THE SIMULATION OF DROPLET MOTION BY USING LATTICE BOLTZMANN METHOD

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ABSTRACT

SCMP (Single Component Multiphase) - LBM (Lattice Boltzmann Model) scheme was developed in order to simulate the phenomenon of droplet motion under different conditions. This study more concern on phenomenon of droplet falling from a flat ceiling and the movement of droplet on inclined surface. Various type of parameter such as contact angle, gravitational force and angle of inclined surface are used to interpret the results obtained in order to explain the phenomenon of droplet dynamics. The basic idea of SCMP LBM is incorporating the free energy method in lattice Boltzmann governing equation. The Van Der Waals real gas equation of state is derived to determine different of phases in the system. The new equilibrium distribution f_{eq} is calculated into the SCMP LBM equation. The capillary and gravitational effects are incorporated into SCMP LBM equation via pressure tensor and the new velocity in calculation of equilibrium distribution function, f_{eq}.Both capillary and gravity-driven flow contributes in different regimes of droplet shapes. Good agreement was obtained between the present approach and those previous studies using Navier-Stokes solver and original LBM.

ABSTRAK

Simulasi untuk fenomena pergerakan titisan bendalir kecil dalam pelbagai keadaan telah dibangunkan menggunakan kaedah *SCMP (Single Component Multiphase)* - *LBM (Lattice Boltzmann Model)*. Fenomena titisan bendalir jatuh daripada siling yang rata dan pergerakan titisan di atas permukaan yang condong dititit beratkan. Pelbagai jenis pembolehubah seperti sudut lekapan, daya graviti dan sudut permukaan condong digunakan untuk mentafsirkan fenomena titisan bendalir ini dengan lebih jelas. Idea asas di dalam *SCMP LBM*, adalah dengan menggunakan kaedah tenaga terbebas di dalam persamaan *lattice Boltzmann*. Persamaan gas nyata daripada Van Der Waals diterbitkan untuk menentukan setiap fasa yang berbeza di dalam sistem. Dengan menggunakan pendekatan daripada Brient's, nilai baru fungsi taburan keseimbangan dikira untuk dimasukkan ke dalam persamaan *SCMP LBM*. Kesan kapilari dan graviti telah dimasukkan ke dalam persamaan *SCMP LBM* melalui persamaan tekanan lekapan dan nilai baru halaju di dalam fungsi taburan keseimbangan. Kedua-dua kesan ini memberikan bentuk titisan yang berbeza. Perbandingan keputusan yang diperolehi daripada pendekatan yang dilaksanakan dengan kajian lepas yang menggunakan Navier-Stokes dan original LBM mendapati ianya mencapai persamaan yang ketara jelasnya.

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CHAPTER 1

INTRODUCTION

1.1 Background

At this recent day, simulation is a very important as a tool to predict the answer of the problem in fluid dynamic. The application of computational method promising a good approximating results to the physical world. A lot of works has been done and still in discovering for a better computational method to solve the problem and improving the method that already exist.

There are numerous computational exist in literature. One of them is used to solve the fluid flow problem. Computational fluid dynamics (CFD) is one of the branches of fluid mechanics that uses numerical methods and algorithms to solve and analyze problems that involve fluid flows.

The fundamental law of any CFD problem is the Navier-Stokes equations, which define any single-phase fluid flow. The classical Navier-Stokes equations have been used from 150 years ago which describe viscous fluid flow. These equations can be simplified by removing terms describing viscosity to yield the Euler equations. Further simplification, by removing terms describing vortices yields the full potential equations. They are non-linear partial differential equations which express mass and momentum conservation for fluids and can only be easily solved for only simple cases. These equations can be linearized to yield the linearized potential equations. Solving this equation is a very challenging task. A lot of numerical method was introduced by mathematicians and engineers in CFD, such as Finite Difference Method, Finite Element Method and Finite Volume Method to solve the Navier-Stokes equation numerically.

1.2 Computational fluid dynamics

In conventional computational fluid dynamics (CFD), scientists and engineers describe a fluid flow by introducing a representative control-volume element, on which macroscopic mass and momentum are conserved. The conservation laws of mass and momentum lead to a "macroscopic" mathematical model, governed by the Navier-Stokes equation, which is traditionally discretized and applied to a physical domain of interest. Physical variables such as velocity and pressure at each grid point around the element can be numerically computed.

Computational fluid dynamics (CFD) is the numerical simulation of fluid flows. CFD become essential tool in solving the Navier-Stokes Equation, the continuity equation, the energy equation and equation derived from them. Incompressible Navier-Stokes equation is the heart of the CFD, which represent a local conservation law for the momentum in the system. This equation only partially addresses the complexity of most fluids of interest in engineering applications; it is successfully applied in different areas for predictions of fluid flows.

