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Linear-elastic stress field of notched concrete beam: An application of finite element in theory of critical distances

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Linear-elastic stress field of notched concrete beam: An application of finite element in theory of critical distances

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Abstract. Fatigue failures occur in all structures including brittle and ductile structures. However, the study on fatigue in ductile material like metal and steel has tremendously developed compared to brittle material such as concrete. Fortunately, the Theory of Critical Distance (TCD) is witnessed successfully assess fatigue fracture in concrete. In obtaining one of the outputs using TCD which is critical distance, fatigue limit of concrete that is obtained through laboratory testing and stress field generated using computational analysis engineering software (CAE) are required. In this article, the concern will be on producing the valid and reliable stress field data since inaccurate input into the CAE will result unreliable output that is exposed to errors. In order to guarantee the result is accurate, validation works were conducted in pre-process and post-process phase while analysing finite model using ABAQUS. As the outputs comply accordingly based on the validation works, the critical distance is confidence to be consumed for the subsequent research related to TCD.

1. Introduction

Fatigue failures do occur on all types of materials, be it either ductile or brittle material. Yet, since the tragedy of The Versailles Train Crash of 1842 claimed to be the departure in understanding the mechanism of fatigue, the research of fatigue especially in ductile material progresses rapidly compared to brittle material like concrete [1]. As evidence, there are developed procedures and formulation in examining and making life prediction in ductile material like metal and steel but not for concrete-like material [2]. Thus, in such a challenging situation as it is, the fatigue study has founded the Theory of Critical Distances (TCD) where it is capable to perform fatigue assessment not only on ductile material but also concrete-like material [3]. Conceptually, pertaining to the critical distance, the intersection of at least two core inputs is compulsory based on TCD concept, which will produce the result. The core inputs needed are the endurance or fatigue limit of a material or in this case the material is concrete, and the stress field at the notch tip. One of it is obtain through laboratory works and another one is generated using engineering simulation software. Thus, the questions aroused. When is the verification works of finite element analysis using software is needed while the physical laboratory testing itself is already complicated? What is the condition of validation works being necessary? Therefore, the aim of the

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article to generate the stress field using ABAQUS and examine the verification works while operating engineering simulation works.

2. Finite element modelling

Finite element method functions to solve engineering and mathematical problems in civil and mechanical structures [4]. The solution of finite element method is through solving two or three dimensional space.

In this case, finite element method is meant to analyse three dimensional structure involving boundary problems. In operating Point Method of TCD, two essential parameters are required; endurance or fatigue limit and linear-elastic stress field at the vicinity of the notch [5]. Intersection of fatigue limit and the stress field will result critical distance based on TCD. Fatigue limit could be obtained through few laboratory procedures [6-11]. In order to pursue the TCD, linear-elastic stress field has to be generated using computer-aided engineering (CAE) simulation. It is indeed another option to trace propagating stress through the crackage formed while laboratory testing. However, there are two limitations in extracting the data in term of stresses, which are the stresses collected have to be approximately perpendicular towards the notch tip, and stress intervals might be huge and data might not be sufficient.

Figure 1 illustrates the concept PM of TCD where the black line represents the linear-elastic stress field and the dashed blue line is the single magnitude fatigue limit, which intersection of these parameters will result half of critical distance *L*/2.

