FINITE ELEMENT MODEL OF BIOMAGNETIC FLUID FLOW IN A BIFURCATED CHANNEL WITH AN OVERLAPPING STENOSIS

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DEDICATION

To my treasured family,

Abdullah Mohamad

Norita Abdullah

Amri Abdullah

Hartini Abdullah

Normazleha Abdullah

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Assalammualaikum W.B.T.

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ABSTRACT

The flow of biomagnetic fluid is considerably influenced by the presence of magnetic field especially in the case of spatially varying magnetic field. Blood is known as the most common biofluid where its magnetic properties are governed by the existence of high concentration of hemoglobin, which is a form of iron oxide in mature red blood cells. Biomagnetic fluid dynamics is a study for the relationship between fluid dynamics of biological blood with the magnetic field. The problems of blood flow in the presence of magnetic field with an overlapping stenosis in the straight and bifurcated channel are investigated numerically. In this study, the flow is considered to be steady, two-dimensional, laminar, and electrically nonconducting under the influence of spatially varying magnetic field. The mathematical model for biomagnetic fluid dynamics for isothermal case is constructed based on modified Navier-Stokes equations coupled with the principles of ferrohydrodynamics (FHD) and magnetohydrodynamics (MHD), which consider force arising due to magnetization and Lorentz force, respectively. Based on these assumptions, the nonlinear system of the governing equations is then discretized by employing Galerkin finite element method and the source code is developed by using MATLAB software in order to solve the problem. The numerical result shows that the intensity of the applied magnetic field could alters the behaviour of blood flow at the area near the magnetic source and cause the formation of new vortex. This could be seen from the velocity profile and streamline pattern of both straight and bifurcated artery. When the magnetic intensity is increased, the flow velocity can be seen to be reduced considerably near the stenosis area compared to the velocity of the flow before magnetic field is introduced. This shows that the study on magnetic field on biomagnetic fluid is proven to be beneficial for future development in biomedical area.

ABSTRAK

Aliran bendalir yang bersifat biomagnetik sangat dipengaruhi oleh kehadiran medan magnet terutama dalam kes medan magnet yang mempunyai kedudukan ruang berbeza. Darah dikenali sebagai bendalir biologi di mana sifat magnetnya ditadbir oleh kewujudan kepekatan hemoglobin yang tinggi, yang merupakan bentuk oksida ferum dalam sel darah merah yang matang. Dinamik bendalir biomagnetik adalah kajian bagi perkaitan antara dinamik bendalir bagi darah dengan medan magnet. Masalah aliran darah dengan kehadiran medan magnet dengan stenosis bertindih dalam saluran arteri lurus dan arteri bercabang dikaji secara berangka. Dalam kajian ini, aliran dianggap mantap, dua dimensi, laminar, dan bukan pengkonduksi elektrik di bawah pengaruh medan magnet yang berbeza ruang. Model matematik bagi dinamik bendalir biomagnetik untuk kes isoterma telah dibina berdasarkan persamaan Navier-Stokes yang diubahsuai dengan mengambil kira prinsip ferohidrodinamik (FHD) dan magnethidrodinamik (MHD), yang masing-masing mempertimbangkan daya yang timbul kerana daya pemagnetan dan daya Lorentz. Berdasarkan andaian ini, sistem tak linear dari persamaan menakluk kemudiannya dicerakinkan dengan menggunakan kaedah unsur terhingga Galerkin dan kod pengaturcaraan telah dibangunkan dengan menggunakan perisian MATLAB bagi menyelesaikan masalah tersebut. Hasil daripada kaedah berangka menunjukkan bahawa intensiti medan magnet yang digunakan boleh mempengaruhi sifat aliran darah pada kawasan berhampiran sumber magnet dan menyebabkan pembentukan pusaran baharu. Ini juga dapat dilihat dari profil halaju dan corak garis arus arteri lurus dan bercabang. Apabila kadar intensiti magnet ditingkatkan, halaju bendalir dapat dilihat menunjukkan penurunan berhampiran kawasan stenosis berbanding halaju bendalir sebelum medan magnet diperkenalkan. Ini menunjukkan bahawa kajian mengenai medan magnet pada bendalir biomagnetik telah dibuktikan bermanfaat untuk perkembangan bidang perubatan masa hadapan.

