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Stochastic modelling for network-based GPS positioning

T.A. Musa*, *J. Wang, C. Rizos
Satellite Navigation and Positioning Group, School of Surveying and Spatial Information Systems,
The University of New South Wales, Sydney NSW 2052, Australia
Tel: +61-2-9385 4208; Fax: +61-2-9313 7493; Email:Tajul.Musa@student.unsw.edu.au

C. Satirapod
Department of Survey Engineering, Faculty of Engineering,
Chulalongkorn University, Bangkok Thailand 10330
Tel. +66-2-2186662; Fax. +66-2-2186653; Email: chalemchorn.s@chula.ac.th

ABSTRACT

Over the past few years the concept of network-based positioning has been developed in support of longer baseline processing compared with 'traditional' single reference station positioning. In fact network-based positioning enables the generation of so-called ‘virtual measurements’, which can significantly improve positioning results. Even though the virtual measurements are generated from the stochastic network estimates, the error propagation into the user position solution has not been investigated in any detail. The aim is to understand how the unique stochastic properties of the network corrections propagate into the uncertainties of the estimated parameters. Test results indicate that by using the virtual measurements and considering the propagation of the network stochastic properties can provide reliable results, both in terms of ambiguity resolution and baseline component estimation.

KEYWORDS: Network-based positioning, virtual measurement, stochastic modelling, MINQUE

1. INTRODUCTION

Over the past few years, the concept of network-based positioning has been extensively developed so that static and kinematic GPS positioning can be undertaken using longer baselines than is possible using 'traditional' single-base station techniques. Numerical results show that the various implementations that have been tested can significantly reduce the distance-dependent biases in the double-differenced carrier phase and pseudo-range measurements (Wanninger, 2002; Chen, 2000). In fact, the reference receiver network permits the generation of so-called ‘virtual measurements’. To create these virtual measurements so-called network corrections terms need to be calculated using data from at least three reference stations (Dai, 2002).

The least squares estimation technique can be applied to the virtual measurements after constructing appropriate functional and stochastic models. The functional model describes the relationships between the measurements and the unknown parameters. Meanwhile, to interpret the precision of the estimates, the stochastic model needs to be defined, typically via some variance-covariance (**VCV**) matrix. The functional model of GPS measurements is very well accepted, but the definition of the stochastic model remains a somewhat controversial issue. Standard procedures can construct the **VCV** through the error propagation law, typically assuming all measurements have the same variance. (This assumption of course will make the processing much easier, but in fact all measurements are subject to different noise levels and therefore cannot be assigned the same precision).

In the case of virtual measurements, the unique stochastic properties of the network are not taken into account. Even though the virtual measurements are generated from the stochastic network estimates, their propagation into the user position has not been investigated in any detail. A better understanding of the stochastic properties of these virtual measurements could lead to an improvement in the ambiguity success rate, and hence ensure fast and reliable ambiguity resolution for network-based positioning.

In this paper, the propagation of the stochastic properties of the network through the virtual measurement will be investigated. A method known as the Simplified MINQUE (Satirapod, 2002) has been used in variance component estimation for the virtual measurements. The following sections will give a brief discussion on baseline processing, variance component estimation, and the generation of the virtual measurements. The stochastic model of the virtual measurements will then be discussed, followed by the description of some experiments and results.

2. DOUBLE DIFFERENCE EQUATIONS AND LEAST SQUARES ESTIMATION FOR CARRIER PHASE MEASUREMENT

Data differencing techniques are well accepted for GPS data processing. The double-differenced observable reduces many common errors and biases resulting in a simplified functional model. For epoch *i* with two receivers tracking *k* satellites, the double-differenced measurement equation can be formed (Hoffman-Wellenhof *et al.*, 1994):

$$\mathbf{DD}_{(i)} = \mathbf{C}\ \mathbf{SD}_{(i)} \tag{1}$$

where **SD** are the single-differenced observables, **Dd**are the double-differenced observables, and **C** is the differencing operator; given by

$$\overset{k-1,k}{\mathbf{C}} = \begin{bmatrix} -1 & 1 & 0 & \dots & 0 \\ -1 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ -1 & 0 & 0 & \dots & 1 \end{bmatrix} \tag{2}$$