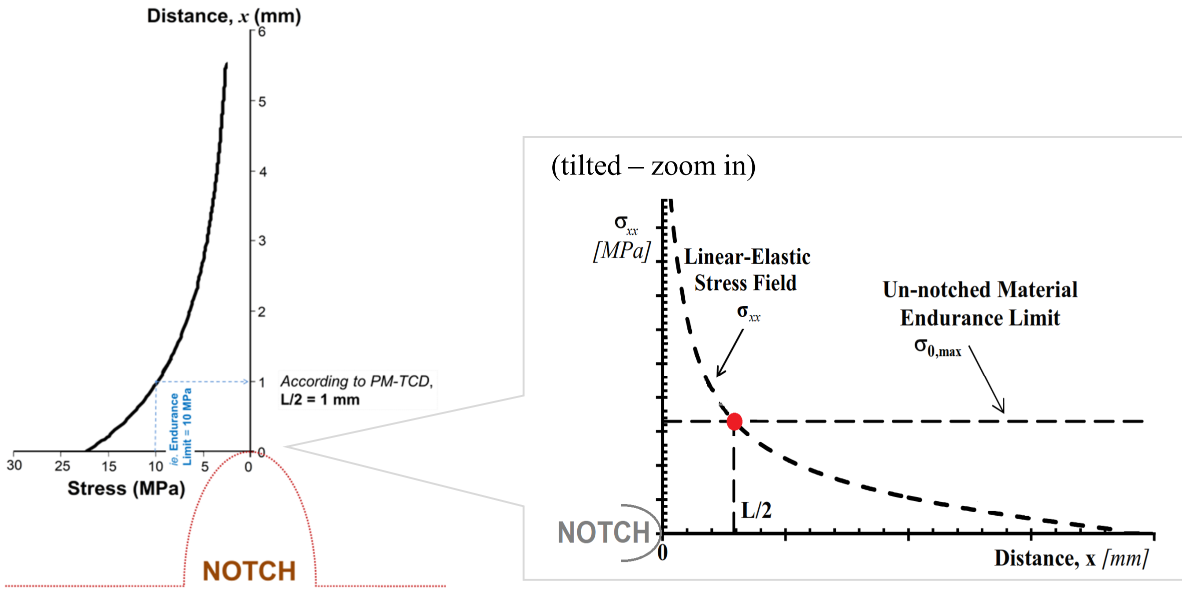

Figure 1. Illustration of Point-Method of TCD concept.

3. Parallelization of finite element model to the physical specimen

3.1. Size of specimen

Based on laboratory works to obtain fatigue limit, plain concrete beam sized 1065 x 110 x 100 mm is designated based on recommendation and related guidelines produced as in Figure 2 [11-12]. To make it comparable to the physical specimen, the model in the ABAQUS is made identical size as showed in Figure 2. As to study fatigue and fracture, the model has to disobey continuum mechanics model which the specimen is all equally perfect. In analyzing fracture mechanic which TCD itself falls under the field, the specimen has to possess discontinuity, notch or crack. Based on Taylor's advice and standard recommendations, a notch of about 30 mm which is not more than 1/3 of its depth is introduced as shown in Figure 3. The configuration of testing will be normal three-point bending test with a centre cyclic loading.

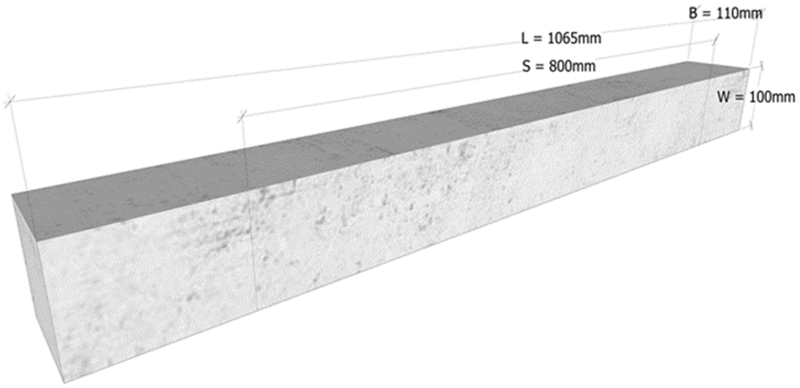

Figure 2. Dimensions for plain concrete beam specimen for fatigue laboratory testing.

