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Quadratic Convective Nanofluid Flow at a Three-Dimensional Stagnation Point with the g-Jitter Effect

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ABSTRACT

The nonlinear density variation with temperature happens in many thermal applications like solar collectors, energy production, heat exchangers, and combustions, and it gives a significant impact on heat transfer and fluid flows. Thus, a nonlinear convective of an unsteady stagnation point flow under the influence of gravity modulation in the presence of water-based nanoparticles alumina (Al_2O_3) is studied here. Suitable variables are utilized to reduce the highly coupled nonlinear governing equations into a system of dimensionless simple partial differential equations. The Keller-box method is then applied to solve the consequent governing equations. Velocity and temperature profiles for various values of pertinent parameters are displayed graphically and discussed. The results indicate that the quadratic convection has enhanced the fluid flow and heat transport. Furthermore, the nonlinear convection parameter and the nanoparticles volume fractions have delivered a positive effect on the skin friction and the rate of heat transfer.

1. Introduction

The efficiency of heat exchangers, solar collectors, nuclear reactors, the cooling system of massive machines, oil extraction, and turbine blades significantly depends on the rate of heat transfer and the fluid flow [1-3]. Furthermore, in certain industrial processes such as food processing, polymer processing, wire/fiber coating, and biomedicine, these factors affect the quality of the final product [4,5]. Linear Boussinesq approximation is normally applied to describe the fluid motion in those processes but substantial errors are always detected. This is because in such thermal systems the temperature gradient is remarkably high, thus the fluid field and the heat transport are dominantly influenced by the nonlinear density variation with temperature in terms of buoyancy force. To be more realistic, Goren [6] had proposed a quadratic relation for the density variation with temperature as $\Delta\rho = -\rho\beta\left(T - T_w\right)^2$. Then, Vajravelu and Sastri [7] recommended expressing the nonlinear behavior by using the Taylor series expansion up to the second order, and it is given as

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$$\rho(T)=\rho(T)\big|_{T=T_w}+(T-T_w)\frac{\partial\rho}{\partial T}\bigg|_{T=T_w}+\frac{1}{2!}(T-T_w)^2\frac{\partial^2\rho}{\partial T^2}\bigg|_{T=T_w}+\mathrm{O}\left(T_w^3\right),$$

$$-\frac{\Delta\rho}{\rho}=\rho(T)-\rho(T)\big|_{T=T_w}=\beta_0(T-T_w)+\beta_1(T-T_w)^2.$$

where $\beta_0=\frac{\partial\rho}{\partial T}\Big|_{T=T_w}$ and $\beta_1=\frac{1}{2!}\frac{\partial^2\rho}{\partial T^2}\Big|_{T=T_w}$.

RamReddy *et al.,* [4] had investigated the nonlinear mixed convection micropolar fluid flow over an inclined plate that was embedded in a non-Darcy porous medium. The influence of the convective boundary condition on the fluid field and heat and mass transfer was considered. The authors detected that the thickness of the temperature and concentration boundary layer was decreased respectively with an increment of the nonlinear thermal and solute parameters. Furthermore, the nonlinear either temperature or concentration parameter has enhanced the skin friction. Then, RamReddy and Naveen [5] extended the work by taking into consideration of the thermal radiation and activation energy effect. They concluded that the physical quantities have been prominently altered by the interested parameters during the quadratic convection flow as compared with the linear convection. The effect of the density variation with temperature and the Newton boundary conditions on a micropolar fluid flow driven by a moving vertical flat plate with the magnetic dipole and thermal radiation has been acknowledged by Mahanty *et al.,* [8]. The finding revealed the velocity profile has an increasing function with the nonlinear convection parameter but an opposite tendency was noticed for the microrotational velocity profile.

On the other hand, many efforts have been done in improving the thermal properties of the heat transfer fluid since it is a substantial part of many engineering and industrial applications [9,10]. In 1995, Choi and Eastman [11] introduced nanofluid which was prepared by dispersing nanoparticles into the base fluid to enhance the thermal behavior of the conventional heat transfer fluid. Then, Choi *et al.,* [12] established an experiment regarding the heat characteristic of the nanofluid. The authors observed that the rate of heat transfer and the thermal conductivity of the base fluid is significantly increased when a tiny number of nanoparticles were suspended into the liquid. In view of this, Kumar *et al.,* [10] studied the nonlinear convection flow of a Jeffrey nanofluid flow through a vertical plate with double diffusion and Cattaneo-Christov heat flux. The rate of heat and mass transfer as well as the skin friction were noticed to be improved by the nonlinear convection parameters.