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LIST OF ABBREVIATIONS

MHD	-	Magnetohydrodynamics
FHD	-	Ferrohydrodynamics
BFD	-	Biomagnetic Fluid Dynamics
FEM	-	Finite Element Method

LIST OF SYMBOLS

x	-	Axial coordinate
y	-	Radial coordinate
$R_1(x)$	-	Radii of the outer wall
$R_2(x)$	-	Radii of the inner wall
a	-	Radii of the mother artery
r_1	-	Radii of the daughter artery
r_0	-	Radii of curvature for the lateral junction
r'^0	-	Radii of curvature for the flow divider
d	-	Onset of the stenosis
l_0	-	Length of the stenosis at a distance d from the origin
x_1	-	Location of the onset and offset of the lateral junction
x_2	-	Location of the offset of the lateral junction
x_3	-	Apex
τ_m	-	Maximum height of stenosis
β	-	Half of the bifurcation angle
ρ	-	Density of blood
u	-	Axial velocity component
v	-	Radial velocity component
w	-	Tangential velocity component
t	-	Time
f_x	-	Body force per unit mass of the fluid element
μ	-	The respective molecular viscosity coefficient
λ	-	The second viscosity coefficient
τ_{ij}	-	Stress in the $j-$direction applied on a plane perpendicular to the $i-$axis
Δ	-	The symmetrical rate of deformation tensor
m	-	Fluid consistency coefficient
n	-	Flow behaviour index

$\mathbf{F}_m$	-	Electromagnetic force
J	-	Current density
$\mathbf{B}$	-	Magnetic flux intensity
$\mathbf{E}$	-	Electric field intensity
σ	-	Electrical conductivity
$\mathbf{B}_0$	-	External magnetic field
$\mathbf{B}_1$	-	Induced magnetic field
B_0	-	Magnitude of B_0
Ω	-	Domain
$\mathbf{n}$	-	Unit vector outward normal to the outflow boundary
P	-	Pressure
Γ_h	-	Boundary corresponding to Dirichlet condition
Γ_g	-	Boundary corresponding to Neumann condition
h	-	Length of the inlet
u_r	-	Averaged mean inflow velocity
Re	-	Reynolds number
M	-	Hartmann number
C_h	-	Finite element partition
K	-	Triangular element
$Q_m(x,y)$	-	Weighting function for continuity equation
$N_l(x,y)$	-	Weighting function for momentum equations
B_{cx}	-	Natural boundary conditions of $x-$momentum equations
B_{cy}	-	Natural boundary conditions of $y-$momentum equations
$N^{e_i,j}$	-	Nodal shape functions j in element e_i for velocity components
$Q^{e_i,j}$	-	Nodal shape functions j in element e_i for pressure components
A_K	-	Area of triangular element K
e_i	-	Triangular element
u^{e_i}, v^{e_i}	-	Spatial variations of velocity components within an element e_i

p^{e_i}	-	Spatial variations of pressure component within an element e_i
u_j, v_j	-	Velocities at corner nodes j
p_j	-	Pressure at corner nodes j
n_q	-	Number of quadrature points (η_j, ξ_j)
K	-	Stiffness matrix
U	-	Vector of degree of freedom
F	-	Force vector
J	-	Jacobian matrix
τ	-	Convergence tolerance
$\tau(\mathrm{Re}_K)$	-	Stabilization parameter
h_K	-	Size of triangular element K
$\eta(\gamma)$	-	Viscosity function
δu	-	Small correction on u
δv	-	Small correction on v
δp	-	Small correction on p
b	-	Iteration
N_e	-	Number of domain element
τ_w	-	Wall shear stress
ΔP	-	Pressure drops
P_a	-	Pressure at the starting point of stenosis
P_b	-	Pressure at the end point of stenosis

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CHAPTER 1

INTRODUCTION

1.1 Research Background

Current worldwide increase in cardiovascular disease has triggered a clinical interest on the study of flow in blood. Cardiovascular diseases are the leading cause of death in Malaysia and globally. Based on statistics (World Health Organization, 2018) ischemic heart disease was the principal cause of death in 2016. Cardiovascular system is an organ system empowered by heart with the help of blood vessels that functions to distribute blood all over the body enabling the cell to receive the nutrients and nourishment from the blood cell itself. Consequently, any disorder that occur to these vital organs may lead to cardiovascular complications lead to fatalities (World Health Organisation, 2017).

Blood, blood vessels and heart are three vital components that made up human cardiovascular or circulatory system. Blood circulates through a network of vessels throughout the body by action of the pumping of the heart. The main function of blood is to carry and transport necessary substances such as oxygen, carbon dioxide, nutrient and waste product to the targeted area. While the structure of heart is a four chambers muscular organ which are two upper atria also acting as the receiving chamber and two lower ventricles function mainly as the discharging chamber since the walls are much thicker compared to atria. The right heart (right atrium and right ventricle) receives blood from the body and pump it to the lungs while left heart delivered the blood received from the lungs throughout the body (Health Encyclopedia, 2019).

In average human body contains about 4 to 5 liters of blood. Particularly, blood is a liquid connective tissue that consist of 4 basic components, which are red blood cells (RBCs) or erythrocytes, white blood cells (WBCs) or leucocytes, platelets and plasma membrane. Each component contributes in helping the blood to fulfil it roles.

The composition of blood and the characteristics of each components are shown in Figure 1.1.