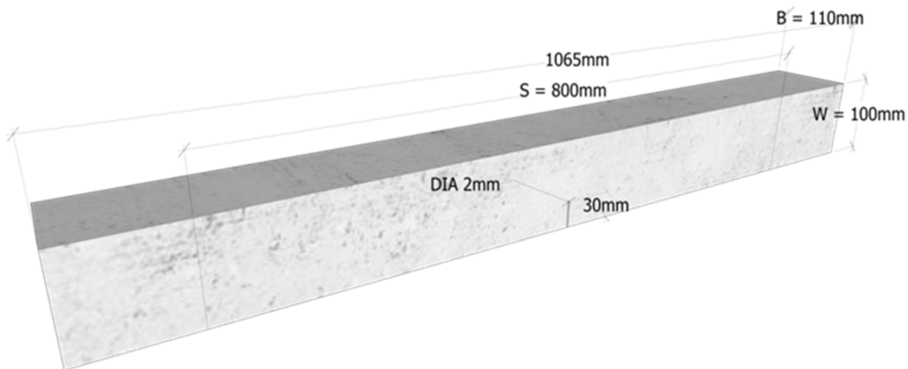

Figure 3. Dimensions for 30-mm (31.68 mm) notched concrete beam specimen for engineering simulation purposes.

3.2. Property assignment

First and foremost, in assigning property, two compulsory properties have to be known; in this case these properties were obtained from laboratory testing – the properties are modulus of elasticity, E and Poisson's ratio, ν. The other beam properties were adopted from laboratory testing into ABAQUS for the purpose of imitating as close as laboratory works are maximum bending stress and displacement/deflection at failure.

In order to obtain the modulus of elasticity, conventionally ASTM C469 was utilised. Based on the standard, the use of extensometer and compressometer are necessary. Thus, obeying the standard the test for modulus of elasticity for concrete of water-to-cement ratio 0.3, 0.4, and 0.5 configured in Figure 4. The cylindrical specimen dimension was 300 mm in height and 150 mm of radius. The specimen prepared in accordance to ASTM C31 where the height shall be doubled to its radius [13].

The significance of using ASTM guidelines is that to synchronise with the laboratory testing for three-point bending test under cyclic loading. Three-point bending test under cyclic loading utilised ACI as well. Both ASTM and ACI are American standards whereby the origin of testing configuration will definitely be related and similar. The synchronisation is important because fatigue limit obtained will intersect with linear-elastic stress field generated by ABAQUS as shown in Figure 1. Hence, the core property like elasticity has to be aligned.

The raw information obtained in the lab was force in *Y*-axis and displacement in *X*-axis. The force is later converted into stress by dividing the area of top cylindrical specimen which is circle in shape. On the other hand, for strain, the progressive displacement is divided by its original height.

IOP Conf. Series: Materials Science and Engineering 1144 (2021) 012037 doi:10.1088/1757-899X/1144/1/012037

Table 1 are the inputs which obtained from laboratory works and inserted into ABAQUS accordingly.

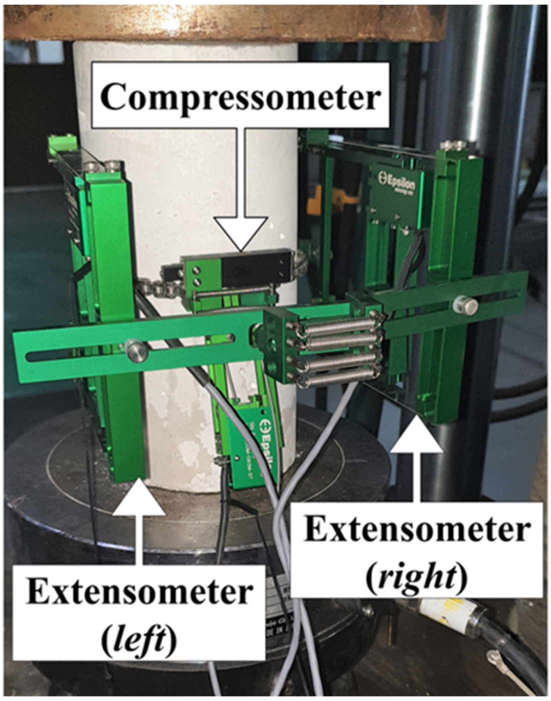

Figure 4. Modulus of Elasticity testing configuration based on ASTM C469.

Table 1. Laboratory data for ABAQUS inputs.