Ahmad *et al.,* [13] had compared the prescribed surface temperature and heat flux conditions for a bi-directional unsteady nanofluid flow over an extending flat surface with the Cattaneo-Christov heat flux. The impact of the Brownian motion and thermophoresis of the nanoparticles were concerned. The author observed that the immigrant of the nanoparticles and thermophoresis constraint has enhanced temperature profiles. Then, the impact of the temperature-dependent viscosity on doubly stratified MHD nanofluid flow using the Cattaneo–Christov heat flux model was deliberated by Ali *et al.,* [14]. The study observed that the viscosity parameter has reduced the nanofluid velocity profile but increased the temperature distribution. Newly, Khan *et al.,* [15] had discussed the application of the nanofluid in the bio-fuel industry. The homotopic technique was applied to theoretically study the bioconvection of couple stress nanofluid and gyrotactic microorganisms over a periodically accelerated surface. Furthermore, Aldabesh *et al.,* [16] investigated the characteristic of a Casson fluid that was driven by two parallel stretchable disks in the existence of a magnetic field, nanoparticles, thermal radiation, and variable thermal conductivity. More interesting research on the nanofluid can be found in the studies by Rusdi *et al.,* [17], Mohammed *et al.,* [18], Nayak *et al.,* [19], and Ahmad *et al.,* [20,21]. However, in the above-mentioned research about the nanofluid, the influence of the quadratic convection is not concerned.

Recently, Mahanthesh and Mackolil [1] have applied the quadratic Boussinesq approximation to study the heat transfer of a nanofluid past a vertical plate with quadratic thermal radiation. The numerical solutions illustrated the nanofluid velocity near the surface of the plate increases with the increment of the quadratic convection parameter but the trend reverses at the region far away from the surface. Besides, Mahanthesh *et al.,* [22] studied the nonlinear convective Sakiadis flow of a dusty fluid with the presence of nanoparticles alumina toward a vertical flat plate. Moreover, the effect of the quadratic thermal radiation was also concerned. The solutions depicted the distribution of the velocity and temperature is strongly affected by the quadratic convection. The heat transport was also detected to be enhanced by the volume fraction of the nanoparticles. The impact of the quadratic thermal radiation and nonlinear convection on a hybrid nano liquid in an annulus has been discussed by Thriveni and Mahanthesh [9]. They observed that the velocity of the fluid and the skin friction coefficient at the wall were enhanced by the quadratic convection parameter. Then, Thriveni and Mahanthesh [2] extended the previous work done by considering the suction/injection, nonuniform heat source, and radial magnetic field. Sensitivity analysis was conducted by the authors and detected that the skin friction is markedly affected by the quadratic convection parameter. Thriveni and Mahanthesh [3] extended the work again to take into consideration the impact of temperature-dependent thermal conductivity and viscosity in a micro-size annulus. Similar outputs were observed in the micro-annulus, the nonlinear convection parameter has a significant effect on the velocity profile as well as the skin friction.

In some manufacturing processes such as food processing, paper production, glass fiber production, and polymer extrusion, there occurs a large local pressure in the system which has induced the stagnation point flow [23]. Thus, the study of the stagnation point flow is essential in the applications for a high quality of the end product. Malvandi *et al.,* [23] have deliberated the effect of Navier's slip velocity on the stagnation point nanofluid flow over a stretching horizontal plate. The heat transport and Cu-water nanofluid flow near a stagnation point on a horizontal stretching/shrinking cylindrical has been acknowledged by Sulochana and Sandeep [24]. Furthermore, Rehman *et al.,* [25] had discussed the heat and mass transport of a MHD water-based nanofluid along an exponentially stretched surface near a three-dimensional stagnation point. Bachok *et al.,* [26] observed that the velocity profile of the fluid was enhanced by the stagnation point but the temperature curve was reduced. In the study, the authors have considered three kinds of nanofluid and their characteristics when flow near a three-dimensional stagnation point. A more interesting investigation on the stagnation point nanofluid flow is given in the studies by Uddin *et al.,* [27], Ghasemian *et al.,* [28], Mustafa *et al.,* [29], Hayat *et al.,* [30], and Soomro *et al.,* [31].

In a microgravity environment, an effect that is known as g-jitter is detected. This effect is driven by the orbiter maneuvers, crew activities, vibrations of mechanical, and the drag of the solar [32,33]. Uddin *et al.,* [34] had observed that the heat transfer in a nanofluid flow over a moving sheet under the influence of convective heat flux was enhanced by the gravity modulation. Then, Rawi *et al.,* [35] used the Keller-Box method to investigate a g-jitter driven mixed convection second-grade nanofluid flow over an inclined stretching sheet with considering the different shapes of the nanoparticle. They observed that the skin friction and heat transfer rate are increased by the frequency of oscillation and decreased by the amplitude of modulation for all types of nanoparticles. Ali *et al.,* [33] have discussed the impact of the gravity modulation on a mixed convection micropolar fluid flow over an incline vertical stretching sheet with the presence of the magnetic field. The author found that the disturbance of the gravity modulation has enhanced the oscillatory skin friction with greater amplitude parameters.

Some researchers have studied the g-jitter impact on a nanofluid flow at a stagnation point. The impact of the gravity modulation near a forward stagnation point flow of a two-dimensional symmetry body had been pioneeringly studied by Rees and Pop [36]. Then, Shafie *et al.,* [37] extended the work by considering a three-dimensional stagnation-point region. The author found that the g-jitter induced flow was markedly affected by the curvature of the stagnation point. However, in these two pioneers' works, the nanofluid is not concerned. Recently, Kamal *et al.,* [38] revisited the problem by investigating the g-jitter effect on a three-dimensional stagnation point nanofluid flow with heat. The authors noticed that the g-jitter has a significant influence on heat and mass transfer. On the other hand, the finding revealed that the fluid velocity was dominantly affected by the stagnation point. Further, Kamal *et al.,* [32] extended the research by taking into account the impact of thermal radiation. More analysis on the nanofluid flow near a stagnation point under the effect of gravity modulation can be referred to Kasim *et al.,* [39], and Kamal *et al.,* [40].