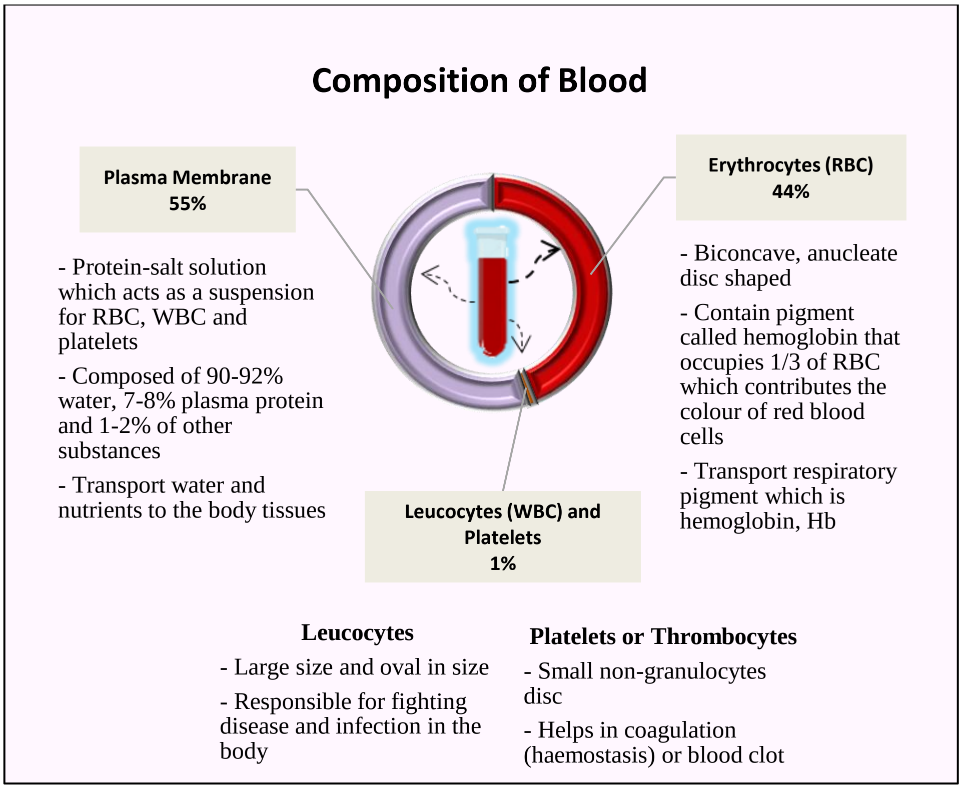


Figure 1.1 The composition of blood

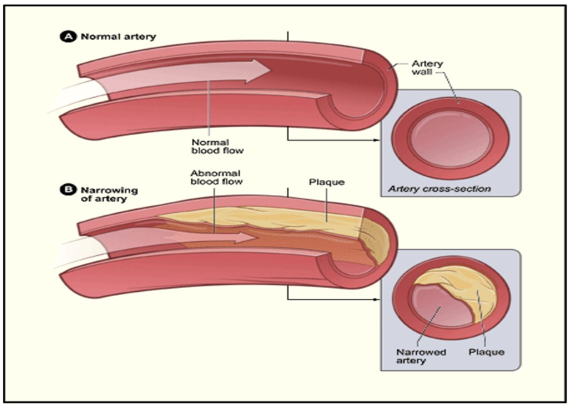
Around 20% of oxygen were carried by haemoglobin and transported to the body tissue and lungs respectively. The hemoglobin is a metalloprotein containing iron and makes up to 96% of dry weight of the red blood cells (Lakna Panawala, 2017). Plasma membrane transported these blood cells along with the nutrients, waste product, antibodies, clotting materials and chemical. Another function of blood is to protect the body from infection and developing body immune system with the help of leucocytes and plasma protein. Platelets in the other way aid the blood in the process of hemostasis to prevent any unnecessary blood loss during injury by forming blood clot and ultimately covers the wound (American Society of Hematology, 2019). The blood is transported through an elastic tube called blood vessels known as artery, veins and capillaries. They have different features and functions in accordance with their nearby structure as shown in Table 1.1.

Table 1.1 Characteristics of blood vessels

Types/ Characteristics	**Arteries**	**Capillaries**	**Veins**
Functions	Carry blood away from the heart	Carry blood very close to the cells of the tissues of the body	Carry blood return to the heart
Blood Types	Oxygenated blood except pulmonary artery	No specifications	Deoxygenated blood except pulmonary vein
Pressure	High blood pressure	No specifications	Low blood pressure
Wall of Blood Vessels	Thicker, elastic and muscular walls	Thin layer of endothelium	Thinner and less elastic walls
Figure			

Ischemic or coronary heart disease occurred due to the atherosclerosis which is the hardening of arterial wall forming a stenosis or plaque. From Table 1.1, it is shown that the wall of arteries is thicker than veins because they received blood at high pressure due to its location near to the heart. In addition to that, having lumen with small diameter helps maintain the pressure of blood moving through the system. Therefore, any abnormalities i.e. stenosis, that occur in the arterial wall could disturb the normal flow of blood, limiting the amount of transported oxygenated blood needed and eventually lead to heart attack or worse, death. Figure 1.2 (a) illustrate the differences between healthy (normal) artery and artery having plaque inside it.