Applying the error propagation law, the **VCV** for the double-differences are written as:

$$\mathbf{VCV}\ (\mathbf{DD})_{(i)} = \mathbf{C}.\mathbf{VCV}\ (\mathbf{SD})_{(i)}\ .\mathbf{C}^{T} = \sigma^{2}\,\mathbf{CC}^{T} \tag{3}$$

where

$$\mathbf{VCV(SD)} = 2\sigma^{2}\,\mathbf{I} \tag{4}$$

is the **VCV** of single-differences, σ^2 is the variance of the one-way carrier phase measurement with expectation value zero, under the assumption that the phase errors show a random behavior and follow the normal distribution, and **I** is the identity matrix. In matrix form, Equation (3) can be written as:

$$\mathbf{VCV(DD)}_{(i)} = \sigma^{2}\begin{pmatrix} 4 & 2 & . & 2 \\ 2 & 4 & . & 2 \\ . & . & . & . \\ 2 & 2 & . & 4 \end{pmatrix} \tag{5}$$

Note that the **VCV** of the double-differences in Equation (5) implies that they are mathematically correlated, having the same variance and are statistically independent in time and space. In standard least squares theory, a set of linearised double-differenced observables can be formed (Blewitt, 1998):

$$\mathbf{z} = \mathbf{Ax} + \mathbf{v} \tag{6}$$

where **z** is the column vector of observed-minus-computed observations, **A** is the design matrix, **x** is the column vector of the unknown parameters, and **v** is a column vector of errors. Assuming the expectation (E) of **v** is zero, the **VCV** as constructed in Equations (3) and (5) now can be written as:

$$E(\mathbf{vv}^{T}) = \mathbf{VCV_x} = \mathbf{W}^{-1} \tag{7}$$

where **W** and $\mathbf{VCV_x}$ is the weight and variance matrix of the observables. The least squares estimator of the unknown parameter **x** is:

$$\hat{\mathbf{x}} = (\mathbf{A}^{T}\mathbf{WA})^{-1}\mathbf{A}^{T}\mathbf{Wz} \tag{8}$$

To compute Equation (8), approximate values of the unknown parameters are needed. Equation (8) is dependent on the design matrix **A**, **VCV** and the set of observations **z**. The adjusted observables and least squares residuals can be computed:

$$\hat{\mathbf{z}} = \mathbf{A}\hat{\mathbf{x}} \tag{9}$$

$$\hat{\mathbf{v}} = \mathbf{z} - \hat{\mathbf{z}} \tag{10}$$

It is usual after computing the least squares residual to also compute the quantity $\hat{\sigma}^2$, the unit variance (e.g., Cross, 1983):

$$\hat{\sigma}^2 = \frac{\hat{\mathbf{v}}^{\mathrm{T}}\mathbf{W}\hat{\mathbf{v}}}{f} \tag{11}$$

where f is the number of degrees of freedom. Equation (8) has the following statistical properties:

$$E(\hat{\mathbf{x}}) = E(\mathbf{x}) = \mathbf{x} \tag{12}$$

$$E(\hat{\mathbf{x}}\hat{\mathbf{x}}^{\mathrm{T}}) = \hat{\sigma}^2(\mathbf{A}^{\mathrm{T}}\mathbf{W}\mathbf{A})^{-1} \equiv \mathbf{VCV}_{\hat{\mathbf{x}}} \tag{13}$$

Thus $\mathbf{VCV}_{\hat{\mathbf{x}}}$ is dependent only on **A** and $\mathbf{VCV}_{\mathbf{x}}$. Once the functional and stochastic models have been specified, one is already in a position to know the precision of the least squares result. It also implies that if one is not satisfied with this precision, one can change it by changing **A** and/or $\mathbf{VCV}_{\mathbf{x}}$ (Teunissen, 1998).

3. VARIANCE COMPONENT ESTIMATION

Tiberius *et al.* (1999) have reported that by ignoring stochastic modelling when processing GPS observations, a lower position precision may result. Their research has shown that measurement precision is dependent on satellite elevation angle, cross-correlation and the time correlation function. The satellite elevation angle and the signal-to-noise ratio (SNR) have been widely used as quality indicators for GPS observations (e.g., Euler and Goad, 1991; Han, 1997; Langley, 1997; Spilker, 1996). However, such quality indicators may not always reflect reality (Satirapod, 2002).

A rigorous statistical method for estimating **VCV** components known as the Minimum Norm Quadratic Unbiased Estimation (MINQUE) has been developed by Rao (1979). Wang (1999) has demonstrated the use of MINQUE procedure to directly estimate elements of the **VCV** of the double-differenced measurements without making any assumptions (as is typical for the standard stochastic modelling procedure) or through complicated functional factors. Moreover, the reliability of the resolved ambiguities and the relative efficiency of baseline estimation were shown to have been significantly improved through the use of the MINQUE procedure (Wang, 1999).