With the entire above comprehensive discussion, we notice that the significance of the quadratic convection on the heat transfer and nanofluid flow near a three-dimensional stagnation point has not been considered yet in the literature. Motivated by Kamal *et al.,* [32] and Kamal *et al.,* [38], the novelty of the present study is to investigate the impact of the quadratic Boussinesq approximation on the stagnation point nanofluid flow under the effect of gravity modulation. The model proposed by Vajravelu and Sastri [7] is utilized to present the nature of the quadratic convection flow. Suitable variables are then applied to convert the highly nonlinear governing equations which are depending on four independent variables into more simple nonlinear dimensionless differential equations that only have two independent variables. The Keller-box method is then used to solve the fluid system along with the concerned non-slip boundary conditions. The effect of the relevant parameters such as the nonlinear thermal parameter and the curvature of the stagnation point on the velocity and temperature profiles as well as the skin friction and the heat transfer rate are graphically displayed and analyzed.

2. Mathematical Formulation

An unsteady incompressible viscous three-dimensional stagnation point nanofluid flow driven by fluctuation gravity is considered. The $z-$axis is directed normally to the body surface as depicted in Figure 1, whereas the *x*-axis and *y*-axis are defined along the surface of the body. Assumes the origin (see Figure 1) has a zero local velocity and it is known as the stagnation point N. Following Buongiorno [41], the nanofluid has been treated as two-component mixtures (base fluid and nanoparticles) with the following assumptions:

i. incompressible flow
ii. no-chemical reaction
iii. viscous dissipation and radiative heat transfer are negligible
iv. base fluid and nanoparticles are in thermal equilibrium so that no slip occurs between them.

The flow of a water-based nanofluid with the presence of alumina (Al_2O_3) nanoparticles is taken into account. The fluid flow is modeled by utilizing the non-slip boundary condition. The constant temperature at the surface body is T_w and T_∞ attains the temperature of the ambient fluid. The temperature difference $(T_\infty - T_w)$ is assumed to be sufficiently large, thus the variation of the density with the temperature is in a quadratic function. By using the quadratic Boussinesq approximation as proposed by Vajravelu and Sastri [7], the governing equations that govern the present flow and heat transfer for a three-dimensional stagnation point nanofluid in the presence of the g-jitter effect can be expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\mu_{nf}\frac{\partial^2 u}{\partial z^2}+g(t)\rho_{nf}ax\left[\beta_0\left(T-T_\infty\right)+\beta_1\left(T-T_\infty\right)^2\right], \tag{2}$$

$$\rho_{nf}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\mu_{nf}\frac{\partial^2 v}{\partial z^2}+g(t)\rho_{nf}by\left[\beta_0\left(T-T_\infty\right)+\beta_1\left(T-T_\infty\right)^2\right], \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_{nf}\frac{\partial^2 T}{\partial z^2}, \tag{4}$$

subject to [38,40]

$$\begin{aligned}
&t<0: u=v=w=0, T=T_\infty \text{ for any } x, y \text{ and } z,\\
&t\geq 0: u=v=w=0, T=T_w \text{ on } z=0, x\geq 0, y\geq 0,\\
&\qquad u=v=w=0, T=T_\infty \text{ as } z\to\infty, x\geq 0, y\geq 0.
\end{aligned} \tag{5}$$

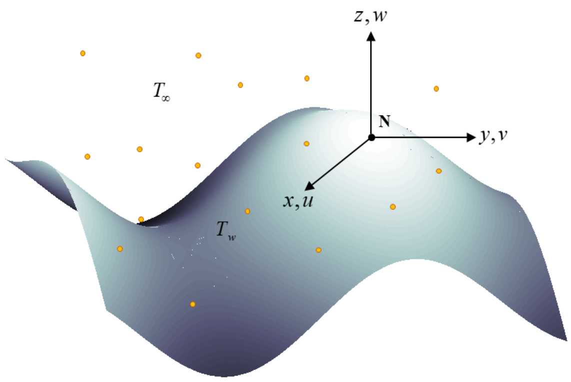


Fig. 1. Physical model representation in the Cartesian coordinate system

The gravity modulation is measured dependent on time and is denoted as [38,40]

$$g(t)=g_0\left[1+\varepsilon\cos\left(\pi\omega t\right)\right] \tag{6}$$

where g_0 is the mean of the gravitational acceleration, the amplitude of the gravity modulation is denoted by ε and ω is the frequency of oscillation, and t is the dimensional time, u,v and w are presenting the velocity component along the x,y,z direction respectively and T represents the dimensional temperature. Then, parameters a and b are applied to characterize the stagnation point with the relation $|a|\geq|b|$ and $a>0$. The suffix $(*)_{nf}$ indicates the thermophysical properties of the nanofluid. The notation ρ,μ,β_0,β_1 and α define the density, the dynamic viscosity, the linear volumetric thermal expansion coefficient, the nonlinear volumetric thermal expansion coefficient, and the thermal diffusion of the nanofluid. The expression of thermophysical characteristic of nanofluid has been defined in terms of the base fluid and nanoparticles properties, given as follows [40],