(a)



(b)

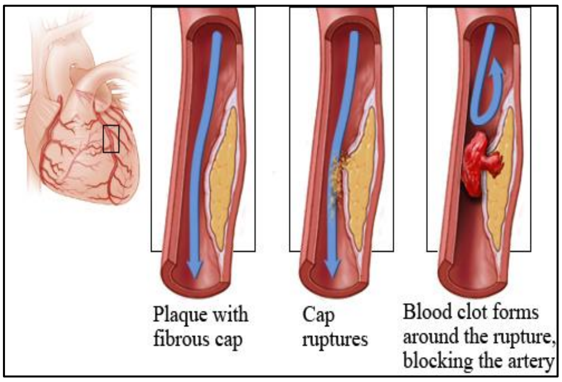


Figure 1.2 (a) Comparison of the normal artery and stenosed artery (Katz et al., 2016) and (b) Formation sequence of the ruptured blood clot (Health Encyclopedia, 2019).

Based on Young (1968), Biswas (2000), Biswas and Chakraborty (2009) and Sankar and Lee (2009), two major complications that causes by atherosclerosis are the atheromatous plaques and an aneurysm. The plaque formation cf., Figure 1.2 (a) is due to the accumulation of fatty materials such as cholesterol and other substances that exist in blood. Over time when the fibrous cap is at the thinnest, the plaque ruptured and thrombosis process will take place producing blood clot as shown in Figure 1.2 (b) (Jacob et al., 2018). The shrink or heal clot leave behind a stenosis (arterial hardening) causing constriction of the artery. As a result, the blood supply demands cannot be achieved and initiate many types of disease such as coronary artery disease, heart attack, kidney failure and stroke (Tegos et al., 2001).

Abnormality such as stenosis in blood vessels highly influenced the hemodynamic of flow (Srivastava, 1995). It is discovered that the flow in stenotic region is different than one in normal arteries where the increasing severity of stenosis also cause the change in flow field (Mustapha et al., 2010). Numerous authors have contributed to the study of blood flow problem in stenosed artery with various type of flow model. In the early stage of studies on stenosed artery, blood used to be modelled as Newtonian fluid. Then, Chakravarty et al. (1996) stated that blood behaves as Newtonian fluid in wider arteries while behaves as non-Newtonian in small arteries. Various studies in this area (Ismail et al., 2008; Varshney et al, 2010; Roy et al., 2018), could provide the researcher with more understanding on the behavior of blood flow in vessels.

For example, the arising of interest in the effect of fundamental body forces on the blood flow in arteries lead to the improvement of study model with the addition of gravitational force and magnetic field. Besides, it is commonly known that most of the cardiovascular disease treatment tools utilize the concept of magnetic field. This is due to the natural attribute of blood having a unique iron containing protein known as hemoglobin and it is able to orient in the presence of magnetic field. Some of the applications of magnetic field are magnetic devices for cell separation and magnetic drug targeting which use to deliver drugs to the targeted cells for tumor patient treatment.

1.2 Problem Statement

Recognizing the importance of a healthy lifestyle in human life, especially the cardiovascular systems has initiate massive studies in bio-medical area. In this thesis, fluid dynamic study coupled with electromagnetic force is studied in order to understand the haemodynamic of blood flow in a constricted channel with the presence of spatially varying magnetic field. Hence, a mathematical model that describe the flow is demonstrated based on the interaction between Navier-Stokes equations and magnetic field equations. Understanding the effect of magnetic force on blood flow characteristics helps to provide a better view on the roles of blood dynamical factor to the progression of arterial diseases.

It is known that constricted or stenosed artery may lead to a serious cardiovascular disease since the flow of blood is being disturbed from performing their designated task. This indicates that the geometry of the artery is a crucial factor that should be considered in studying blood flow characteristics. Since blood vessel consists of single and bifurcated vessels, both geometries are studied to observe the effects of magnetic force on the flow of blood with the existence of overlapping stenosis. To solve the proposed problem in this study, Galerkin weighted residual finite element method is applied. This is because finite element method (FEM) is known for its ability to solve the problem with complex geometry and provides a faster rate of convergence.

1.3 Objectives of Study

The main purpose of this study is to develop a mathematical model based on the biomagnetic blood flow in a channel by using Galerkin weighted residual finite element method. In addition, it is worth to note that this study considered the fluid to be isothermal in order for the mathematical model to be more simplified. Specifically, the objectives are to:

a) Develop the mathematical model of biomagnetic fluid dynamic of blood flow in a channel with an overlapping stenosis

b) Generate the Matlab source code for biomagnetic fluid flow in a straight and bifurcated channel discretized by Galerkin weighted residual finite element method.

c) Investigate the effects of biomagnetic fluid flow in the presence of an overlapping stenosis in a straight and bifurcate channel.

