# Focal Region Ray Tracing for Slanted Incident Rays on Spherical Reflector Antenna 

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#### Abstract

Multi beam antenna system is required at base station for fifth-generation (5G) mobile communication and also to cover large portions of field of view for the satellite antenna. A spherical reflector antenna is promising to give high gain directivity of each independent beams for its multiple beam capabilities. The study of the feed elements system is carried out by analysing the focus of slanted incident rays in receiving mode from $0^{\circ}$ to $30^{\circ}$ at step angle $10^{\circ}$. Simple analytical equations are derived to find the incident and reflected rays. This focal region of spherical reflector is studied in MATLAB program. Accuracy of the MATLAB results are validated by examining the focal region using an electromagnetic simulator, FEKO. The locus of feed of spherical reflector antenna for certain angles of the incident rays are presented in this study.


## 1. Introduction

The proposed millimeter waves in 5G have high bandwidth potential that can transmit large amount of data in shorter time. However, higher frequencies transmission has less penetrating capability and small coverage distance [1]. Thus, beamforming capability is required to support massive multipleinput multiple-output (MIMO) application in small cell radius at approximately 200 cm [2]. Multiple beams antenna can serve a large number of users simultaneously and support multicast transmissions in broadcasting services [3]. Hence, providing spherical reflector antenna as promising candidate with high directional beam and wide-angle scanning performances due to its symmetrical surface [4].

The primary issue with spherical reflector is the spherical aberration, particularly in large aperture reflector. Based on the previous configuration by Ishimaru, et. al in [5], double spherical reflectors antenna that has the same center point for main and sub-spherical had shown that spherical aberration can reduce antenna gain. A continuation of the research is carried out in [6] to clarify the caustic positions of spherical reflector. Thereby, analysis on focal region ray tracing is useful to study the aberration effect and determination of feed position is important for multi beam radiation pattern.

In this paper, focal region ray tracing of spherical reflector for slanted incident rays at certain angles, from $10^{\circ}$ to $30^{\circ}$ is studied and illustrated by MATLAB program. The accuracy of MATLAB results are validated by using an electromagnetic simulator, FEKO. The focal region ray tracing is developed in FEKO by employing Ray Launching - Geometrical Optics (RL-GO) solver for the incident and reflected rays in receiving mode condition. The off axis focal region is illustrated in MATLAB and the intersection coordinates of the desired reflected rays are found to determine the adequate feed position.

## 2. General Geometry of Spherical Reflector

The geometry of spherical reflector is rotationally symmetrical around the axis of symmetry, $y$-axis as shown in figure 1 . While $z$-axis corresponds to the radial length of the spherical reflector. The incident rays are excited in receiving mode and measured from a longitudinal distance at spherical center, $C_{M}$. The ray path of slanted incident rays (black) at given angle, $\Delta \varphi$ is illustrated with respect to its horizontal incident condition (blue) with total reflection, $\varphi$ at given radial length. The parameters of the spherical reflector are shown in table 1.

Table 1. Spherical Reflector Antenna Configuration.

| Antenna Parameter | Dimension |
| :---: | :---: |
| $C_{M}, r_{M}$ | 200 mm |
| $\varphi_{o}$ | $80^{\circ}$ |
| $\Delta \varphi$ | $0^{\circ}, 10^{\circ}, 20^{\circ}, 30^{\circ}$ |



Figure 1. Two-dimensional (2D) geometry of spherical reflector and ray path of parallel incident rays.

## 3. Focal Region Ray Tracing in MATLAB

In order to find the focal region of the reflected rays, simple equations for slanted incident ray at given angle, $\Delta \varphi$ subjected to its horizontal incident ray, is derived below. Two conditions of horizontal incident ray at $\varphi_{o}$ and $\varphi_{1}$ are shown as the examples and its caustic point is indicated by $\bigcirc$ mark. The cartesian coordinate for slanted incident ray $\left(y_{M}, z_{M}\right)$ is expressed in the equations (1) to (3) below.

$$
\begin{equation*}
y_{M}=C_{M} \tag{1}
\end{equation*}
$$

$\varphi$ is presented by the total reflection angle of its horizontal condition such as $\varphi_{o}$ or $\varphi_{1}$.

$$
\begin{gather*}
z_{M}=R_{M}-\Delta z  \tag{2}\\
z_{M}=\left(r_{M} \sin \left(\frac{\varphi}{2}\right)\right)-\left(r_{M} \cos \left(\frac{\varphi}{2}\right) \tan (\Delta \varphi)\right) \tag{3}
\end{gather*}
$$

Referring to the law of reflection as the fundamental of ray tracing concept, the tangent line is erected to a normal point where the reflection line is drawn; sphere radius $r_{M}$ from centre $C_{M}$. The incident ray is reflected from the spherical reflector at its radial position $R_{M}$, either at $R_{M O}$ or $R_{M 1}$.