$$\mu_{nf} = \frac{\mu_f}{\left[1-\left(\phi_{Al_2O_3}\right)\right]^{2.5}},\ \alpha_{nf} = \frac{k_{nf}}{\left(\rho c_p\right)_{nf}},\ \rho_{nf} = \left(1-\phi_{Al_2O_3}\right)\rho_f + \phi_{Al_2O_3}\rho_{Al_2O_3},$$
$$\left(\rho\beta\right)_{nf} = \left(1-\phi_{Al_2O_3}\right)\left(\rho\beta\right)_f + \phi_{Al_2O_3}\left(\rho\beta\right)_{Al_2O_3},$$
$$\left(\rho c_p\right)_{nf} = \left(1-\phi_{Al_2O_3}\right)\left(\rho c_p\right)_f + \phi_{Al_2O_3}\left(\rho c_p\right)_{Al_2O_3}, \tag{7}$$
$$\frac{k_{nf}}{k_f} = \frac{k_{Al_2O_3} + 2k_f + 2\left(\phi_{Al_2O_3}k_{Al_2O_3}\right) - 2k_f\phi_{Al_2O_3}}{k_{Al_2O_3} + 2k_f - \left(\phi_{Al_2O_3}k_{Al_2O_3}\right) + k_f\phi_{Al_2O_3}},$$

where, ϕ is the nanoparticle volume fraction. k and c_p are respectively denoted the thermal conductivity and the specific heat capacity. The alumina and fluid components in the nanofluid are defined by the subscript $(*)_{Al_2O_3}$ and $(*)_f$ respectively. Table 1 gives the thermophysical properties.

Table 1
Thermophysical properties

Physical Properties	Al_2O_3	Water
Density ρ, $\left(\mathrm{kg\cdot m^{-3}}\right)$	3970	997.1
Specific heat capacity c_p, $\left(\mathrm{J\cdot kg^{-1}\cdot K^{-1}}\right)$	765	4179
Thermal conductivity k, $\left(\mathrm{W\cdot m^{-1}\cdot K^{-1}}\right)$	40	0.613
Thermal expansion β, $\left(\mathrm{K^{-1}}\right)$	0.85	21

Eq. (1) to Eq. (5) are then transformed into a simple and dimensionless system by using the semi-similar transformation technique. As defined in the studies by Shafie *et al.,* [37] and Kamal *et al.,* [40], the following semi-similar variables are utilized.

$$\eta = Gr^{1/4}az,\quad \tau = \Omega t,\quad t = \upsilon a^2 Gr^{1/2}t^*,$$
$$u = \upsilon a^2 x Gr^{1/2} f',\quad v = \upsilon a^2 y Gr^{1/2}h',\quad w = -\upsilon a Gr^{1/4}\left(f+h\right), \tag{8}$$
$$\Omega = \frac{\omega}{\upsilon a^2 Gr^{1/2}},\quad \theta = \frac{\left(T-T_\infty\right)}{\left(T_w-T_\infty\right)},\quad Gr = \frac{g_0\beta_0\left(T_w-T_\infty\right)}{a^3\upsilon^2}, \lambda = \frac{\beta_1\left(T_w-T_\infty\right)}{\beta_0},$$

where the differentiation with respect to η is denoted by the prime at the top of the function f and. θ is the dimensionless variables for temperature. Gr is the thermal Grashof number, Ω is the frequency of oscillation, υ is the fluid kinematic viscosity, λ is the nonlinear thermal parameter. The transformed dimensionless governing equations that depending on two independent variables are

$$C_2\Omega\frac{\partial f'}{\partial\tau} - C_1 f''' - C_2(f+h)f'' + C_2 f'^2 - C_3\left[1+\varepsilon\cos\left(\pi\tau\right)\right]\left(\theta+\lambda\theta^2\right) = 0, \tag{9}$$

$$C_2\Omega\frac{\partial h'}{\partial\tau} - C_1 h''' - C_2(f+h)h'' + C_2 h'^2 - cC_3\left[1+\varepsilon\cos\left(\pi\tau\right)\right]\left(\theta+\lambda\theta^2\right) = 0, \tag{10}$$

$$\Omega\frac{\partial\theta}{\partial\tau}-\frac{C_4}{C_5\,\mathrm{Pr}}\theta''-\left(f+h\right)\theta'=0, \tag{11}$$

where Pr is the Prandtl number and

$$\begin{aligned} &c=b/a,\ C_1=\frac{1}{\left(1-\phi_{Al_2O_3}\right)^{2.5}},\ C_2=1-\phi_{Al_2O_3}+\frac{\phi\rho_{Al_2O_3}}{\rho_f},C_3=1-\phi_{Al_2O_3}+\frac{\phi\left(\rho\beta\right)_{Al_2O_3}}{\left(\rho\beta\right)_f},\\ &\quad C_5=1-\phi_{Al_2O_3}+\frac{\phi\left(\rho c_p\right)_{Al_2O_3}}{\left(\rho c_p\right)_f},\ C_4=\frac{k_{Al_2O_3}+2k_f+2\left(\phi_{Al_2O_3}k_{Al_2O_3}\right)-2k_f\phi_{Al_2O_3}}{k_{Al_2O_3}+2k_f-\left(\phi_{Al_2O_3}k_{Al_2O_3}\right)+k_f\phi_{Al_2O_3}}. \end{aligned} \tag{12}$$

subjected to the dimensionless boundary condition

$$\begin{aligned} &f\left(\eta,0\right)=f\left(0,\tau\right)=f'\left(0,\tau\right)=0,\ \ h\left(\eta,0\right)=h\left(0,\tau\right)=h'\left(0,\tau\right)=0,\\ &\qquad\theta\left(\eta,0\right)=0,\ \ \theta\left(0,\tau\right)=1,\\ &\quad f'\to 0,\ \ h'\to 0,\ \ \theta\to 0,\ \ \text{as } \eta\to\infty. \end{aligned} \tag{13}$$