1.4 Scope of Study

The fluid flow models are based on the Newtonian Navier-Stokes equations where the blood flow is assumed to be to-dimensional, steady and laminar; a fluid that travels smoothly in regular paths. In addition to that, the fluid is flowing under the presence of magnetic field since this study focus on biomagnetic fluid flow in a channel artery. The flow is isothermal and the magnetic field involves are based on the force arising due to spatially varying magnetic field (magnetic gradient). The geometry of stenosis chosen here is an overlapping stenosis which presented in both straight and bifurcated channel arteries. Bifurcated artery is an artery that consists of a single mother and daughter arteries. In this study, the mathematical model is then discretized by using finite element Galerkin weighted residual method and the results are computed by using MATLAB software. The validation of the source code is made by comparing the benchmark problem for lid driven cavity case. Although there exists a limitation in the study due to the negligibility of heat transfer, stability term is added to cope with this issue. Finally, the characteristics of blood flow considered in this study includes the velocity profile, streamlines pattern and pressure drop.

1.5 Significance of Study

This main focused for this research is on the flow characteristics for biomagnetic fluid inside the straight and bifurcated channel with an overlapping stenosis. It is important to note that the presence of magnetic field did influence the fluid flow especially in the case of spatially varying magnetic field (Tzirtzilakis, 2005). Hence, further study that considered different types of channel with the presence of overlapping stenosis might provide a more details and wide evaluation on haemodynamics alterations of blood flow.

Another significance that can be considered here are the application of finite element formulation method to solve the biomagnetic fluid dynamics (BFD) problem. It is advantages to use finite element method due to its efficiency in handling complex geometry such as bifurcated channel geometry. Besides, BFD is the study on the interaction of biological fluid with electromagnetics force which results in a lot of benefits in real-life situation. Such examples are medical devices that make use of magnetic field such as magnetic drug targeting which proven useful in targeting and delivering the drugs to the targeting spot with the use of external magnetic field. This shows that exploring deeper in this research area could prove to be helpful in advancing biomedical technology area.

1.6 Thesis Outline

This thesis started with the introduction of the research. In this chapter, background of the research is briefly introduced followed by the problem statement, objectives, scopes and significance of the research.

The remainder of the thesis is organized in four chapters. Chapter 2 present the literature review which consists of the discussion on previous and current study related to this research. This chapter begin with the introduction of blood as biofluid and followed by the study of magnetic effect of blood which are ferrohydrodynamics and magnetohydrodynamics. Then, the finite element Galerkin method for solving the nonlinear problem is discussed with all the problems related to this method of solution.

The derivation involves in this study is detailed in Chapter 3. This includes the derivation of the governing equations and magnetic field equations. Subsequently, Chapter 4 introduced the modeling of the study using Galerkin weighted residual method formulation. The numerical flow of the solution starts with non-dimensional equations and discretization of the model is provided and the source code validation is also presented.

Chapter 5 focused on the results of the study. The effects of biomagnetic fluid dynamics (BFD) in two types of channel which are straight and bifurcated channel are presented. This chapter present the results on the effect of magnetic field in both channels. Finally, the conclusion of this study is summarized in the last chapter, Chapter 6. This follows by a section with some of recommendation for future works.

REFERENCES

American Society of Hematology (2019). Blood Basics. Retrieve from: https://www.hematology.org/Patients/Basics/

Abdullah, A., AI-Rashed, A., Alnaqi, A., & Hossain, M. A. (2016). Numerical Simulation of Biomagnetic Fluid Flow in an Oscillating Lid Driven Cavity. International Journal of Applied Engineering Research, 11(23), 11259-11265.

Anderson John, D. (1995). Computational fluid dynamics: the basics with applications. *International edition: McGraw-Hill*.

Aziz, M., & Yadav, K. S. (2016). Pathogenesis of atherosclerosis. *Med Clin Rev*, *2*(3).

Babuška, I., Andersson, B., Guo, B., Melenk, J. and Oh, H. (1996). Finite Element Method for Solving Problems with Singular Solutions. *Journal of Computational and Applied Mathematics. 74*(1-2), 51-70.

Bäck, M., & Hansson, G. K. (2017). Basic mechanisms of atherosclerosis. *Chronic Coronary Artery Disease: A Companion to Braunwald's Heart Disease E-Book*, 45.

Bathe, K. J. (1996). Finite Element Procedures. USA: Prentice-Hall, Inc.

Baptista, C. A., Didio, L. J., & Prates, J. C. (1991). Types of Division of the Left Coronary Artery and the Ramus Diagonalis of the Human Heart. *Jpn Heart J Japanese Heart Journal*, *32*(3), 323-335.

Biswas, D. (2000). Blood Flow Models: A Comparative Study. New Delhi: Mittal Publications.

Biswas, D. and Chakraborty, U. S. (2009). Pulsatile Flow of Blood in a Constricted Artery with Body Acceleration. Applied Mathematics, 4(2): 329–342.