Water-cement ratio	Modulus of Elasticity, E (MPa)	Poisson ratio, ν	Maximum Bending Stress (MPa)	Maximum Deflection at Failure (mm)
0.3	39008.361	0	5.0	0.50
0.4	34662.222	0	5.5	0.55
0.5	32707.543	0	6.0	0.60

Since the research involved concrete mix with three different water-cement ratios of 0.3, 0.4, and 0.5, the procedure will be repeated according to the water-cement ratio. Figure 5 shows the stress versus strain plot for concrete with water-cement ratio 0.3. The typical formula used to calculate modulus of elasticity is stress divided by the strain. Thus, the slope of the stress-strain graph is actually representing the modulus of elasticity.

Based on the data extracted from Figure 5, the value of stresses for Y_1 and Y_2 are 15.28 MPa and 19.18 MPa respectively. Meanwhile for X_1 and X_2 are strains resultant to the Y_1 and Y_2 are 0.00059 and 0.00069 correspondingly. Therefore, the modulus of elasticity for concrete with water-cement ratio 0.3 is 39008 MPa.

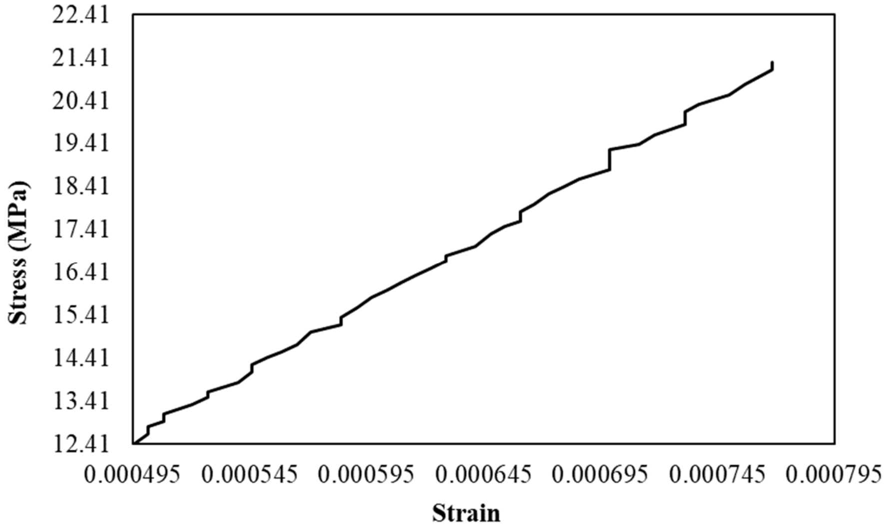

Figure 5. Stress-Strain graph for concrete with water-cement ratio 0.3.

The techniques and calculation are repeated for concrete water-cement ratio 0.4 and 0.5. Modulus of elasticity for water-cement ratio 0.4 and 0.5 are 34662 MPa and 32707 MPa respectively.

To ensure the concrete tested for modulus of elasticity is valid, the average compressive strength for all batches was checked and it is confirmed that they fall within the allowable margin set in the concrete mix design as shown in Table 2. On top of that, the trend compressive strength and modulus of elasticity corresponded correctly with the preference given by Eurocodes and British Standards [14].

Table 2. Targeted and achieved characteristic strength for water-cement ratio 0.3, 0.4, and 0.5.

Characteristic Strength / Water-Cement Ratio	0.3	0.4	0.5
Targetted Characteristic Strength (MPa) – calculated in concrete mix design form	68	52.4	39
Achieved Characteristic Strength (MPa) – obtained from laboratory testing	69.028	54.390	48.490
Allowable Deviation Margin = +/- 10 MPa	1.028	1.990	9.429

3.3. Maximum principal stress

The concrete beam model has been assigned with all basic properties has already ample to achieve the main purpose of running finite element. The purpose of operating notched concrete beam model using finite element is to obtain the maximum principal stress field perpendicular to the vicinity of notch tip. To obtain the stress field perpendicular to the notch tip, it is sufficient for the model to run up to its failure criterion which is not beyond its elastic properties. Hence, plasticity in concrete is not necessary to be assigned.

The stress field generated by FEA using the maximum principal strength. Few literatures suggested to utilise maximum principal strength concept as it suits and provide more sense in the condition such as stress field at the notch vicinity [5,15]. Maximum principal stress will denotes flexural strength perpendicular to the direction of loading applied [16].

