Wireless power transmission and its annexure to the grid system

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Abstract: Transmission or distribution of 50 or 60 Hz electrical energy from the generation point to the consumers' end without any physical wire has yet to mature as a familiar and viable technology. Apart from a few demonstrations or feasibility studies the research effort thus far on wireless power transmission (WPT) is inadequate. Moreover, the reported works have not disclosed any design methodology, and this has restricted further advancement in the research on WPT. A unified, consistent and coherent mathematical model for WPT has been synthesised. The output obtained from the model when applied to a case has been compared against the technical specifications partially reported in the literature by a feasibility study of the same case. In addition, it is shown that a potential application of WPT would be in annexing a grid system for radial feed to a demand site in certain scenarios. Also, the specific aspects upon which further research should focus have been identified so that WPT annexure can be viable in the discussed scenarios. The findings are expected to renew interest in WPT among the public in general and among the researchers and the environmentalists in particular.

1 Introduction

Wireless communication using radio frequency (RF) waves or microwave is a very familiar topic. However, wireless power transmission (WPT) is yet to be so, though its history [4] dates back to the late nineteenth century. The year 1888 was marked by a demonstration on wireless transmission of pulsed power generated at 500 MHz. Since then, only a few demonstrations or feasibility studies on WPT have been published [1-4] at discrete time intervals. Among these, the most significant is a feasibility study reported [4] by a French research group in 1997 for a terrestrial (ground-to-ground) WPT project. Its aim was to deliver 10 kW power from the grid system, through AC-DC-microwave-DC-AC conversion, to a 0.7 km distant small tourist spot across a mountain in La Reunion Island, an overseas territory of France. The project is termed 'Grand-Bassin WPT model' after the name of the tourist village.

It is noteworthy that since 1969 (the year marked by the successful landing of the first manned space craft on the moon surface) until now, the main thrust of WPT has been on the concept of space-to-ground (extraterrestrial) transmission of energy using microwave beam. This concept aims towards tapping the round-the-clock available solar energy from outer space in the form of photovoltaic DC power by a low earth orbit satellite. The tapped energy would be converted into a microwave beam for transmission to the designated earth stations where it would be rectified into DC using a device termed 'Rectenna' (receiving and rectifying antenna). A pilot project based around this concept, named 'SPS (Solar Power Satellite) 2000', is still under preliminary investigation by the Japanese Space and Astronomical Science Society. Nevertheless, for a number of reasons the chances of the terrestrial WPT becoming an affordable reality are brighter than its extraterrestrial counterpart. Apart from the time frame for implementation, huge size of rectennas and requirement for energy storage between two passes of the satellite, the other crucial points which bear against the extraterrestrial WPT are affordability, simplicity, controllability, maintainability, reliability, commercial exploitability, and health and safety issues.

However, the reported works on terrestrial WPT [1-4] have not revealed the design method and the complete technical information, and also have not addressed the full-scale potential of WPT as compared with the alternatives, such as a physical power distribution line or a stand-alone terrestrial photovoltaic plant.

In this paper, the classical theories on individual microwave devices such as magnetron (for DC to microwave conversion), transmission and reception antennas, and RF diodes (for rectifying microwave power back into DC) have been integrated coherently and consistently so as to come up with a unified mathematical model for a WPT system. This model has then been used to determine various parameters and device sizes that would be required should the same amount of power as in the Grand Bassin project be wireless delivered over the same distance. The parameters and device sizes obtained from the mathematical model are compared with the partial specifications reported [4] by the Grand Bassin feasibility study. This is followed by a consideration of the prospects, limitations and improvements involved in WPT becoming viable in annexing a grid in certain scenarios.

2 WPT system

Fig. 1 shows the block diagram of a conceptual WPT system annexed to a grid.

