NONLINEAR DYNAMIC MODELLING OF FLEXIBLE BEAM STRUCTURES USING NEURAL NETWORKS

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Abstract

This paper investigates the utilisation of back propagation neural networks (NNs) for modelling flexible beam structures in fixed-free mode; a simple repsentation of an aircraft wing or robot arm. A comparative performance of the NN model and conventional recursive least square scheme, in characterising the system is carried out in the time and frequency domains. Simulated results demonstrate that using NN approach the system is modelled better than with the conventional linear modelling approach. The developed neuro-modelling approach will further be utilized in the design and implementation of suitable controllers, for vibration suppression in such system.

***Keywords**: System identification, neural Networks, flexible beam, non-linear system.*

1. Introduction

In the process of identification a suitable model is developed that exhibits the same input/output characteristics as the controlled process (plant) [1]. Once a model of the physical system is obtained, it can be used for solving various problems such as, to control the physical system or to predict its behaviour under different operating conditions. A number of techniques have been devised by researchers to determine models that best describe input-output behaviour of a system. In many cases, intelligent techniques, including neural networks (NNs) are used in determining models that best represent the behaviour of non-linear systems that might be difficult to obtain using traditional approaches [2].

Recently NNs have become an attractive tool for use in constructing models of non-linear processes. This is because NNs have an inherent ability to learn and approximate non-linear functions. This therefore provides a possible way of modelling non-linear processes effectively [3,4]. Back propagation (BP) NNs are the most prevalent NN architectures for control applications because they have the capability to 'learn' system characteristics through non-linear mapping [3,4,5]. Their learning and update procedure is intuitively appealing because it is based on a relatively simple concept- if the network gives the wrong answer, the weights are corrected so that the error is lessened and as a result, future responses of the network are more likely to be corrected. When the network is given an input, the updating of activation value propagates forward from the input layer of processing units through each internal layer, to the output layer of processing units. The output units then provide the network's response. When the network corrects its internal parameters, the correction mechanism starts with the output units and propagates backward through each internal layer to the input layer.

In this investigation, supervised learning with BP NNs is used, where a network is trained, based on a comparison of the output and the target, until the network output matches the target, see Figure 1. The learning rule specifies how the parameters should be updated to minimize a prescribed error measure. Adaptive NN is actually used for system identification, and the task is to find an appropriate NN architecture and a set of parameters which can best model an unknown target system that is described by a set of input-output data pairs [6].

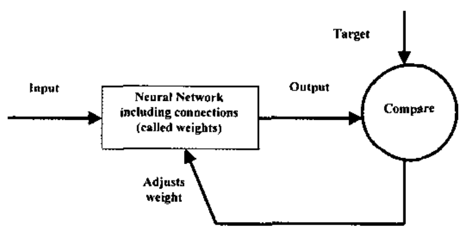

Figure 1: Neural-network training

This paper investigates the utilisation of backpropagation multi-layer perceptron (BPMLP) NNs based on one-step-ahead (OSA) prediction technique for

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modelling a single-input single-output (SISO) flexible beam system; a simple representation of an aircraft wing or robot arm. This is shown in Figure 2. Figure 3 shows the basic four steps involved in identifying a dynamic system.

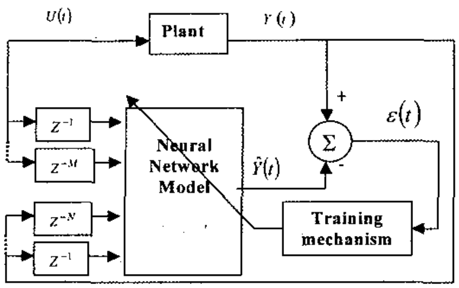

Figure 2: System identification using neural network model

Once a model of the system is obtained, it is necessary to verify if the model is adequate to represent the system. This is achieved by doing some validation tests such as correlation tests and calculating the mean squared error (MSE). In this work, the results are presented in both time and frequency domains. The performance of system identification using NN is compared with that of the parametric identification using recursive least square (RLS) technique.

An important stage in a system identification process is the selection of the type and characteristics of plant excitation signal [7]. In order to allow non-linear dynamics of the system be incorporated within the model, a pseudo random binary sequence (PRBS) signal covering the dynamic range of interest of the system was used in training the NN as well as in estimating the parameters using RLS scheme. The developed neuro-modelling approach will further be utilized in the design and implementation of suitable controllers.