Botnar, R., Rappitsch, G., Scheidegger, M. B., Liepsch, D., Perktold, K. and Boesiger, P. (2000). Hemodynamics in the carotid artery bifurcation: *Journal of Biomechanics. 33*(2), 137-144.

Caro, C. G., Fitz-Gerald, J. M., & Schroter, R. C. (1971). Atheroma and Arterial Wall Shear Observation, Correlation and Proposal of a Shear Dependent Mass Transfer Mechanism for Atherogenesis. *Proceedings of the Royal Society B: Biological Sciences, 177*(1046), 109-133.

Capron, L., & Wyplosz, B. (2002). Basic Mechanisms of Atherosclerosis: From Inflammation to Infection. In *Coronary Circulation and Myocardial Ischemia* (pp. 11-19). Springer, Berlin, Heidelberg.

Chakravarty, S., & Mandal, P. K. (1996). A nonlinear two-dimensional model of blood flow in an overlapping arterial stenosis subjected to body acceleration. *Mathematical and computer modelling*, *24*(1), 43-58.

Chakravarty, S., & Mandal, P. (1997). An analysis of pulsatile flow in a model aortic bifurcation. *International Journal of Engineering Science*, *35*(4), 409-422.

Cowling, T. G. (1957). Magnetohydrodynamics, Interscience Publ. *Inc., New York*.

Dumser, T., Borsch, M., & Wonhas, C. (2013). Coronary artery disease in aircrew fatalities: morphology, risk factors, and possible predictors. *Aviation, space, and environmental medicine*, *84*(2), 142-147.

Epstein, S. (2015). SpingerOpen blog. Magnetic drug targeting and nanoparticle size. Retrieve from: http://blogs.springeropen.com/springeropen/2015/11/12/magnetic-drug-targeting-nanoparticle-size/

Erturk, E., Corke, T. C., & Gökçöl, C. (2005). Numerical solutions of 2-D steady incompressible driven cavity flow at high Reynolds numbers. *International journal for Numerical Methods in fluids*, *48*(7), 747-774.

Erwan Hafizi Kasiman (2012). *Mixed formulations for Navier Stokes equations with magnetic effect in rectangular channel.* MEng thesis, Universiti Teknologi Malaysia, Skudai.

Falk, E. (2006). Pathogenesis of atherosclerosis. *Journal of the American College of Cardiology*, *47*(8 Supplement), C7-C12.

Fortin, M., & Brezzi, F. (1991). *Mixed and hybrid finite element methods*. New York: Springer-Verlag.

Furlani, E. P. (2010). Magnetic biotransport: analysis and applications. Materials, 3(4), 2412-2446.

Galkina, E., & Ley, K. (2009). Immune and inflammatory mechanisms of atherosclerosis. *Annual review of immunology*, *27*.

Ghia, U. K. N. G., Ghia, K. N., & Shin, C. T. (1982). High-Re solutions for incompressible flow using the Navier-Stokes equations and a multigrid method. *Journal of computational physics*, *48*(3), 387-411.

Gray, W. H., & Schnurr, N. M. (1975). A comparison of the finite element and finite difference methods for the analysis of steady two dimensional heat conduction problems. *Computer Methods in Applied Mechanics and Engineering*, *6*(2), 243-245.

Haik, Y., Chen, J. C., & Pai, V. M. (1996). Development of bio-magnetic fluid dynamics. In *Proceedings of the Ninth International Symposium on Transport Properties in Thermal Fluid Engineering* (pp. 25-28). HI, Jun.

Haik, Y., Pai, V., & Chen, C. J. (2001). Apparent viscosity of human blood in a high static magnetic field. *Journal of Magnetism and Magnetic Materials*, *225*(1-2), 180-186.

Health Encyclopedia (2019). How a Heart Attack Happens. Retreive from: https://healthy.kaiserpermanente.org/health-wellness/health-encyclopedia/he.zm2431.

Higashi, T., Yamagishi, A., Takeuchi, T., Kawaguchi, N., Sagawa, S., Onishi, S., & Date, M. (1993). Orientation of erythrocytes in a strong static magnetic field. *Blood*, *82*(4), 1328-1334.

Ismail, Z., Abdullah, I., Mustapha, N., & Amin, N. (2008). A power-law model of blood flow through a tapered overlapping stenosed artery. *Applied Mathematics and Computation*, *195*(2), 669-680.

Insull Jr, W. (2009). The pathology of atherosclerosis: plaque development and plaque responses to medical treatment. *The American journal of medicine*, *122*(1), S3-S14.

Jacob et al., (2018). Mechanisms of Plaque Formation and Rupture. Acute Coronary Syndromes Compendium, (DOI: 10.1161/CIRCRESAHA.114.302721), pp. 1852-1862.

Jamalabadi, M. Y. A., Daqiqshirazi, M., Nasiri, H., Safaei, M. R., & Nguyen, T. K. (2018). Modeling and analysis of biomagnetic blood Carreau fluid flow through a stenosis artery with magnetic heat transfer: A transient study. *PloS one*, *13*(2), e0192138.