The 50 (or 60) Hz AC power tapped from the grid lines is stepped down to a suitable voltage level (e.g. 415 volts) for
Fig. 1  Conceptual model for a WPT system annexed to a grid rectification into DC. This is then supplied to an oscillator-fed magnetron. Inside the magnetron [5, 6] electrons are emitted from a central terminal called a cathode. A positively charged anode surrounding the cathode attracts the electrons. Instead of travelling in a straight line, the electrons are forced to take a circular path by a high-power permanent magnet. As they pass by resonating cavities of the magnetron, a continuous pulsating magnetic field (electromagnetic radiation in microwave frequency range) is generated. After the first round of cavity-to-cavity trip by the electrons is completed the next one starts, and this process continues for as long as the magnetron remains energised. Fig. 2 shows the formation of a re-entrant electron beam in a typical six-cavity magnetron. The output voltage of the rectifier that interfaces the magnetron with the AC grid decides the magnetron anode DC voltage. This in turn controls the radiation power output. The frequency of the radiation is adjusted by varying the inductance or capacitances of the resonant cavities. The usual value of this frequency is 2.45 GHz.

Fig. 2  Re-entrant electron beam in a six-cavity magnetron

The microwave power output of the magnetron is channelled into an array of parabolic reflector antennas for transmission to the receiving end H-dipole antennas. To compensate for the large loss in free space propagation and boost at the receiving side the signal strength as well as the conversion efficiency, the antennas are connected in arrays. Moreover, arrayed installation of the antennas will necessitate a compact space. The widely used parabolic reflector and H-dipole antennas are chosen because they are commercially available, and have high gain with proven performance in signal communication.

A series-parallel assembly of Schottky diodes, having a low standing power rating but good RF characteristics, is used at the receiving end to rectify the received microwave power back into DC. Since the consumers' load is AC the DC power is inverted into 50 (or 60) Hz AC.

A simple radio control feedback system operating in FM (Frequency Modulation) band provides an appropriate control signal to the magnetron for adjusting its output power level with fluctuation in the consumers' demand at the receiving side. Should there be a total loss of load, the feedback system would switch off the supply to the oscillator and magnetron at the sending end.

3 Mathematical model

Magnetron:
With the availability of sufficient filament (cathode) power and requisite anode voltage, the power at the output (n-th) cavity for an n-cavity magnetron can be expressed [5, 6] as follows:

\[ P_n \approx \left( \prod_{m=1}^{n-1} A_m \right) P_1 \quad (1) \]

The power at the first cavity \( P_1 \) is as follows, and is also termed ‘noise power’ because of its small magnitude.

\[ P_1 = \frac{kT_1 f}{Q} \quad (2) \]

where \( k = 1.38054 \times 10^{-23} \text{ J/K} \) is the Boltzmann constant, \( T_1 \) is the temperature (in °Kelvin) of Cavity-1, \( Q \) is the 'Quality' factor of cavity resonator, and \( f \) is the generated radiation frequency.

For an n-cavity magnetron and a re-entrant electron beam the overall gain is given by:

\[ \prod_{m=1}^{n-1} A_m \approx (1 + \varepsilon)^{T/T} \quad (3) \]

where \( \varepsilon \) is about 0.01, and \( T \) is the output stabilising time in the order of μs.

Antennas:
The microwave power \( P_n \) at wavelength \( \lambda \) and frequency \( f \) available at the interaction gap of the magnetron output cavity resonator is transferred to and radiated by an array of transmitting antennas each having a gain \( G_r \). The power obtained \( (P_r) \) at the array of receiving antennas, each having a gain \( G_r \), and distanced from the transmitting array by \( d' \), is given by [7] by Friis free-space
The dimensions of the antenna elements can be obtained from the area of the antenna array. The chosen power density in watts/m² in the vicinity of the receiving antenna can be estimated [7] as follows:

\[ \text{Power Density} = \frac{W}{A_y} \]  

(6)

where \( W = \text{Power} \) is the power in watts obtained at each element of the receiving antenna array and \( A_y \) is the total area of the array.

Equation (6) can also be extended for the transmitting antenna array when \( W' \) would imply power (watt) radiated by each antenna element and \( A_y' \) would be replaced by \( A_t \), i.e. the total area of the transmitting antenna array.

