.3 Problem Statement

Based on the wave propagation in FBG and the theory of soliton, lead to some problems such as:

- i. How potential well in solitary waves can exist?
- ii. What is the impact of increasing nonlinearity in FBG towards the potential well plot?
- iii. How is the energy transferred along the propagating modes for potential well?

1.4 Aims and Objectives

This research aims to optimize and characterised the nonlinear parameters in terms of the potential energy. A mathematical model on solitary wave will be developed to show the existence of bright solitary wave. The equations will be derived based on the coupled mode theory. A MatLab coding will be developed to solve these equations. This model will also be used to monitor energy of soliton along the propagation of solitary wave in FBG. The motion of a particle moving in FBG represents the pulse propagation in the grating structure of fiber optics. In order to describe the photon motion, the function of potential energy is depicted via modelling and the simulation.

1.5 Scope of the study

This study is developed from a model of optical soliton in FBG using the coupled-mode theory including the Kerr nonlinearity, group velocity dispersion (GVD) and self phase modulation (SPM). This study also included the applications of Split Step Fourier Method (SSM) in order to solve Nonlinear Schrodinger Equation numerically. On the other hand, Associated Legendre equation also applied to the soliton equation.

1.6 Research Methodology

This study covers two main areas, namely, modelling and simulation on the existence of optical soliton in grating structure in FBG. Figure 1.1 shows the flow and steps undertaken to conduct this research.

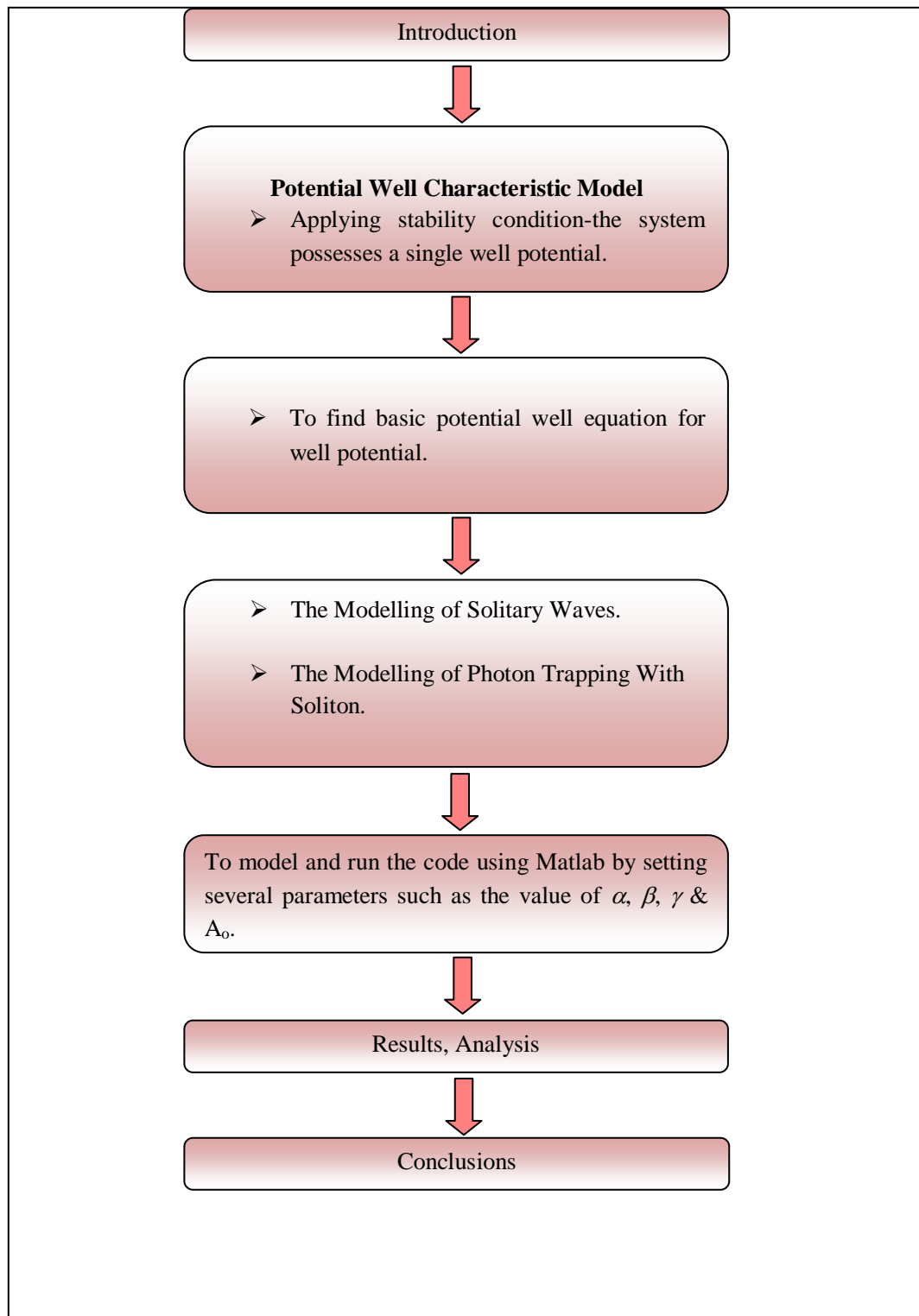


Figure 1.1: Flow Chart of Methodology

1.7 Significance of the Study

This research will contribute towards the research areas of nanophotonics and optical solitons. The more specific significance of this study are as follows:

- i. Manage to control photon in certain gap of soliton.
- ii. Able to trap soliton in propagation of FBG.