In order to execute test and allowing concrete beam model to behave alike to the real one, crack must be permissible to occur. Few options can be set to permit a concrete model to crack i.e. Element Elimination Technique (EET) like Concrete Smeared Cracking, Crack under Extended Finite Element Method (X-FEM) custom etc. These options were in "Assigning property" step.

However, in assigning properties, elasticity mode is chosen and only core properties were inserted – modulus of elasticity, Poisson's ratio, maximum principal stress and displacement at failure. These

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properties were good enough to make the model to turn out to be concrete-alike, but the problem will arise when the loading started to employ, the model will not crack and will only deflects to the limit which in real case, while reaching the maximum limits, the concrete beam cracks.

In the model, crack assignment is configured under X-FEM criterion over other option as mentioned above. X-FEM is chosen to allow crack to propagate in concrete beam model because it allows local augmentation in certain area. In the case of this research, crack is targeted to occur starting at the notch tip which is localised and did not expect the crack to randomly exist. Secondly, it runs as a mesh-free function where it is understandable when crack occur, the mesh of the model will definitely be disturbed and reduces the accuracy. X-FEM has special attributes that will coordinate with the mesh as the fracture propagates. Thirdly, X-FEM solves problems on path-basis which what the research need – analysing stress with respect to distance from the notch tip [17]. Finally, X-FEM works fine in triangular mesh and that is the reason mesh is chosen in triangular shape to apply onto the beam model [18].

Concrete Smeared Cracking of Element Elimination Technique (EET) is no doubt is accurate to introduce crack in concrete model. However, Concrete Smeared Cracking of EET is less suitable to execute and provide the information that the study needs because although it has decent concept in executing cracks in model with concrete-like material, it is quite tricky in plotting stress along the crack path from the notch tip. The way it analyses is the element which involves in crack will be deleted like in Figure 6 [17]. If the elements were deleted along the crack path until it reaches the ultimate tensile strength, it is more difficult to plot the stress field perpendicular to the crack path because stresses are picked based on the nodes which is more effective.

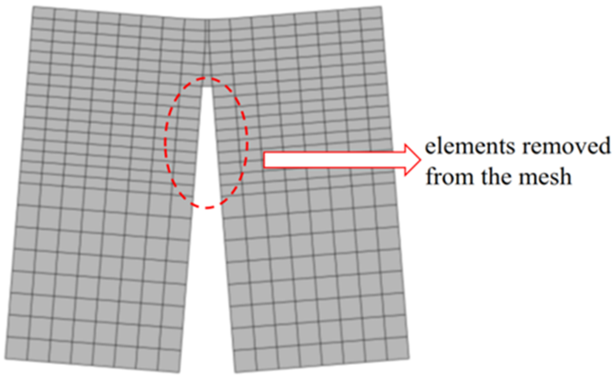

Figure 6. Crack method using Concrete Smeared Cracking of EET [17].

3.4. Mesh convergence analysis

The purpose of mesh convergence analysis is basically to determine the most stable and effective mesh size for the model created. Mesh convergence study can be performed by selecting a parameter which is the essence in running the ABAQUS and analyse using different mesh sizes [19]. The ultimate parameter might be in the single form, meaning that the single parameter could clearly portrait by comparing the parameter with different mesh sizes directly. However, another form of conveying the mesh convergence analysis which takes place in the research is describing the mesh sizes through the convergence in the stress field.

Thus, Figure 7 shows the stress field plot from the notch tip of 31.68-mm notched concrete beam of concrete mix with water-cement ratio 0.3 with five sizes of mesh sizes. The mesh sizes applied on the notched beam model were 1, 2, 2.5, 3, and 4 mm. The analysis has tried even smaller mesh sizes like 0.5 mm and 0.8 mm. However, the deviation and inconsistency of these mesh sizes for stress field compared to other mesh sizes is obvious and intolerable. Thus, the mesh convergence analysis will only involve mesh sizes which are within comparable. The uniform time of data collection to plot stress field for all mesh sizes for concrete specimen model is approximately at time step T=50.