For a set of r double-differenced measurements at epoch i, the **VCV** (Equation 5) can also be written as:

$$\mathbf{VCV} = \sum_{j=1}^{k} \boldsymbol{\theta}_j \mathbf{T}_{ji} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12} & \cdots & \sigma_{1r} \\ \sigma_{12} & \sigma_{22}^2 & \cdots & \sigma_{2r} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{r1} & \sigma_{r2} & \cdots & \sigma_{rr}^2 \end{bmatrix} \tag{14}$$

where $\boldsymbol{\theta}$ is the vector of the unknown variance components, **T** are the so-called accompanying matrices and $k=r(r+1)/2$ is the number of unknown **VCV** components. The **VCV** components can be estimated as (Rao, 1979):

$$\boldsymbol{\theta} = \mathbf{S}^{-1}\mathbf{q} \tag{15}$$

where the matrix $\mathbf{S} = \{S_{ij}\}$ is

$$S_{ij} = \text{Trace } \{\mathbf{RT}_i\mathbf{RT}_j\} \tag{16}$$

and the vector $\mathbf{q} = \{q_i\}$ is

$$q_i = \mathbf{z}^T\mathbf{RT}_i\mathbf{Rz} \tag{17}$$

and

$$\mathbf{R} = \mathbf{VCV}^{-1}[\mathbf{I} - \mathbf{A}(\mathbf{A}^T\mathbf{VCV}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{VCV}^{-1}] \tag{18}$$

In addition, the MINQUE computation is burdened with the requirement to compute the matrix **R**, which may require a huge computer memory, for processing 6 satellites and 15 second sampling interval in 60 minutes session length, MINQUE require 11250 kilobytes memory usage (Satirapod 2002). A simplification of the MINQUE procedure can be obtained by neglecting the off-diagonal elements of matrix R. This simplified procedure is shown to produce results that are close in value to those derived using the rigorous MINQUE procedure. Furthermore the computational load is much less than for the case of the MINQUE procedure. A complete discussion of the MINQUE and simplified MINQUE procedures can be found in Wang (1999, 2002) and Satirapod (2002).

4. GENERATING THE VIRTUAL MEASUREMENTS

The idea behind network-based positioning is to eliminate most/all of the orbit bias, ionospheric delay and tropospheric delay. If there are several reference stations whose positions are known with high precision, a linear combination model of the single-differenced observables can be formed (Han and Rizos, 1996; Han, 1997):

$$\begin{aligned}\sum_{i=1}^{n}\alpha_i\Delta\phi_i = \sum_{i=1}^{n}\alpha_i\Delta\rho_i + \sum_{i=1}^{n}\alpha_i\Delta d\rho_i - c\sum_{i=1}^{n}\alpha_i\Delta dT_i + \lambda\sum_{i=1}^{n}\alpha_i\Delta N_i - \sum_{i=1}^{n}\alpha_i\Delta d_{ion,i} + \\ \sum_{i=1}^{n}\alpha_i\Delta d_{trop,i} + \sum_{i=1}^{n}\alpha_i\Delta d_{mp,i} + \varepsilon_{\sum_{i=1}^{n}\alpha_i\Delta\phi_i}\end{aligned} \tag{19}$$

where n is the number of reference stations, i indicates the i^{th} reference station, Δ is the single-differencing operator, ϕ is the carrier phase observation, p is the satellite position vector minus the station position vector, dp is the effect of orbit error, dT is the receiver clock error with respect to GPS time, d_{ion} is the ionospheric delay, d_{trop} is the tropospheric delay after model correction, d_{mp} is the multipath on the carrier phase measurement, ε is the carrier phase observation noise, λ is the wavelength of the carrier wave, N is the integer ambiguity and α_i is the weight for the i reference station.

Using only an approximate value of the user's position, a proper weight (α) can be determined that is inversely proportional to the distance from the reference stations (X_i) to the user (X_u), satisfying the following conditions:

$$\sum_{i=1}^{n} \alpha_i (X_u - X_i) = 0 \tag{20}$$

and

$$\sum_{i=1}^{n} \alpha_i = 1 \tag{21}$$

Figure 1 illustrates the geometric situation. This network-based concept was first introduced by Wu (1994) and extended by the Satellite Navigation and Positioning (SNAP) research group (Han, 1997; Chen, 2001; Dai, 2002). In the event that there are three or more reference stations, another constraint should be added to make sure it satisfies Equations (20) and (21). This will also provide a unique solution for the weights as the number of reference stations is increased. The constraint is:

$$\sum_{i=1}^{n} \alpha_i^2 = \min \tag{22}$$

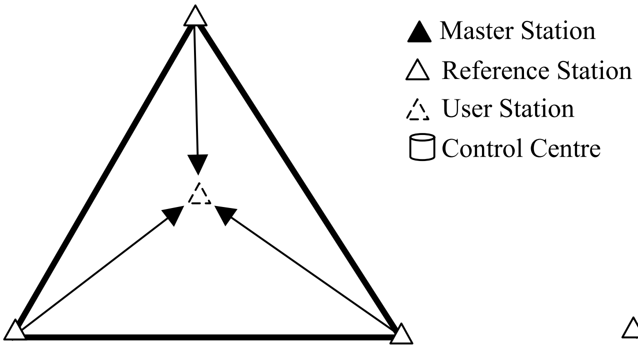

Figure 1. Network approach

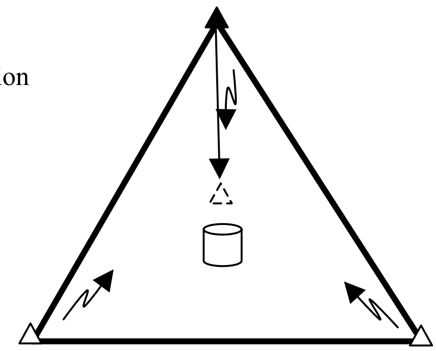

Figure 2. Network approach with one master control and control centre

If one reference station is selected as a master station (m), from the n possible reference stations, Equations (20) and (21) can be written as:

$$\sum_{i=1}^{n} \alpha_i \sum_{i=1}^{n-1} \Delta X_{i,m} = \Delta X_{u,m} \tag{23}$$

The master station should be selected so that it is the nearest to the user, which also reduces the residual ionospheric effects (even though these effects are already mitigated from the network estimates). A suggestion can be made by establishing a communication between the reference stations to one control centre for network data processing. Figure 2 illustrates the concept in Equation (23). Through a least squares condition estimation process, and satisfying Equation (22), a set of parameters can be determined:

$$\boldsymbol{\alpha} = (\mathbf{B}\mathbf{B}^{\mathrm{T}})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{L} \tag{24}$$

where **B** is the design matrix and **L** is [1 $\Delta X_{u,m}$ $\Delta Y_{u,m}$]^T. Through Equations (23) and (24), the double-differenced functional model can be written as:

$$\Delta\nabla\phi_{u,m} - [\alpha_1 V_{1,m} + \ldots + \alpha_{n-1} V_{n-1,m}] = \Delta\nabla p_{u,m} + \lambda\Delta\nabla N_{u,m} + \varepsilon_{\sum_{i=1}^{n}\alpha_i\Delta\nabla\phi_i} \tag{25}$$

where V is defined as the double-difference residual vectors from master station to other reference stations after the ambiguities (N) have been resolved and $[\alpha_1 V_{1,m} + \ldots + \alpha_{n-1} V_{n-1,m}]$ is the network corrections term. In fact Equation (25) may also be denoted as a 'virtual measurement' after applying the network correction term.

5. STOCHASTIC MODELLING

Even though the functional model for the virtual measurement (Equation 25) using the network estimates can eliminate or mitigate the common biases, residual biases still exist and contribute to the noise terms. Dealing with these residual biases is very challenging if one tries to use the functional model approach. An alternative approach is to account for them within the stochastic model. Han (1997) has proposed a real-time stochastic model estimation procedure based on an exponential function of the satellite elevation angle for the virtual measurement. The standard deviation of one-way L1 observations for satellite j is written as:

$$\sigma^j = s.[a_0 + a_1.\exp(-e^j / e_0)] \tag{26}$$

where e^j is the elevation angle, a_0, a_1 and e_0 are approximated by constants, experimentally determined from different kinds of GPS receivers (e.g., Euler and Goad, 1991; Han, 1997), and s is a scale factor which will weight the contribution of carrier phase measurements, and is assumed to be the same over a short period of measurements.

A proper standard deviation for the virtual measurement should be specified that will make the estimation result more realistic. It is necessary to understand how the stochastic network estimates propagate into the user parameter estimation. Basically the idea is to understand how the unique stochastic properties of the network corrections propagate into the uncertainty of components which form the network correction parameters. This propagation is best explained using the **VCV**.