1.8 Organization of the Study

This report is organized as follows. Chapter 1 is the research framework. This chapter contains some discussion on the introduction to our study, a description to the problem, the objectives of the study, the scope of study, the significance of the study and finally the chapter organization. Chapter 2 will brief about the theory that connects to this work embracing past research that has been done related to the study. Chapter 3 will elaborate a complete account on the research methodology that is used in this study. It will cover simulations related to mathematical modelling of FBG. Chapter 4 is the report on the results and its analysis. Chapter 5 gives the conclusions followed by recommendations for future works.

REFERENCES

- Abdullaev, F. Kh. and Garnier, J. “*Bright Solitons in Bose-Einstein Condensates*”.
- Aly R. Seadawy (2011). Exact Solutions of a Two-Dimensional Nonlinear Schrödinger Equation, *Applied Mathematics Letter* .25, 687-691.
- Amir Rosenthal and Moshe Horowitz (2006). Analysis and Design of Nonlinear fiber Bragg Gratings and Their Application for Optical Compression of Reflected Pulses. *Optics Letters*. Vol. 31, No. 9.
- Asim Shahzad and Zafrullah, M. (2000). Solitons Interaction and Their Stability Based On Nonlinear Schrödinger Equation. *International Journal of Engineering & Technology IJET*. 9, 156-158.
- Benjamin J. Eggleton and Slusher, R. E., (1996). Bragg Grating Solitons. *Physical Review Letters*, Volume 76-10, 1627-1630.
- Carr, L. D. and Castin, Y. (2002), Dynamics of a matter-wave bright soliton in an expulsive potential. *Physical Review A*. 66.
- Chamorro-Posada, P. and McDonald, G. S., (2004). Exact Analytical Helmholtz Bright And Dark Solitons. *Advanced Optoelectronics and Lasers*. 5582, 154-161.
- Chen, Z., Mitchell, M., Segev, M., Coskun, T., Christodoulides, D.N., (1998). *Phys Rev Lett Science* 280, 889-892.

- Christodoulides, D. N., Coskun, T., Mitchell, M., Segev, M. (1998), *Phys Rev Lett*, 80, 5113-5116.
- G. Bosco (2000). Suppression of Spurious Tones Induced by the Split-Step Method in Fiber Systems Simulation. *IEEE Photonics Technology Letters*, Vol. 12, No. 5.
- Haryana Mohd Hairi , (2009). Nonlinear Parametric Study of Photon in a Fibre Bragg Grating. *Physics Procedia*. 2, 81–85.
- Hill, K.O. and Meltz, G., (2002). Fiber Bragg grating technology fundamentals and overview. *Journal of Lightwave Technology*, 1263 – 1276.
- Jared C. Bronski and Mordechai Segev (2001). Mathematical Frontiers in Optical Solitons. *PNAS*. vol. 98 no.23, 12872-12873.
- Juan Belmonte-Beitia and Pedro J Torres (2008). Existence of Dark Soliton Solutions of the Cubic Nonlinear Schrodinger Equation with Periodic Inhomogeneous Nonlinearity. *Journal of Nonlinear Mathematical Physics, Spain*. Volume 15, 65–72.
- Ken Steiglitz and Darren Rand, (2009). Photon trapping and transfer with solitons. *Physical Review A*. 79, 021802R.
- Lakoba, T.I. (2010). *Stability analysis of the split-step Fourier method on the background of a soliton of the nonlinear Schrodinger equation*. University of Vermont, Burlington, USA.
- Mandelik, D., Morandotti, R., Aitchison, J. S., and Silberberg, Y. (2004). Gap Solitons in Waveguide Arrays. *Physical Review Letters*, Volume 92, No 9.

- Natalia M. Litchinitser, Benjamin J. Eggleton, and David B. Patterson, (1997). Fiber Bragg Gratings for Dispersion Compensation in Transmission: Theoretical Model and Design Criteria for Nearly Ideal Pulse Recompression. *Journal of Lightwave Technology*. Vol. 15, No. 8.
- Oleg V. Sinkin and Ronald Holzlohner, (2003). Optimization of the Split-Step Fourier Method in Modeling Optical-Fiber Communications System. *Journal of Lightwave Technology*. Vol. 21,1246-1250.
- Ostrovskaya, E. A. and Kivshar, Y. S, (2002). Quantum Electron. *IEEE J. Sel. Top.* 8, 591-593.
- Sadhan K. Adhikari (2005). Bright solitons in coupled defocusing NLS equation supported by coupling: Application to Bose-Einstein condensation. *Physics Letters A*. Volume 346, Issues 1-3, 179-185.
- Senthilnathan, K. (2003). Bright and Dark Bragg Solitons in Fiber Bragg Grating. *IEEE Journal of Quantum Electronics*, Vol.39, No.11.
- Senthilnathan, K. and Porsezian, K. (2003). Symmetry-Breaking Instability in Gap Soliton. *Optics Communications*. 227, 295-299.
- Shapira, Y. P. and Horowitz, M. (2009). Pulse Propagation In A Fiber Bragg Grating Written In A Slow Saturable Fiber Amplifier. *Optics Letters*. 34(20), 3113-3115.
- Wen-Jun Liua and Bo Tian (2010), Beijing University of Posts and Telecommunications, *Annals of Physics*, Volume 325, Issue 8, 432-436.
- Yoany Rodríguez García, Jesús M. Corres, and Javier Goicoechea, (2010). Vibration Detection Using Optical Fiber Sensors. *Journal of Sensors*. Volume 5, 908-1001.

Yuri S. Kivshar (1999). Soliton stability in birefringent optical fibers: analytical approach. *J. Opt. Soc. Am.* Vol. 7, 756-800.

Zabusky, N. J. and Kruskal, M. D. (1965). *Phys. Rev. Lett.* 15, 240-243.