The interesting physical quantities in the present communication are the skin friction in the *x*-direction C_{fx}, the skin friction in the *y*-direction C_{fy}, and the Nusselt number Nu and are written as,

$$\begin{aligned} &C_{fx}=\mu_{nf}\left(\frac{\partial u}{\partial z}\right)_{z=0}\Big/\left(\rho_f\upsilon^2a^3x\right),\ C_{fy}=\mu_{nf}\left(\frac{\partial v}{\partial z}\right)_{z=0}\Big/\left(\rho_f\upsilon^2a^3y\right),\\ &\qquad Nu=-a^{-1}k_{nf}\left(\frac{\partial T}{\partial z}\right)_{z=0}\Big/k_f\left(T_w-T_\infty\right). \end{aligned} \tag{14}$$

Therefore, the dimensionless forms of these physical quantities can be presented as

$$\begin{aligned} &C_{fx}/Gr^{3/4}=f''\left(\tau,0\right)/\left(1-\phi_{Al_2O_3}\right)^{2.5},\ C_{fy}/Gr^{3/4}=h''\left(\tau,0\right)/\left(1-\phi_{Al_2O_3}\right)^{2.5},\\ &\qquad Nu/Gr^{1/4}=-\left(k_{Al_2O_3}/k_f\right)\theta'\left(\tau,0\right). \end{aligned} \tag{15}$$

3. Numerical Solution

The Keller-box method is applied to solve the Eq. (9) to Eq. (11) subject to boundary conditions (13). This scheme is detail explained in Cebeci and Bradshaw [35]). It is a flexible, powerful, unconditional, and accurate technique for solving a parabolic boundary layer fluid problem [42]. The numerical procedure was conducted using the Fortran language platform and the solution was found to converge in less than 20 seconds which is found to be very effective. The calculation is attained by the following algorithmic steps.

i. The high order governing Eq. (8) to Eq. (10) are reduced into a system of first-order partial differential equations.
ii. Apply the finite central differences formula to discretize the first-order partial differential equations.
iii. Use of Newton's method to linearize the resulting algebraic equations.
iv. Represent the algebraic system in the form of matrix-vector.
v. Solve the obtained system by the block tridiagonal elimination method.

The algorithm for using the Keller-box method was also shown in Figure 2 in terms of flow charts for better understanding. In numerical calculation, the proper step size $\Delta\eta$ and η_∞, which denoting the boundary layer thickness, are determined through a trial and error approach. Generally, we start the calculation by using a small value of boundary layer thickness and then successively increase the value until the computed interested physical quantities are independent from η_∞. Once the suitable value of η_∞ is obtained, we use it to determine the proper step size $\Delta\eta$. In the present study, accurate numerical results are achieved by using a step size $\Delta\eta = 0.05$ and η_∞ to be 5 to 6. Moreover, the step size for time $\Delta\tau$ is taken as 0.01. The convergence criterion ε has fixed to be 10^{-5}. In this study, the value selected for the parameter considered is actually taken from the previous study conducted. For example, the value of Pr taken here is 6.2 that correspond to Pr for water at room temperature as we applied water as the base fluid in this study [43].