**RF Rectifier:**

The microwave power \( P_r \) obtained at the receiving antenna array is fed to a number of parallel blocks of series-connected Schottky diodes [8] for conversion into DC power \( (P_{dc}) \) at a voltage \( V_{dc} \). For a conversion efficiency \( \eta_r \), the DC power available is:

\[ P_{dc} = \eta_r P_r \]  

(7)

The number of Schottky diodes (each diode rated \( V_d \) volts and \( P_d \) watts) connected in series in each block is \( n_s \) given by:

\[ n_s = \frac{V_{dc}}{V_d} \]  

(8)

The number of parallel blocks of series-connected Schottky diodes is \( n_p \) given by:

\[ n_p = \frac{P_{dc}}{n_s P_d} \]  

(9)

**Inverter:**

The DC power output from the Schottky diode block \( (P_{dc}) \) is fed to a three-phase six-valve inverter for conversion into three-phase AC power \( (P_{ac}) \) at 50 or 60 Hz at a line-to-line voltage \( V_L \). For an inverter efficiency \( \eta_i \), the AC power available for load is:

\[ P_{ac} = \eta_i P_{dc} \]  

(10)

The obtained line-to-line AC voltage at the load end [9] is:

\[ V_c = \frac{\pi P_{ac}}{3\sqrt{2}} \]  

(11)

**Overall efficiency:**

The overall efficiency of the WPT system is \( P_{ac}/P_r \) \((P_r \) is the AC power input from grid to the magnetron). This may also be obtained as the product of all the component efficiencies e.g. the AC-DC-microwave conversion (i.e. rectifier plus magnetron) efficiency \( \eta_r \), microwave beam transmission efficiency \( \eta_t \), RF-DC conversion (i.e. Schottky diode block) efficiency \( \eta_s \) and DC-AC conversion (inverter) efficiency \( \eta_i \):

\[ \eta_{ac} = \eta_r \eta_s \eta_i \]  

(12)

### 4 Comparison of the mathematical model with the published model

The Appendix (Section 8) presents the various input data considered (Section 8.1) and computations made (Section 8.2) for the application of the developed mathematical model to the Grand-Bassin case. Table 1 shows a comparison of the device sizes and parameters obtained from the mathematical model with those available from the Grand-Bassin literature [4] model.

The comparison shows that the mathematical model provides device sizes and parameters for the Grand-Bassin WPT project which are closer to those available in the literature. The power density of 50 W/m² (i.e. 5 mW/cm²), which is only 50% of the maximum permissible limit [10] for exposure to microwave radiation, has been used in both models. However, the literature model has used a different version of parabolic reflector transmission antenna, which has yet to be time-tested, documented and commercialised.

### 5 Discussion

WPT as a radial annexure to the grid may be a potential option in some situations, such as (i) supplying a site across a river where extending the grid by a physical overhead line or a submarine cable is not a feasible option due to the river depth and soil condition, and (ii) feeding a site across a preserved mountain or forest where earth excavation or logging for erection of a transmission line would impair the site's aesthetic value and the landscape, or affect the local flora and fauna. Naturally, such sites usually have a limited load growth and a peak demand ranging from a few kilowatts to several hundred kilowatts.

An alternative option in the above scenarios may be the installation of a stand-alone photovoltaic (PV) electricity plant. However, it is well-known that even in tropical and equatorial countries the availability of sunshine on the earth surface is subject to the vagaries of nature. Moreover, a battery backup, even if just for the night-time, is also expensive.

The authors of this paper have made a preliminary cost estimation for each of the three options i.e. physical grid extension, PV system, and WPT annex to the grid. This estimation indicates that for delivering the same amount of power, the physical grid extension, where possible, is still the least costly among the three options at the current market price. The estimated cost of installing a WPT annexure to the grid is almost the same as that of an isolated PV system supported by limited battery backup. However, the added advantage from WPT is that it can ensure a reliable supply round-the-clock due to its annexing of the grid. On the contrary, a stand-alone PV system cannot guarantee a dependable supply, especially at night, in the event that it substantially calls on its limited battery backup during the daytime due to inadequate sunshine at the site.