The direction along which these incident rays will be reflected are then identified in MATLAB by deriving equations of lines, $y=m x+c$. The MATLAB results of equation (1) and (3) are shown in the following figure 2 .


Figure 2. Focal region ray tracing for $\Delta \varphi$ equal to (a) $0^{\circ}$, (b) $10^{\circ}$, (c) $20^{\circ}$, and (d) $30^{\circ}$ respectively.
From figure 2 , the caustic curve for different angle of $\Delta \varphi$ is different. With the increasing angle of $\Delta \varphi$, the reflected rays unveil a larger curvature of caustic curve on the lower part (negative z-axis) of the spherical reflector. The reflected rays' concentration becomes asymmetric around the y -axis if $\Delta \varphi$ is more than $0^{\circ}$. Hence, at bigger angle of $\Delta \varphi$, the focal region becomes smaller by considering only few
rays that are reflected towards the same direction of focusing area. Therefore, only fewer coordinate of feeds to select from smaller focal region. The thick red line indicates the contribution curvature of the spherical reflector focal region. Two possible coordinate of feeds in figure 2(d) are closer with each other compared to $2(\mathrm{a}), 2(\mathrm{~b})$ and $2(\mathrm{c})$ for $\Delta \varphi$ equal to $0^{\circ}, 10^{\circ}$ and $20^{\circ}$ respectively that have larger focal region. These are obtained from two intersection points of reflected rays at 0 mark in the focal region by using polyxpoly syntax.

## 4. Focal Region Settings in FEKO

In FEKO, focal region ray tracing of spherical reflector is conducted by using RL-GO ray launching settings. Ray contributions are examined for direct and reflected conditions. The spherical reflector is modelled by importing coordinates from MATLAB structure through polyline method. Next, a free space region medium for the spherical reflector with length equal to 200 mm is created as the boundary of the ray tracing as shown in figure 3. These geometries are set to solve with RL-GO solution under properties option. A plane wave source is added into the configuration by adjusting the setting for loop over multiple directions $(\theta)$ at given $\Delta \varphi$ as shown in figure 4.


Figure 3. 3D geometry of spherical reflector


Figure 4. Plane wave source setting for focal region ray trace at $\theta$ either $100^{\circ}, 110^{\circ}$, or $120^{\circ}$ for $\Delta \varphi$ equal to $10^{\circ}, 20^{\circ}$, or $30^{\circ}$ respectively.

The plane wave excitation is significant to produce the incident rays and then will be reflected inwards by the spherical reflector surface. A focal region can be seen from the collection of intersection reflected rays in the air region. Plane wave direction from few angles, $\Delta \varphi$, are shown in figure $5(\mathrm{a})$, 5(b) and 5(c).


Figure 5. Shifted plane wave at $\Delta \varphi$ equal to (a) $10^{\circ}$, (b) $20^{\circ}$, and (c) $30^{\circ}$ respectively.

Figure 5 clearly shows the upward movement of the focal region from $10^{\circ}$ to $30^{\circ}$. Both MATLAB and FEKO ray tracing results are compatible and reflected rays are intersected at the same locations. Measure distance tool in POSTFEKO is used to confirm the distance of intersection points in MATLAB to choose the potential feed positions.

## 5. Locus of feed

From focal region analysis, various possibilities of feed position can be determined for multiple beam operations. Two approximate feed positions from the focal region are determined for the multi beam mechanism as shown in table 2 and figure 6 .

Table 2. Coordinate of Feed Positions

| $\begin{gathered} \Delta \varphi \\ \left({ }^{\circ}\right) \end{gathered}$ | Feed position (1) |  | Feed position (2) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \mathrm{y}- \\ \text { coordinate } \\ (\mathrm{mm}) \end{gathered}$ | Zcoordinate (mm) | $\begin{gathered} \mathrm{y}- \\ \text { coordinate } \\ (\mathrm{mm}) \end{gathered}$ | Zcoordinate (mm) |
| 0 | 69.45 | 0.00 | 99.99 | 0.00 |
| 10 | 86.28 | 20.05 | 101.51 | 17.36 |
| 20 | 100.00 | 36.39 | 106.02 | 34.20 |
| 30 | 112.06 | 50.77 | 113.39 | 50.00 |



Figure 6. Locus of feed positions with spherical reflector

## 6. Conclusion

Focal region ray tracing at slanted incident angle is derived and solved by MATLAB program which later is proven by FEKO simulation. However, there are many possibilities of feed position at lower shifted angle of plane wave due to larger focal region. Since the spherical aberration is moving, the feed determination is only applicable for the upper part of the spherical reflector, which then an offset spherical reflector is considered for good multi beam performance. Future work is needed to ensure good multibeam radiation patterns from transmitting mode of the feeder antenna on offset spherical reflector.

## References

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