Consider the standard deviation of the one-way carrier phase for k satellites is given by σ_k from the geometric correlations of the linear combination model in Equation (19), assuming first that each single-differenced observable has equal standard deviation and is independent. The **VCV** of the linear combination is (Han, 1997):

$$\mathbf{VCV}_{\sum_{i=1}^{n}\alpha_i\Delta\phi_i} = \left(1 + \sum_{i=1}^{n}\alpha_i^2\right)\cdot\sigma_k^2 \tag{27}$$

It can be seen from Equation (27) that the network geometry is embedded within the **VCV**. Having the double-differencing operator as **C** in Equation (2), the variance-covariance matrix of the double-differenced observables can be constructed by applying Equation (3) to Equation (27):

$$\mathbf{VCV}_{\sum_{i=1}^{n}\alpha_i \Delta\nabla\phi_i} = \mathbf{C}_k \cdot \mathbf{VCV}_{\sum_{i=1}^{n}\alpha_i \Delta\phi_i} \cdot \mathbf{C}_k^T \quad (28)$$

Following the same procedure as used to obtain Equation (27), if the **VCV** of the master-to-user station can be found as $\mathbf{VCV}_{\Delta\nabla\phi_{u,m}}$, the **VCV** of the virtual measurement in Equation (25) can also be derived:

$$\mathbf{VCV}_{\sum_{i=1}^{n}\alpha_i \Delta\nabla\phi_i} = \cdot \frac{\left(1+\sum_{i=1}^{n}\alpha_i^2\right)}{2} . \mathbf{VCV}_{\Delta\nabla\phi_{u,m}} \quad (29)$$

Following from Equation (29), it is evident that the $\mathbf{VCV}_{\sum_{i=1}^{n}\alpha_i \Delta\nabla\phi_i}$ is dependent on the unique geometric correlation of the network. By having a rigorous statistical method such as MINQUE, or the simplified MINQUE, for estimating the $\mathbf{VCV}_{\Delta\nabla\phi_{u,m}}$ components in Equation (29), the estimation result for the virtual measurements will probably be more realistic.

6. THE EXPERIMENTS

The static positioning technique has been used in these experiments. Only single-frequency (L1) GPS data has been used and processed using the SNAP GPS processing software. The data was downloaded from the Southern California Integrated GPS Network (SCIGN) website (http://www.scign.org). Three different data periods (day of year (DoY) 221 00, 222 00 and 227 02) were used in these experiments with a 30s observation rate. Figure 3 shows part of the network, which consists of three sites as reference stations (FXHS, FMTP, QHTP), and another two sites as user stations (CSN1, CMP9). The coordinates of all the stations (Table 1) were obtained using the Scripps Coordinates Update Tool (SCOUT) (SOPAC, 2002). This service computes the coordinates by using the three closest located reference sites and the precise GPS ephemerides (Janssen and Rizos, 2003). These coordinates will be considered as known coordinates and used in the data processing.

SITE	**X (m)**	**Y (m)**	**Z (m)**
FXHS	-2511943.6388	-4653606.7722	3553873.9778
FMTP	-2545459.7204	-4612207.1586	3584252.1200
QHTP	-2486712.3456	-4629002.0822	3604537.5090
CSN1	-2520225.8551	-4637082.4402	3569875.3624
CMP9	-2508505.9552	-4637175.0256	3579499.8619

Table 1. ITRF2000 coordinates of SCIGN

The baseline processing was divided into two parts; first the fiducial reference baselines (FXHS-FMTP and FXHS-QHTP) were determined, and the inner baselines (FXHS-CSN1 and FXHS-CMP9) specific to the user sites. All the baselines were processed relative to one master station (FXHS). The fiducial baselines were processed in order to generate the network correction terms. Using coordinates of the reference stations (Table 1) and approximate user station coordinates (about ~100m in error), the network coefficients (α) were calculated using Equation (24). Table 2 lists the calculated α_1 and α_2 coefficients for user sites CSN1 and CMP9 with respect to fiducial baselines FXHS-FMTP and FXHS-QHTP. The network correction terms (see Equation 25) can be generated using these coefficients values and the double-difference residuals (after fixing the ambiguities) of the fiducial baselines. To ensure successful ambiguity resolution amongst the fiducial baselines, observation data sets with session lengths of 2hr (DoY 221 00), 3hr (DoY 222 00), 2hr (DoY 227 02) and 2.5hr (DoY 227 02) were processed.

SITE	α_1	α_2
CSN1	0.333	0.113
CMP9	0.178	0.373

Table 2. Network coefficients (α) calculated from coordinates at user stations (approximate) and reference stations

The inner baselines were processed with different session length (Table 3) using the following baseline processing methods:

A. Standard baseline processing with a stochastic model expressed by Equation (5).
B. Standard baseline processing and the simplified MINQUE stochastic model (Equation 15).
C. Virtual measurements with a stochastic model as in method A.
D. Virtual measurements with a stochastic model as in method B.