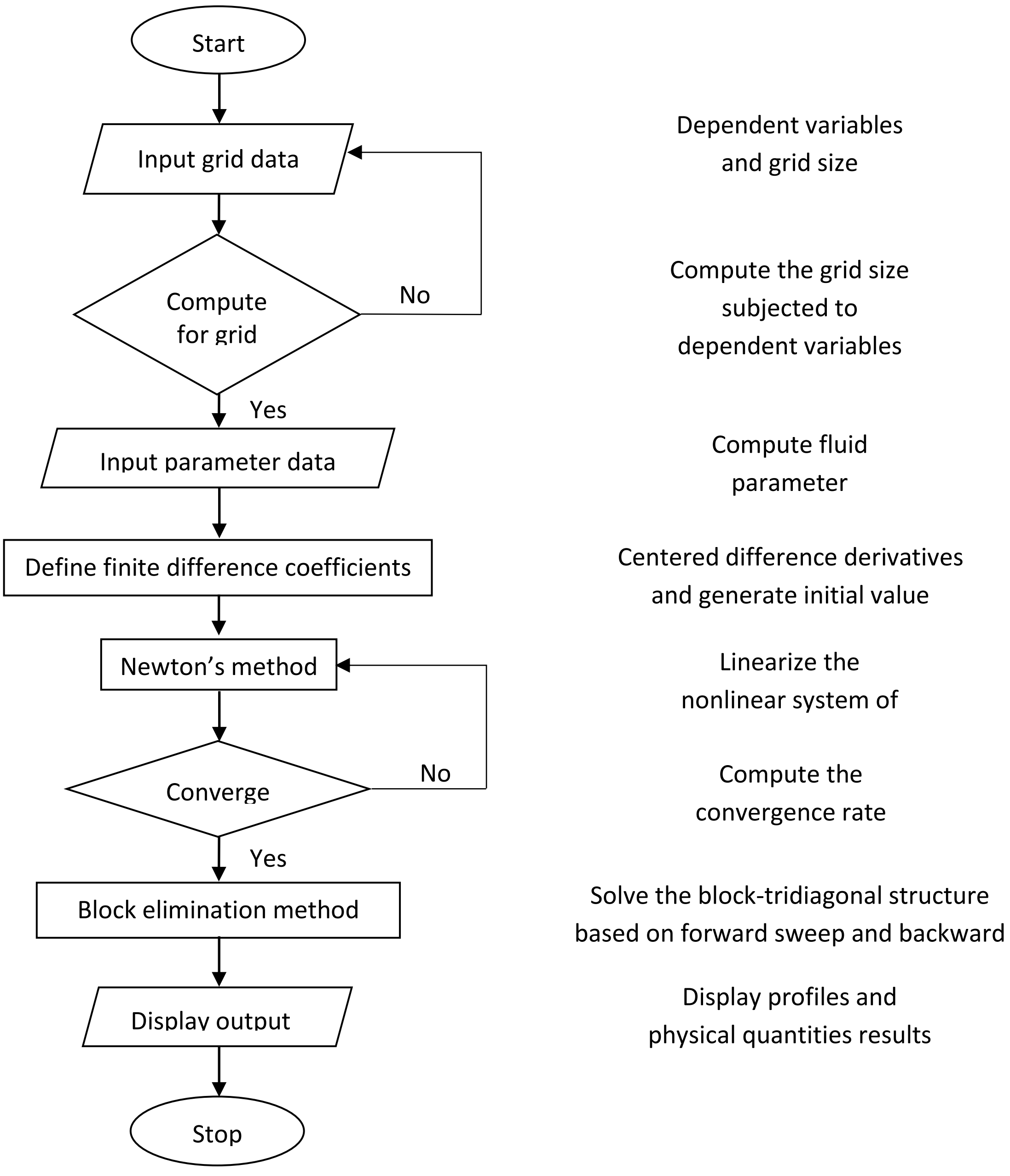


Fig. 2. Flow diagram for the Keller box method

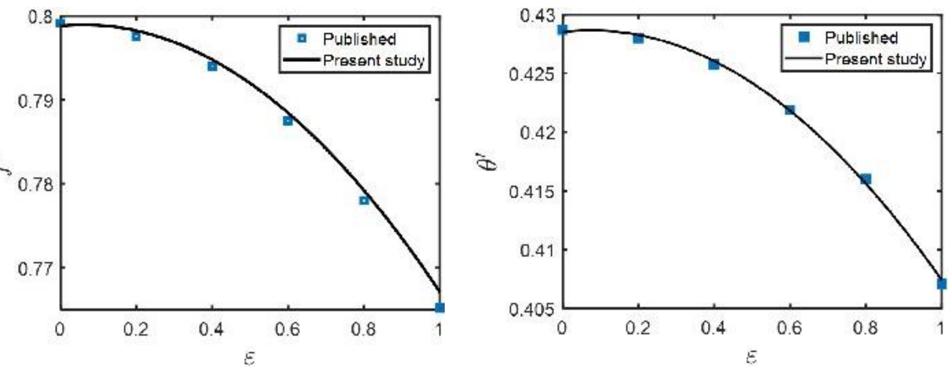
4. Results and Discussion

The impact of the nonlinear thermal parameter, the amplitude of gravity modulation, and volume fraction of the nanoparticle on the fluid velocity and temperature are carried out. A comparison study was conducted by comparing present results with a published work to measure the accuracy of the method and algorithm applied in solving the system of equations. Table 2 shows the comparison between the solutions of Kamal *et al.,* [40] and the results obtained in the present study with $c = 0.5, \Omega = 0.2, \Pr = 0.72$ and $\phi = 0$. Here f'' and h'' are the skin friction coefficient subjected to x and $y-$direction while θ' is the Nusselt number. The results of the present study are in very good agreement with the outputs provided in the literature. In addition, the residual error curves are obtained in Figure 3 between the previous study by Kamal *et al.,* [40] and the present study. It can be seen that the published data fit accordingly with the current research.

Figure 4 elucidates that the nonlinear thermal parameter increases the velocity profile but decreases the temperature profile. In the distribution of the fluid velocity, dual nature is detected. At $\eta > 1.8$, the rise in λ has diminished the velocity of the fluid. However, a reverse tendency is observed at the region near the body surface (see Figure 4(a)). This is because the nonlinear thermal parameter is directly proportional to the temperature gradient at the body surface; consequently, more buoyancy force is generated and then enhances the velocity profile. Moreover, the increment of the parameter has reduced, respectively, the thickness of the momentum boundary layer and thermal boundary layer. Besides, the rate of skin friction and the rate of heat transfer are significantly improved by the nonlinear thermal parameter as depicted in Figure 5. The nature of the quadratic convection has also been investigated by Mahanthesh and Mackolil [1] for a nanofluid flow over a vertical plate by using the modified Buongiorno model. The authors have observed an equivalent behavior of the parameter on the fluid velocity and temperature. Practically, the influence of the quadratic convection should be considered when designing the cooling system in a nuclear reactor or heat exchanger since it has a predominant impact on the fluid field and heat transport. The physical quantities result in terms of skin friction and the Nusselt number with various values of the nonlinear thermal parameter was also presented in Table 3.