Katz et al., (2016). Care Coronary Artery Disease (CAD). Retrive from: https://wildirismedicaleducation.com/courses/coronary-artery-disease-continuing-education.

Kumar, A. (2011). Performance and Analysis of Blood Flow through Carotid Artery. *Int. J. Eng. Bus. Manag. International Journal of Engineering Business Management.* 1.

Lakna Panawala (2017). What is the function of hemoglobin in the human body. *Research Gate*, pp. 1-7.

Libby, P. (2000). Changing Concepts of Atherogenesis. *J Intern Med Journal of Internal Medicine. 247*(3), 349-358.

Libby, P., & Theroux, P. (2005). Pathophysiology of coronary artery disease. *Circulation*, *111*(25), 3481-3488.

Lim Yeou Jiann (2017). *Finite element method for fluid flow of modelling of the descemet membrane detachment and rhematogeneous retinal detachment.* PhD thesis, Universiti Teknologi Malaysia, Skudai.

Liu, Y. L., Chen, D., Shang, P., & Yin, D. C. (2019). A review of magnet systems for targeted drug delivery. *Journal of Controlled Release*.

Loukopoulos, V. C., Bourantas, G. C., Labropoulos, D., & SPA, V. N. (2016). Numerical study of magnetic particles concentration in biofluid (blood) under the influence of high gradient magnetic field in microchannel. In *7th European Congress on Computational Methods in Applied Sciences and Engineering: ECCOMAS Congress 2016* (pp. 1084-1092). National Technical University of Athens.

Loukopoulos, V. C., & Tzirtzilakis, E. E. (2004). Biomagnetic channel flow in spatially varying magnetic field. *International Journal of Engineering Science*, *42*(5-6), 571-590.

Mandal, M. D., Manna, N., & Chakrabarti, D. S. (2010). Numerical Study Of Blood Flow Through Different Double Bell Shaped Stenosed Coronary Artery During

The Progression Of The Disease, Atherosclerosis. *International Journal of Numerical Methods for Heat & Fluid Flow, 20*(6).

Martin-Lorenzo, M., Alvarez-Llamas, G., McDonnell, L. A., & Vivanco, F. (2016). Molecular histology of arteries: mass spectrometry imaging as a novel ex vivo tool to investigate atherosclerosis. *Expert review of proteomics*, *13*(1), 69-81.

Mekheimer, K. S. (2008). Effect of the induced magnetic field on peristaltic flow of a couple stress fluid. *Physics Letters A*, *372*(23), 4271-4278.

Merriam-Webster on Bifurcate (2016, February 26). Retrieve from: http://www.merriam-webster.com/dictionary/bifurcate.

Milner, J. S., Moore, J. A., Rutt, B. K., & Steinman, D. A. (1998). Hemodynamics of human carotid artery bifurcations: Computational studies with models reconstructed from magnetic resonance imaging of normal subjects. *Journal of Vascular Surgery, 28*(1), 143-156.

Mota, R., Homeister, J. W., Willis, M. S., & Bahnson, E. M. (2001). Atherosclerosis: Pathogenesis, Genetics and Experimental Models. *eLS*, 1-10.

Muraca, E., Gragmina, V. and Fragomeni, G. (2009). Mathematical Model of Blood Flow in Carotid Bifurcation. *COMSOL Conference 2009*. Milan.

Mustapha, N., Mandal, P. K., Johnston, P. R., & Amin, N. (2010). A numerical simulation of unsteady blood flow through multi-irregular arterial stenoses. *Applied Mathematical Modelling*, *34*(6), 1559-1573.

Norliza Mohd Zain (2018). *Magnetohydrodynamics effects on Generalized Power law fluid model of blood flow in a stenosed bifurcated artery*. MPhil Thesis, Universiti Teknologi Malaysia, Skudai.

Pandey, K. M., Thakur, R., Hazarika, A., Ashutosh, T., & Gogoi, D. (2014). Non-Newtonian Computational Fluid Dynamics (CFD) Modelling on Left Coronary

Artery with Multiple Blockages. *AMM Applied Mechanics and Materials, 592-594*, 1924-1929.

Pathophysiology, Neuro4Students. (2010, March 13). Retrieve from: https://neuro4students.wordpress.com/pathophysiology/.

Pauling, L., & Coryell, C. D. (1936). The magnetic properties and structure of hemoglobin, oxyhemoglobin and carbonmonoxyhemoglobin. *Proceedings of the National Academy of Sciences*, *22*(4), 210-216.

Perktold, K. and Rappitsch, G. (1995). Computer Simulation of Local Blood Flow and Vessel Mechanics in a Compliant Carotid Artery Bifurcation Model. *Journal of Biomechanics. 28*(7), 845-856.

Perktold, K., Peter, R., Resch, M., & Langs, G. (1991). Pulsatile non-Newtonian blood flow in three-dimensional carotid bifurcation models: A numerical study of flow phenomena under different bifurcation angles. *Journal of Biomedical Engineering, 13*(6), 507-515.