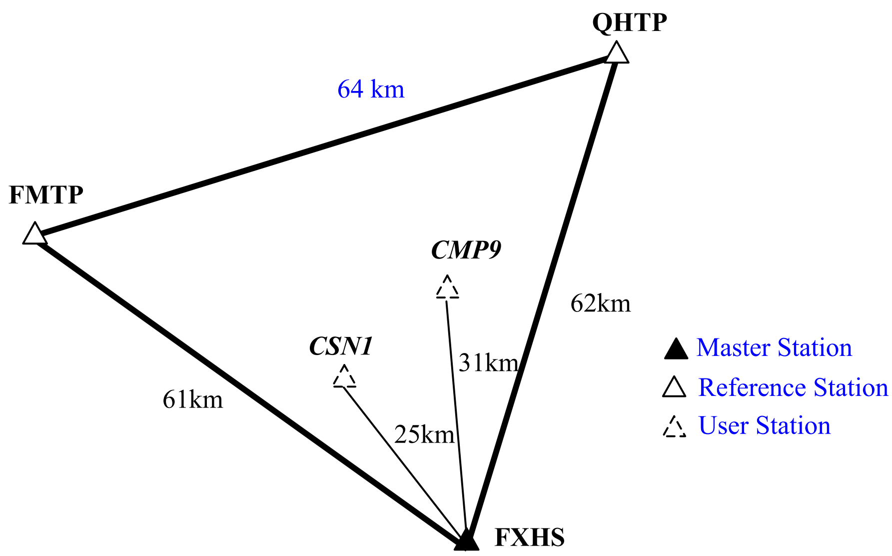

Figure 3. Test network, part of the SCIGN

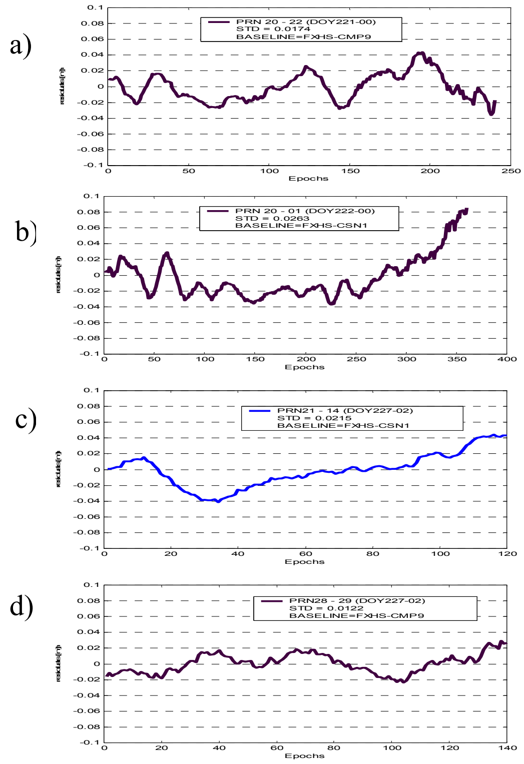

Figure 4.1. Method A

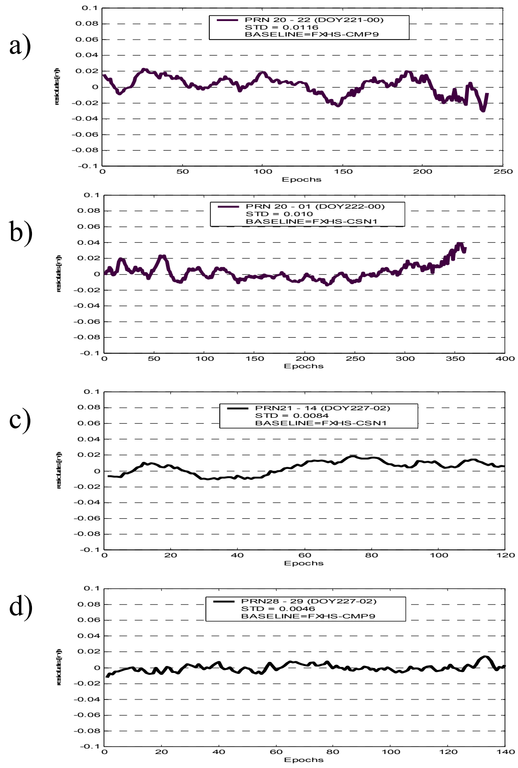

Figure 4.2. Method C

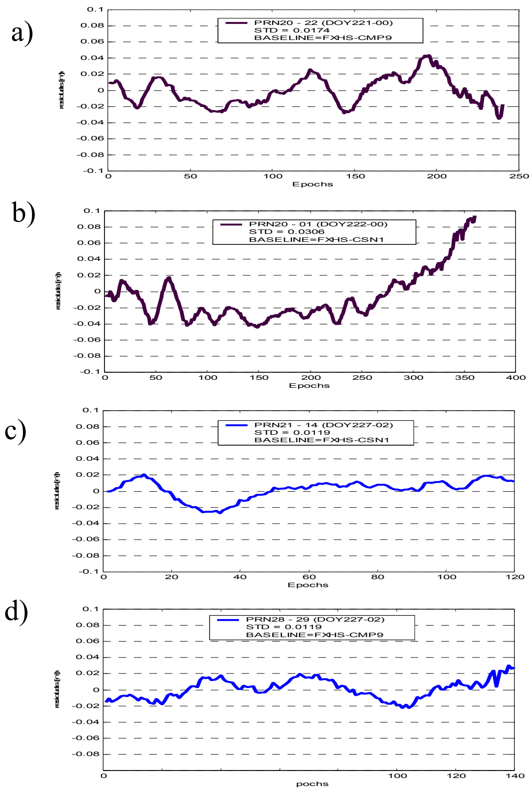

Figure 4.3. Method B

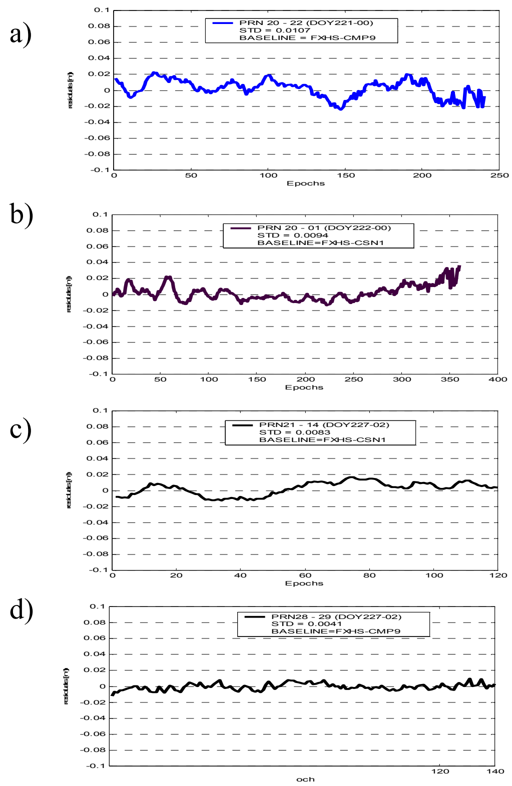

Figure 4.4. Method D

Figure 4. Double-differenced residuals (inner baselines) for the same satellite pairs processed using method A, B, C and D. Session length is 2hr (DoY 221 00), 3hr (DoY 222 00), 1hr (DoY 227 02) and 70 minutes (DoY 227 02). Observations rate is 30s.

Day of Year (DoY)	Inner Baseline	Session Length
221 2000	FXHS-CMP9	2 hr
222 2000	FXHS-CSN1	3 hr
227 2002	FXHS-CSN1	1 hr
227 2002	FXHS-CMP9	70 min

Table 3. Inner baseline processing

Figure 4 shows the double-difference residuals for the inner baselines for the same satellite pairs, processed using each of the four methods. The time series in Figures 4.1 and 4.3, which refer to the method A and B, indicate not much difference between the two methods. The standard deviation of the time series for both methods varies from ±0.01m to ±0.03m (Figure 4.1 and 4.3). After applying the virtual measurement (in method C and D), both results show a significant improvement, as indicated in Figures 4.2 and 4.4, where the standard deviation is now reduced to between ±0.004m and ±0.01m.

Baseline	Method	F-ratio	W-ratio
FXHS-CMP9 (2hrs)	A	2.088	10.464
	B	2.421	12.398
	C	6.984	33.167
	D	9.796	39.878
FXHS-CSN1 (3hrs)	A	4.288	29.050
	B	3.753	24.853
	C	21.126	75.523
	D	25.533	83.104
FXHS-CSN1 (1hrs)	A	1.263	2.661
	B	1.036	0.398
	C	2.975	11.604
	D	4.720	19.333
FXHS-CMP9 (70 min)	A	2.000	9.643
	B	1.818	9.252
	C	6.175	21.954
	D	6.720	23.186

Table 3. Ambiguity discrimination test for inner baseline; DoY 221 00, DoY 222 00 and DoY 227 02 with different session length