It should be noted that the radial feed of bulk power (several hundred kW) through wireless transmission from the grid to a site would require higher ratings for magnetrons and Schottky diodes.
Table 1: Comparison between the mathematical and the literature models of the Grand-Bassin WPT system for delivery of 10 kW to a 0.7 km distant load

<table>
<thead>
<tr>
<th>No.</th>
<th>Device or parameters</th>
<th>Mathematical model</th>
<th>Literature model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Load-end AC (50Hz) power output</td>
<td>10 kW</td>
<td>10 kW</td>
</tr>
<tr>
<td>2.</td>
<td>Inverter</td>
<td>12.5 kVA rated output based on a power factor of 0.8</td>
<td>Size not available</td>
</tr>
<tr>
<td>3.</td>
<td>RF to DC converter</td>
<td>367 parallel blocks with each block comprising 23 Schottky diodes in series (each diode rated 25 V, 1.25 kW)</td>
<td>Details not available</td>
</tr>
<tr>
<td>4.</td>
<td>Receiving antenna array</td>
<td>3 elements of H-dipole antenna occupying a length of 18 m and arranged in an array of 3 x 1.</td>
<td>6 elements of H-dipole antenna occupying a length of 17 m.</td>
</tr>
<tr>
<td>5.</td>
<td>Transmission antenna array</td>
<td>16 elements of Parabolic Reflector antenna arranged in an array of 4 x 4.</td>
<td>15 elements of Multi-foci Parabolic Reflector antenna</td>
</tr>
<tr>
<td>6.</td>
<td>Magnetron</td>
<td>14.5 kW rated output, 8 cavities with a quality factor Q = 2000</td>
<td>No particulars given</td>
</tr>
<tr>
<td>7.</td>
<td>AC power input from the Grid</td>
<td>17.08 kW</td>
<td>17.5 kW</td>
</tr>
<tr>
<td>8.</td>
<td>Overall efficiency of the WPT system</td>
<td>58.54%</td>
<td>57.0%</td>
</tr>
</tbody>
</table>

Magnetrons available as of now can provide a continuous output of microwave frequency power equal to 25 kW each, such that a reasonable number of them can easily be tandem compounded to get the desired power. Schottky diode rating reported [11] to date, however, is only 1.25 watts and 25 volts each. Such a low rating leads to the requirement of an excessively large number of Schottky diodes that significantly contributes to the WPT system cost and also reduces the RF to DC conversion efficiency. There is also room for further improvement in the overall efficiency of the WPT system, though it is much higher than that of the equivalent PV system. This can be achieved through increased directivity of the antenna arrays and higher conversion efficiency of DC to AC inverters, in addition to higher ratings for Schottky diodes.

6 Conclusions

Research effort towards the development of terrestrial wireless power transmission (WPT) as a fully matured and viable technology remains in its infancy. Though, few experimental prototypes and feasibility studies have been reported, these are unaccompanied by any design method or analytical model. This paper has presented a mathematical model that compares well with the partially disclosed specifications from a published case study. The model can be used as an effective design and simulation tool for investigations into the WPT performance, with the device sizes and parameters changed to achieve the target, while satisfying the safety limit for microwave radiation.

This paper also shows that, where possible, physical extension of the grid system is the least costly power supply option. However, WPT has immense potential in annexing a nearby grid system for radial feed to sites having a limited load growth and a peak demand of up to several hundred kilowatts. Instances where WPT could be used include a site across a river where physical extension of the grid by an overhead line or a submarine cable may not be at all feasible, or a site across preserved mountains or forests where physical line erection or cable laying would affect the local landscape or the flora and fauna.

The major impediment to the viability of wireless transmission of bulk power (in the range of several hundred kilowatts) is the lack of higher ratings for Schottky diodes (microwave to DC converters) to date. Also, the overall efficiency of WPT, though much higher than that of a PV system, needs to be further enhanced. However, a preliminary cost estimation has indicated that installing a WPT system, as a radial annexe to the nearest power grid, would cost about the same as installing an isolated photovoltaic (PV) system with limited battery backup.

Achieving higher rating, efficiency and directivity for Schottky diodes, inverters and arrayed antennas respectively should be the focus of future research on WPT. This will further reduce the cost of WPT and turn it into a commercially exploitable option for supplementing the grid system.