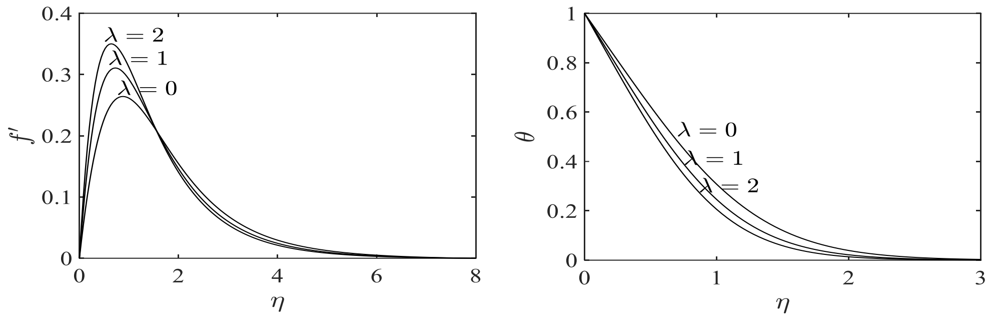
Table 2
Comparison results for skin frictions and Nusselt number with $c = 0.5, \Omega = 0.2,$ $\Pr = 0.72, \phi = 0$ and $\gamma = 0$

ε	Kamal *et al.,* [40]			Present		
	f''	h''	θ'	f''	h''	θ'
0.0	0.7989	0.4264	0.4287	0.7990	0.4262	0.4285
0.2	0.7980	0.4260	0.4280	0.7975	0.4265	0.4282
0.4	0.7946	0.4243	0.4258	0.7943	0.4249	0.4254
0.6	0.7885	0.4212	0.4219	0.7879	0.4209	0.4218
0.8	0.7794	0.4167	0.4160	0.7789	0.4165	0.4161
1.0	0.7669	0.4108	0.4071	0.7656	0.4112	0.4070



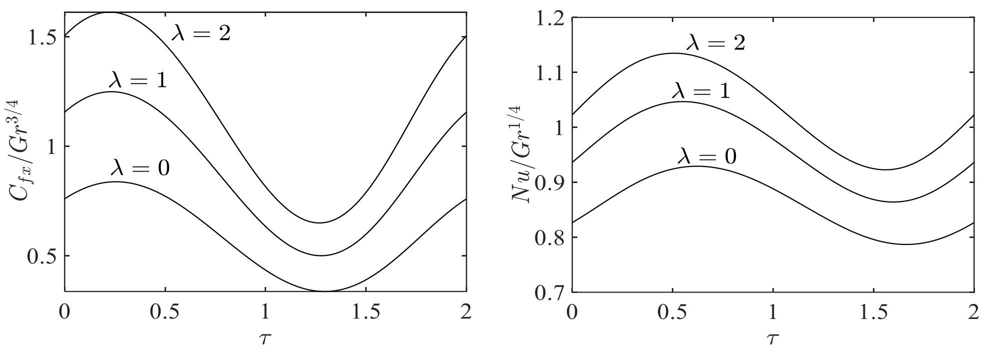
(a) (b)

Fig. 3. Residual curve error results of the present study and Kamal *et al.,* [40]



(a) (b)

Fig. 4. The behavior of (a) velocity profile and (b) temperature profile due to various values of the nonlinear thermal parameter λ with fixed parameter $c=0.5, \Omega=0.2, \phi=0.05, \Pr=6.2, \varepsilon=0.5$



(a) (b)

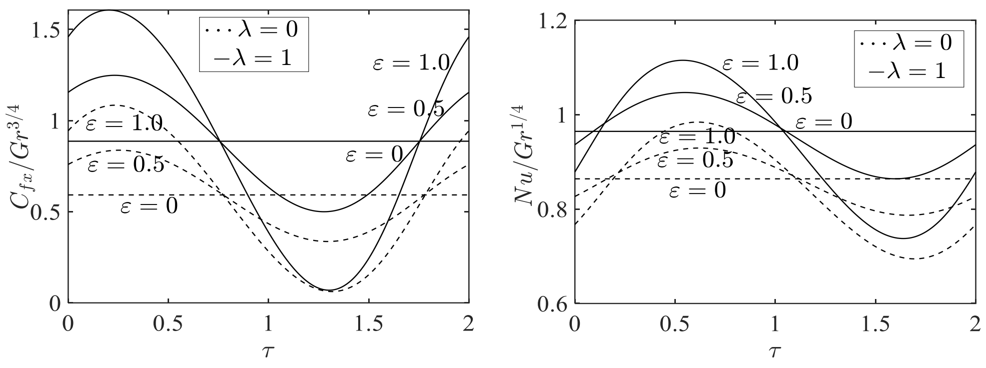
Fig. 5. The physical quantities in (a) skin friction coefficient and (b) Nusselt number of various values of the nonlinear thermal parameter λ with fixed parameter $c=0.5, \Omega=0.2, \phi=0.05, \Pr=6.2, \varepsilon=0.5$