After all the potential candidates passed the ambiguity acceptance test (in the LAMBDA method) the ambiguity discrimination test statistic will be the next step. This statistical test will ensure the most likely integer ambiguity combination is statistically better than the second best. Two ambiguity discrimination test statistics, namely the classic F-ratio (Euler & Landau, 1992) and the W-ratio (Wang *et al.* 1998), were used in this experiment. Usually the critical value of the F-ratio is chosen as 2.0, but there is no theoretical (statistical) basis for using such a fixed value. On the other hand, the W-ratio values are larger than 3 (if the round-off error is less than 0.2 cycles), which indicates a confidence level of 99.9% for the statistic test discriminating between the best and the second best ambiguity combinations, see Wang *et al.* (1998) for details. The larger the values of these statistics, the more reliable the ambiguity resolution. Table 3 lists the results of the ambiguity discrimination tests.

From Table 3, it can be seen that for methods A and B, the F-ratio and the W-ratio passed the critical value except for the 1hr session of DoY 227 02 and the 70 minutes session of DoY 227 02 for method B. These results may be due to systematic errors in the measurement which are not cancelled by methods A and B. The estimated baseline components and their a posteriori standard deviations (Table 4) show no significant difference for method A and B, except for two cases which do not passed the ambiguity discrimination test. Their standard deviations are larger and the baseline length significantly different to those calculated using the coordinates in Table 1 (baseline FXHS-CSN1 is 24447.7599m and baseline FXHS-CMP9 is 30635.0437m).

Baseline	**Method**	**Estimated baseline component (m)**			**Baseline Length (m)**	**Standard Deviation (mm)**		
		X	**Y**	**Z**		**X**	**Y**	**Z**
FXHS-CMP9	A	3437.653	16431.677	25625.939	30635.049	1.1	2.3	1.5
	B	3437.650	16431.660	25625.949	30635.048	1.3	2.0	1.4
(2hrs)	C	3437.655	16431.671	25625.944	30635.049	0.7	1.4	0.9
	D	3437.654	16431.667	25625.947	30635.050	0.8	1.1	0.7
FXHS-CSN1	A	-8282.219	16524.271	16001.420	24447.743	1.3	2.2	1.5
	B	-8282.227	16524.242	16001.431	24447.732	1.1	2.4	1.7
(3hrs)	C	-8282.216	16524.271	16001.420	24447.743	0.5	0.9	0.6
	D	-8282.221	16524.267	16001.423	24447.743	0.5	0.9	0.6
FXHS-CSN1	A	-8282.239	16525.377	16001.088	24448.279	2.9	5.0	3.8
	B	-8282.143	16524.310	16001.320	24447.678	3.3	5.4	3.1
(1hrs)	C	-8282.155	16524.291	16001.338	24447.681	1.7	3.0	2.3
	D	-8282.151	16524.290	16001.339	24447.680	1.8	2.7	1.9
FXHS-CMP9	A	3437.701	16431.760	25625.867	30635.038	1.7	1.7	2.7
	B	3437.704	16431.760	25625.860	30635.033	1.0	0.8	1.2
(70min)	C	3437.701	16431.760	25625.867	30635.038	0.7	0.8	1.2
	D	3437.701	16431.760	25625.866	30635.037	0.5	0.6	0.9

Table 4. Estimated baseline components and standard deviations for inner baseline; DoY 221 00, DoY 222 00 and DoY 227 02 with different session length

However, after applying the network correction terms through the virtual measurements (Equation 25) in method C and D, a significant improvement was achieved where all the results passed the critical value for the ambiguity discrimination tests (F-ratio and W-ratio).

For the baseline lengths, they are closer to the calculated baseline lengths and their standard deviations are smaller compared with method A and B. An advantage of the virtual measurements is that the effects of systematic errors are reduced, improving the precision of the estimated positioning results and the reliability of ambiguity resolution.

7. CONCLUDING REMARKS

Application of network-based positioning techniques has benefited the user with better positioning results compared to traditional single-base station techniques. Many functional models has been developed to improve the positioning results but less research has been done on the stochastic properties of the network. This paper has investigated the propagation of the unique stochastic network through the virtual measurement created after applying the network correction term. By having redundant virtual measurements, their **VCV** has been estimated using the simplified MINQUE procedure. It has been shown from experiments that the proposed virtual measurement with the estimated **VCV** (method D) gives better results, both in terms of ambiguity resolution and baseline component estimation. Note that the stochastic model used does not consider the impact of temporal correlations, which will be a topic for future research. Including this temporal correlation will ensure an even more realistic stochastic model.

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