7 References

1. BROWN, W.C.: 'Experiment in the transport of energy by microwave beam', IEEE International Confe. Rec., 1964, 12, pp. 8-18

8 Appendix

8.1 Input data considered for the mathematical model

General information:

\[ P_{AC} = \text{AC Power delivered to load} = 10 \text{ kW} \]
\[ d = \text{distance between transmission and reception (load)} \text{ points} = 0.7 \text{ km} \]
\[ \text{Power density (for microwave radiation)} = 50 \text{ W/m}^2 \]

Inverter:

\[ \eta_i = \text{DC to AC conversion efficiency} = 0.9 \]
\[ V_L = \text{line-to-line voltage at the load end} = 415 \text{ V} \]
\[ \text{Power factor} = 0.8 \]

Schottky diodes:

\[ \eta_s = \text{RF to DC conversion efficiency} = 0.85 \]
\[ V_d = \text{each diode rated volts} = 25 \text{ V} \]
\[ P_d = \text{each diode rated power} = 1.25 \text{ W} \]

Receiving antennas (H-dipole):

\[ G_r = \text{each antenna gain} = 40 \text{ dB i.e.} 10,000 \]

Transmitting antennas (parabolic reflector):

\[ G_r = \text{each antenna gain} = 40 \text{ dB i.e.} 10,000 \]
\[ \eta_t = \text{transmission efficiency} = 0.9 \]

Magnetron:

\[ \eta_m = \text{efficiency} = 0.85 \]
\[ n = \text{number of cavities} = 6 \]
\[ T = \text{temperature (in 'Kelvin) of Cavity-1} = 500^\circ \text{K} \]
\[ = 226.85^\circ \text{C} \]
\[ T_o = \text{output stabilising time} = 0.28712 \mu\text{s} \]
\[ f = \text{output microwave radiation frequency} = 2.45 \text{ GHz} \]

8.2 Design of the Grand-Bassin case using the presented mathematical model

The considered data are appropriately substituted in corresponding equations to get various outputs in the following sequence.

Equation (10) gives Schottky diode rectifier block’s DC power output \[ P_{DC} = 11.11 \text{ kW} \]
Equation (11) gives Schottky diode rectifier DC output voltage \[ V_{dc} = 560.45 \text{ volts} \]
Equation (8) provides number of series-connected Schottky diodes in each block \[ n_s \geq 23 \]
Equation (9) gives number of parallel blocks of series-connected Schottky diodes \[ n_{ps} \geq 387 \]
Equation (7) provides the microwave power obtained at the receiving antenna array and fed to Schottky diode rectifier \[ P_r = 13.07 \text{ kW} \]. This means for a transmission efficiency \[ \eta_t = 0.9 \], the microwave power obtained from the magnetron and radiated (transmitted) is \[ P_r \geq 14.5 \text{ kW} \].
Equation (5) gives product of the number of transmitting \((x)\) and receiving \((y)\) elements \[ z = 46.55 \] and has been approximated as \( z \approx 48 \). The value of \( x \) is chosen as 16 so that parabolic reflector transmitting antennas, each with a major axis diameter \( D \) can be arranged in a square matrix array of \( 4 \times 4 \) giving an area of \( 4D \times 4D \). Choice of \( x = 16 \) leads to the number of receiving H-dipole antenna elements \( y = 3 \).
Equation (6) when applied for the receiving antenna array gives the total area of \( 87.13 \text{ m}^2 \). Each dipole antenna length is chosen as 6 m and width (feed point to the wire distance) as 5 m so that \( 3 \times 6 \times 5 = 90 \text{ m}^2 \) close to the obtained area.
Equation (6) when applied for the transmitting array gives the total area of \( 18.15 \text{ m}^2 \). This also conforms to the requirement of a larger receiving antenna array than the transmitting array so that the transmitted power can be collected maximally at the receiving apertures. Equating \( 40 \times 40 \) to \( 18.15 \) gives \( D = 1.06 \text{ m} \) so that each parabolic reflector antenna radius is 530 cm.
Equations (3) and (1) provide magnetron noise power (at the 1st cavity resonator) \[ P_i = 8.432 \times 10^{-15} \text{ watts} \].
Equation (2) gives the ‘Quality’ factor of cavity resonator \( Q \geq 2000 \) that is equivalent to a highly selective or narrow band width tuning circuit.
Equation (12) provides the overall efficiency of the WPT system \[ \eta_o = 58.54\% \]. The AC power intake from the grid \( P_{in} \) is equal to \( P_{ac}/\eta_o \).