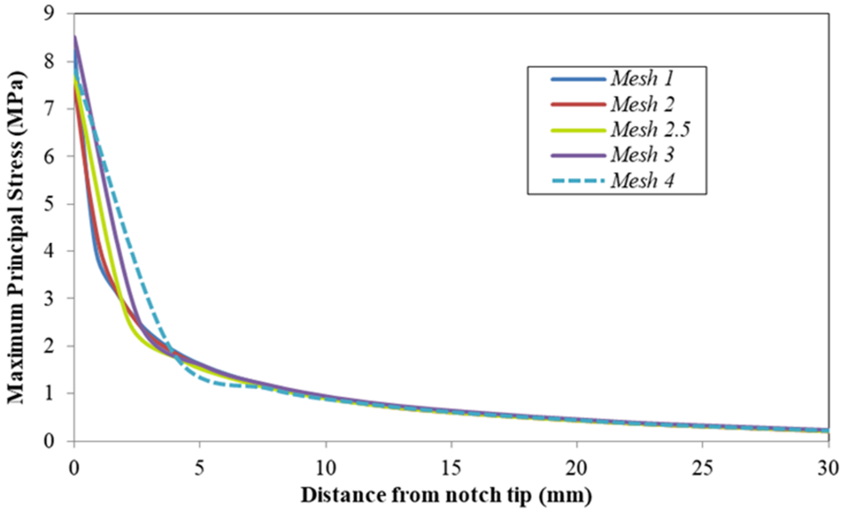

Figure 7. Linear-elastic stress field (mesh convergence test) at vicinity of 31.68-mm notched concrete beam with different mesh sizes.

Mesh sizes that fall in the range of less than 5% marginal difference are mesh sized 2.5 mm and 3 mm as shown in Figure 8. It is important to be precise and crucial at the location which is nearer to the notch tip because that is the place where the critical distance analysis will focus.

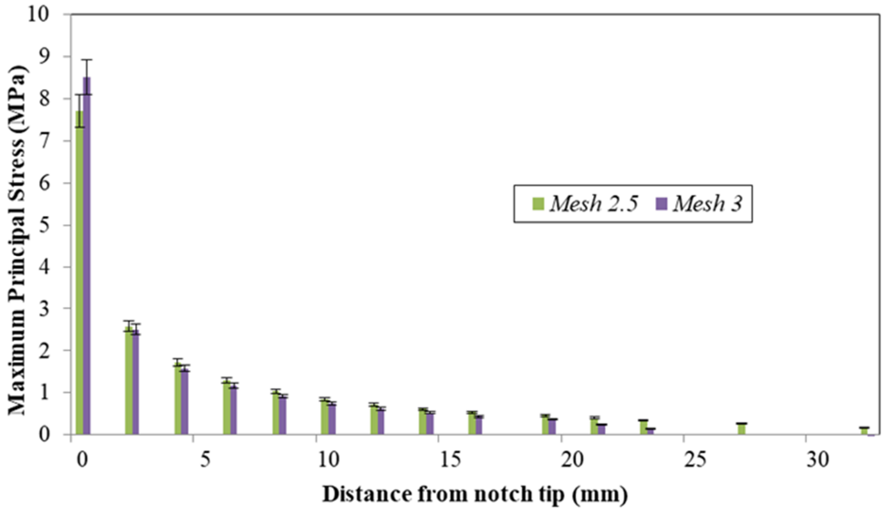

Figure 8. 5% marginal error allowance between mesh size 2.5 mm and 3 mm for 31.68-mm Concrete Notched Beam.

Figure 8 displays a tolerable relationship between mesh size 2.5 mm and 3 mm. It can be said that if it is more than 95% confidence, the mesh has achieved convergence based on these two sizes meshes as both falls in less than 5% difference. Individually, mesh size 2.5 mm will be chosen over 3 mm. It is because the first two points which the nearest to the notch tip shows inconsistency by having an apparent drop compared to the mesh size 2.5 mm.

4. Validation works

The effort of inputting as much data from laboratory work is because ABAQUS is a tool that acts as a calculator to solve engineering problems [20] – the more input data from laboratory into ABAQUS, hence the outputs information from ABAQUS will become more realistic and accurately represent results from the laboratory. Therefore, there is less need for an intense validation analysis [21-22]. Secondly, the output data were iterated from the errors to the laboratory is about 20% until it approaches zero – this is another good validation technique as stated by the numerical organisations [23, 24]. Thirdly, convergence of the aimed data through mesh convergence analysis is another guarantee that the model is stable and the output is acceptable because in the CAE itself has been built incorporating two fundamental elements which are verification and validation factors [25–27].