Table 3
The skin friction and Nusselt number with various values of the nonlinear thermal parameter λ

λ	Skin friction coefficient	Nusselt number
0	0.5898	0.8586
1	0.8825	0.9572
2	1.1452	1.0317

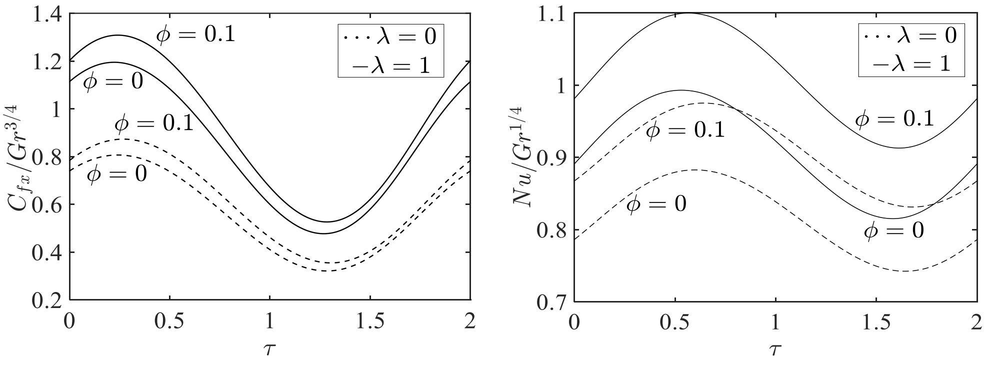
On the other hand, the effect of the amplitude of the gravity modulation on the skin friction and the rate of heat transfer is illustrated in Figure 4. The value of the ε is varied from 0 to 1 since a reverse direction will happen for $\varepsilon > 1$ due to the g-jitter cycle. As seen in Figure 4, when the effect of the g-jitter is omitted, $\varepsilon = 0$, either the skin drag force or the Nusselt number is constant. However, fluctuation behavior is observed for the physical quantities when $\varepsilon > 0$. The amplitude of the gravity modulation increases both the skin friction and the rate of heat transport. A similar outcome was achieved by Kamal *et al.,* [38] and Rawi *et al.,* [44]. Due to this nature, the g-jitter influence through some suitable adjustment can be used as a stabilizer of a certain unstable system likes a heated fluid layer or a chaotic flow.

Furthermore, the presence of the quadratic convection has enhanced the impact of the g-jitter amplitude on the fluid field and heat transition (see Figure 4 and compares the results with $\lambda = 0$ and $\lambda = 1$). Besides, Figure 5 depicts the effect of the volume fraction of nanoparticles on the physical quantities with and without the influence of the nonlinear convection. An ascending value of ϕ has intensified the rate of the heat transfer and the skin friction. An additional force is produced at the surface of the boundary since there is additional friction which is contributed by the alumina nanoparticles. Thus, the skin friction coefficient is increased. The effect is more dominant when the quadratic convective is taken into the account. This observation is essential in designing manufacturing processes or devices that involve a substantial change of temperatures such as frozen food production, solar collector, etc.



(a) (b)

Fig. 6. The physical quantities in (a) skin friction coefficient and (b) Nusselt number various values of the amplitude of modulation ε with fixed parameter $c = 0.5$, $\Omega = 0.2, \varepsilon = 0.5, \mathrm{Pr} = 6.2$



(a) (b)

Fig. 7. The physical quantities in (a) skin friction coefficient and (b) Nusselt number various values of the nanoparticle volume fraction ϕ with fixed parameter $c = 0.5$, $\Omega = 0.2, \varepsilon = 0.5, \mathrm{Pr} = 6.2$

5. Conclusion

The quadratic convection flow of a nanofluid near a three-dimensional stagnation point under the influence of gravity modulation is numerically investigated. The water-based Al_2O_3 nanofluid flow in a microgravity environment with non-slip boundary conditions is considered. The Keller-box method is practiced to compute the solutions and the results are graphically displayed with the help of MATLAB software. The conclusions of the present investigation can be summarized as

i. The nonlinear thermal parameter has a positive effect on the fluid velocity but a reverse behavior on the temperature profile.
ii. The physical quantities of the fluid system fluctuate when the amplitude of gravity modulation $\varepsilon > 0$.
iii. The nonlinear thermal parameter, the amplitude of gravity modulation, and the volume fraction of nanoparticles have enhanced the skin friction and also the rate of heat transfer
iv. The effect of the amplitude of the g-jitter and the volume fraction is more prominent when considering the quadratic convection.

The results of this fundamental study may apply as a pioneering step in the manufacturing industry in producing a better cooling system. The analysis result provides some information that is closely related to wall durability and the heat transfer achievement based on the fluid applied. There are quite some interesting possibilities in extending the fundamental research such as consideration of other effects such as magnetohydrodynamic, porous medium and the chemical reaction. Consideration of geometrical, position and properties held by the boundary body can also provide a different behavior on the fluid.

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