Reddy, J.N. & Gartling, D.K. (2010). The finite element method in heat transfer and fluid dynamics. CRC press.

Reddy, K., Reddy, M., & Reddy, R. (2011). Mathematical model governing magnetic field effect on bio magnetic fluid flow and orientation of red blood cells. *Pac.-Asian J. Math*, *5*, 344-356.

Reddy, K. S., & Yusuf, S. (1998). Emerging epidemic of cardiovascular disease in developing countries. *Circulation*, *97*(6), 596-601.

Rosensweig, R. E. (1985). Ferrohydrodynamics Cambridge University Press Cambridge. *New York*, *Melbourne*.

Roy, M., Sikarwar, B. S., Bhandwal, M., & Ranjan, P. (2017). Modelling of blood flow in stenosed arteries. *Procedia computer science*, *115*, 821-830.

Samuels, O. B., Joseph, G. J., Lynn, M. J., Smith, H. A., and Chimowitz, M. I. (2000). A Standardized Method for Measuring Intracranial Arterial Stenosis. ANJR Am J Neuroradiol. 21: 643-646.

Sankar, D. S. and Lee, U. (2009). Mathematical Modeling of Pulsatile Flow of NonNewtonian Fluid in Stenosed Arteries. Communications in Nonlinear Science and Numerical Simulation, 14(7): 2971–2981.

Seo, T. (2013). Numerical simulations of blood flow in arterial bifurcation models. *Korea-Australia Rheology Journal Korea-Aust. Rheol. J., 25*(3), 153-161.

Shaw, S. (2018). Mathematical Model on Magnetic Drug Targeting in Microvessel. *Magnetism and Magnetic Materials*, 83.

Srivastava, V. P. (1995). Arterial blood flow through a nonsymmetrical stenosis with applications. *Japanese Journal of Applied Physics*, *34*(12R), 6539.

Sud, V. K., & Sekhon, G. S. (1989). Blood flow through the human arterial system in the presence of a steady magnetic field. *Physics in Medicine & Biology*, *34*(7), 795.

Sud, V. K., Sekhon, G. S., & Mishra, R. K. (1977). Pumping action on blood by a magnetic field. *Bulletin of mathematical biology*, *39*(3), 385-390.

Tan Phey Hoon (2019). T*wo dimensional finite element formulation for crude oil nanofluid flow subjected to magnetic field*. MSc thesis, Universiti Teknologi Petronas, Perak.

Tegos, T. J., Kalodiki, E., Sabetai, M. M., & Nicolaides, A. N. (2001). The genesis of atherosclerosis and risk factors: a review. *Angiology*, *52*(2), 89-98.

Türk, Ö., Tezer-Sezgin, M., & Bozkaya, C. (2014). Finite element study of biomagnetic fluid flow in a symmetrically stenosed channel. *Journal of Computational and Applied Mathematics*, *259*, 760-770.

Tzirakis, K., Papaharilaou, Y., Giordano, D., & Ekaterinaris, J. (2014). 'Numerical investigation of biomagnetic fluids in circular ducts.'*International journal for numerical methods in biomedical engineering*, *30*(3), 297-317.

Tzirtzilakis, E. E. (2005). A mathematical model for blood flow in magnetic field, Physics of Fluids, 17(077103), 1-15.

Tzirtzilakis, E. E., & Xenos, M. A. (2013). Biomagnetic fluid flow in a driven cavity. *Meccanica*, *48*(1), 187-200.

Urone, P. P., & Hinrichs, R. (2018). College Physics: OpenStax.

Van de Vosse, F. N., Van Steenhoven, A. A., Janssen, J. D., & Reneman, R. S. (1990). A two-dimensional numerical analysis of unsteady flow in the carotid artery bifurcation. *Biorheology*, *27*(2), 163-189.

Varshney, G., Katiyar, V., & Kumar, S. (2010). Effect of magnetic field on the blood flow in artery having multiple stenosis: a numerical study. *International Journal of Engineering, Science and Technology*, *2*(2), 967-82.

World Health Organisation (2017). Cardiovascular disease (CVD). https://www.who.int/news-room/fact-sheets/detail/cardiovascular-diseases-(cvds).

World Health Organisation (2018). The top 10 causes of death. Retrieve from: https://www.who.int/news-room/fact-sheets/detail/the-top-10-causes-of-death

Yamada, Y., Yoshimura, N. and Sakurai, T. (1968). Plastic Stress-Strain Matrix and Its Application for the Solution of Elastic-Plastic Problems by the Finite

Element Method. *International Journal of Mechanical Sciences. 10*(5), 343-354.

Young, D. F. (1968). Effects of a Time-Dependent Stenosis of Flow through a Tube. Journal of Engineering and Industrial, 90(2): 248–254.

Zborowski, M., & Chalmers, J. J. (2011). *Magnetic cell separation* (Vol. 32). Elsevier.