In understanding the needs in validation, full finite element analysis that needs verification from laboratory works and not vice versa. In this research, finite element analysis is needed to extract a data which will be used to intersect to obtain a fracture characteristic known as critical distance. Last but not least, of those mentioned validation efforts, the end result of critical distance is strictly referred and confirmed with the literature. As in Figure 9, the critical distance obtained was 4.096 mm. The critical distance is later confirmed by the related research to be within the acceptable range [28].

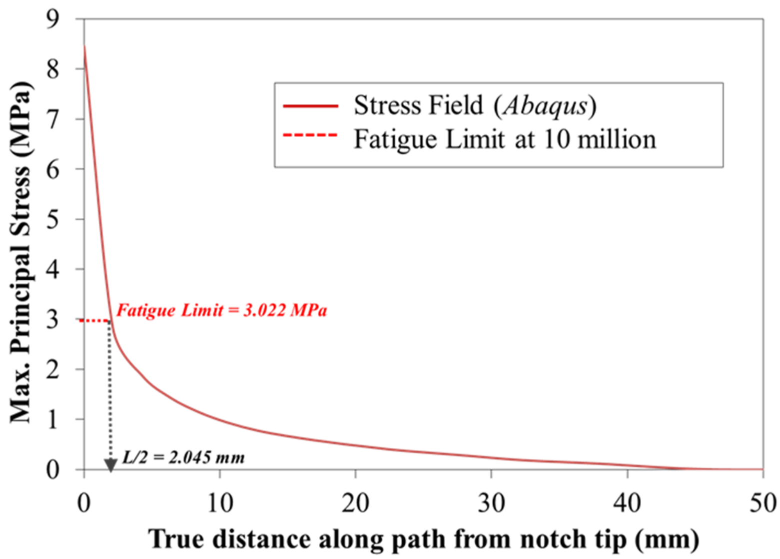

Figure 9. Intersection of Fatigue Limit at 10 million cycles and Stress Field at vicinity of 31.68-mm Notched Concrete Beam.

5. Conclusion

Finite element is studied due to the need in TCD which to generate linear-elastic stress field to be intersected with the fatigue limit of concrete. Generating the stress field needs CAE which involves finite element. The aimed output of TCD in the article is critical distance. Therefore, to ensure the output is correct and reliable, few validation works were executed. The validation works conducted were consuming and verify the inputs from the laboratory data to be inserted into the CAE, iterating the output to ensure the errors are minimal, running mesh convergence analysis, and confirming with the related literatures. Hence, as the outputs have undergone thorough validation works, the research is convinced to consume the data to proceed to another scope of TCD research.

6. References

[1] Santecchia E, Hamouda A M, Musharavati F, Zalnezhad E, Cabibbo M, El Mehtedi M and Spigarelli S 2016 A Review on Fatigue Life Prediction Methods for Metals *Adv. Mater. Sci. Eng.* 1–26

[2] Susmel L 2016 High-cycle fatigue of notched plain concrete *Proc. Structural Integrity XV Portuguese Conf. on Fracture* **2** 3447–3458