The classical approach in CFD, treat of such fluids and describe the new physical properties in terms of transport phenomena related to a new observable, macroscopic property. A PDE is written down for the dynamics of this property then is solved by an appropriate numerical technique. In a fluid with important temperature variations for example, a new observable property, the temperature, is introduced and its dynamics is described by a heat transport equation.

1.3 Lattice Boltzmann Model

In recent years, the lattice Boltzmann method (LBM) has attracted much interest in the physics and engineering communities. As a different approach from the conventional computational fluid dynamics (CFD), the LBM has been demonstrated to be successful in simulations of fluid flow and other types of complex physical system. In particular, this method is promising for simulations of multiphase and multicomponent fluid flow involving complex interfacial dynamics. It is a discrete computational method based upon the Boltzmann equation. It considers a typical volume element of fluid to be composed of a collection of particles that are represented by a particle velocity distribution function for each fluid component at each grid point. It obtains macroscopic flow information based on integration of probability density function.

Unlike other conventional CFD that directly simulates evolution of the macroscopic kinetic equation for the single particle distribution function, the time is counted in discrete time steps and the fluid particles can collide with each other as they move, possibly under applied forces. The rules governing the collisions are designed such that the time-average motion of the particles is consistent with the Navier-Stokes equation.

A major advantage of lattice Boltzmann method is the ease and accuracy with which it enables complicated boundary geometries to be processed, hence, investigating suitable boundary conditions for lattice Boltzmann simulations has become a highly researched area in many engineering and scientific applications. Another advantage of using LBM is the simplicity of programming, the parallelism of the algorithm, and the capability of incorporating complex microscopic interactions. It is an approach that bridges microscopic phenomena with the continuum macroscopic equations. Further, it can model the time evolution of the systems.

1.4 Classical CFD versus Lattice Boltzmann Methods

The conventional simulation of fluid flow and other physical processes generally starts from non linear partial differential equation (PDEs). These PDEs are discretized by either finite differences, finite element, finite volume or spectral methods. The resulting algebraic equations of ordinary differential equation are solved by standard numerical methods.

In LBM, the starting point is a discrete microscopic model governed by Boltzmann equation. The derivation of the corresponding macroscopic equation requires multi-scale analysis [Wolf Gladrow, 2000].

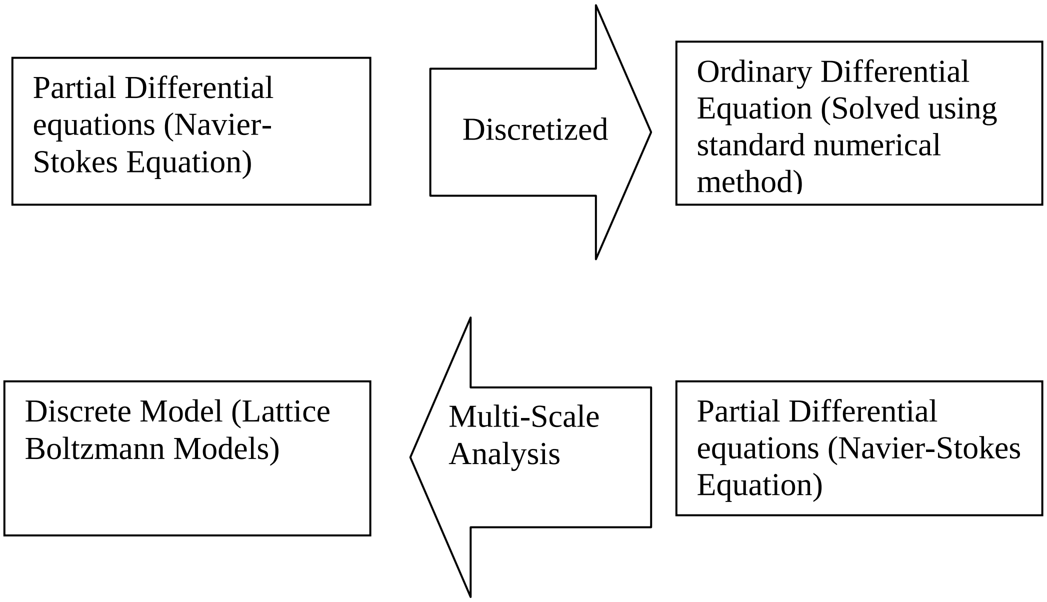

Figure 1.1: Classical CFD versus LBM

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