[3] Susmel L and Taylor D 2007 A novel formulation of the theory of critical distances to estimate lifetime of notched components in the medium-cycle fatigue regime *Fatigue Fract. Eng. Mater. Struct.* **30** 567–581
[4] Harish A 2020 *Finite Element Method – What Is It? FEM and FEA Explained* SimScale
[5] Taylor D 2007 *The Theory of Critical Distances: Basics An Introduction to the Basic Methodology of the TCD. The Theory of Critical Distances: A New Perspective in Fracture Mechanics* Elsevier Science
[6] ACI Committee 215 1992 *215R-74 (92): Considerations for Design of Concrete Structures Subjected to Fatigue Loading (Reapproved 1997)* American Concrete Institute
[7] Toumi A, Bascoul A and Turatsinze A 1998 Crack propagation in concrete subjected to flexural cyclic loading *RILEM 148-SSC Test Methods Strain Softening Response Concr. (Materials Struct./ Matériaux Constr.)* **31** 451–458
[8] TC 89-FTM 1991 *TC 89-FMT Fracture Mechanics of Concrete – Size-Effect Method for Determining Fracture Energy and Process Zone Size of Concrete* RILEM
[9] Guan J, Hu X and Li Q 2016 In-depth analysis of notched 3-p-b concrete fracture *Eng. Fract. Mech.* **165** 57–71
[10] Zhang C, Hu X, Wu Z and Li Q 2018 Influence of grain size on granite strength and toughness with reliability specified by normal distribution *Theor. Appl. Fract. Mech.* **96** 534–544
[11] Guan J, Hu X and Li Q 2016 In-depth analysis of notched 3-p-b concrete fracture *Eng. Fract. Mech.***165** 57–71
[12] TC 89-FMT 1991 *Fracture Mechanics of Concrete – Determination of fracture Parameters of Plain Concrete Using Three-Point Bend Tests* RILEM
[13] C31M-12 2012 *Standard Specification for Making and Curing Concrete Test Specimens in the Field* ASTM International
[14] BS EN 197-1 2011 *Cement Part 1: Composition, Specifications and Conformity Criteria for Common Cements* British Standards Institution
[15] Davies C M, O'Dowd N P, Nikbin K M, Webster G A and Biglari F 2005 Comparison of methods for obtaining crack-tip stress distributions in an elastic-plastic material *J. Strain Anal. Eng. Des.* **40** 431–450
[16] Taylor D 2007 *The Theory of Critical Distances in Detail The History, Background and Precise Definition of the TCD. The Theory of Critical Distances: A New Perspective in Fracture Mechanics* Elsevier Science
[17] Bendezu M A L, Romanel C and Roehl D M 2015 A comparative study on finite element methods for crack propagation in concrete *Proc. XXXVI Iber. Lat. Am. Congr. Comput. Methods Eng.* Rio de Janeiro
[18] Asferg J L, Poulsen P N and Nielsen L 2007 A direct XFEM formulation for modeling of cohesive crack growth in concrete *Comput. Concr.* **4** 83–100
[19] Autodesk-Support 2015 *How to Perform a Mesh Convergence Study* in *Simulation Mechanical* https://knowledge.autodesk.com/support/simulationmechanical/learnexplore/caas/sfdcarticls/sfdcarticles/How-to-Perform-a-Mesh- Convergence-Study.html
[20] Pegg E C and Gill H S 2016 An open source software tool to assign the material properties of bone for ABAQUS finite element simulations *J. Biomech.* **49** 3116–3121
[21] English T 2019 *What Is Finite Element Analysis and How Does It Work?* in *Interesting Engineering, Inc.* https://interestingengineering.com/what-is-finite-element-analysis-and-how-does-it-work
[22] Boulbes R J 2020 *Troubleshooting Finite-Element Modeling with Abaqus* Springer
[23] Mcaulley A 2019 *How to Repeat Loop Until Condition is Met? While or For Loop?* in *MathWorks - MATLAB Answer* https://www.mathworks.com/matlabcentral/answers/476099 how-to-repeat-loop-until- condition-is-met-while-or-for-loop
[24] Elkner J, Wentworth P, Downey A B and Meyers C 2012 *Iteration How to Think Like a Computer Scientist - Learning with Python*
[25] Altair Partner Alliance 2015 *Verification and Validation in FEA*

[26] Chillery M 2019 *The Importance of Mesh Convergence (Part 1)* in *International Association for the Engineering Modelling, Analysis and Simulation Community (NAFEMS)* https://www.nafems.org/join/directory/knowledge-base/the-importance-of-mesh convergence -part-1
[27] Harish A 2020 *What is Convergence in Finite Element Analysis?* https://www.simscale.com/ blog/2017/01/convergence-finite-element-analysis
[28] Taylor D 2016 On the role of microstructure in finite fracture mechanics *Procedia Struct. Integr.* **2** 1999–2005

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