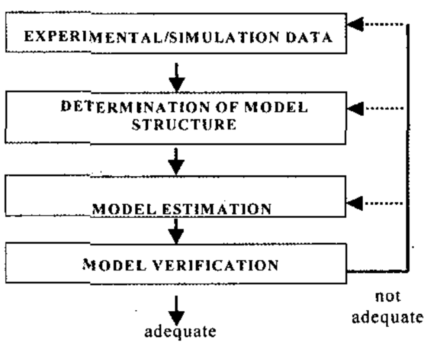

Figure 3: System Identification procedure

2. The Flexible Beam System

Figure 4 shows a flexible beam in fixed-free mode where $U(x,t)$ represents an applied force at a distance x from the fixed end at time t and $y(x,t)$ is the resulting beam deflection from its stationary position at the point where the force has been applied. L is the length of the beam and dx is a differential length of the beam.

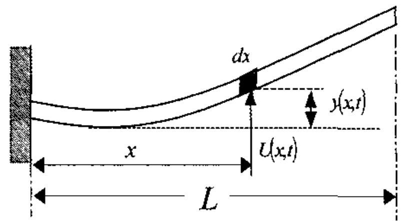

Figure 4: Fixed-free beam

The motion of a beam in transverse vibration in response to an applied force $U(x,t)$ is governed by the well-known fourth-order partial differential equation (PDE) [1]

$$\frac{\mu^2 \partial^4 y(x,t)}{\partial x^4} + \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{1}{m} U(x,t) \qquad (1)$$

where μ is a beam constant given by $\mu^2 = EI$ with I and E representing moment of inertia of the beam and the Young modulus respectively and m is the mass of the beam. The corresponding boundary conditions at the fixed and free ends of the beam are given as:

$$y(0,t) = 0 \quad \text{and} \quad \frac{\partial y(0,t)}{\partial x} = 0 \qquad (2)$$

$$\frac{\partial^2 y(L,t)}{\partial x^2} = 0 \quad \text{and} \quad \frac{\partial^3 y(L,t)}{\partial x^3} = 0 \qquad (3)$$

Note that the model in equation (1) does not incorporate damping. The finite difference (FD) method is used as a numerical solution to the PDE in [7]. This involves a discretization of the beam into a finite number of equal length sections, each of length Δx, and the beam motion (deflection) for the end of each section is considered at equally spaced time steps of duration Δt. Hence, using first order central FD methods to approximate the partial derivative terms in equations (1)-(3) gives [8]:

$$y_{j+1} = -y_{j-1} - \lambda^2 \mathbf{S} y_j + \frac{(\Delta t)^2}{m} U(x,t) \qquad (4)$$

where y_{j+1}, y_{j-1}, y_j are $k \times 1$ matrices representing deflections at grid points 1 to k of the beam at time step j,

$\mathbf{S}$, known as stiffness matrix, depends on the physical parameters and boundary conditions of the beam and $\lambda^2 = \frac{(\Delta t^2)}{(\Delta x^4)}\mu^2$. The stability of the algorithm in equation (4) is satisfied by $0 < \lambda^2 \le 0.25$ [8]. The first five resonance modes of this beam, as obtained through theoretical analysis, are located at 1.875 Hz, 11.751 Hz, 32.902 Hz, 64.476 Hz and 106.583 Hz respectively with first two modes being dominant ones.

3. Model Validation

Model validity tests are procedures designed to detect the adequacy of a fitted model. In practice, the model of the system will be unknown and the detection of an inadequate fit is more challenging. A common measure of predictive accuracy used in control and system identification is to compute the one step-ahead prediction of the system output. This is expressed as

$$\hat{y}(t) = f(u(t), u(t-1), \ldots, u(t-n_u), y(t-1), \ldots, y(t-n_y))$$

where $f()$ is a non-linear function, u and y are the inputs and outputs respectively. The residual or prediction is given by

$$\varepsilon(t-1) = y(t) - \hat{y}(t)$$

Often $\hat{y}(t)$ will be a relatively good prediction of $y(t)$ over the estimation set even if the model is biased because the model was estimated by minimizing the prediction errors. Correlation tests are also used to validate the model. If a model is adequate then the residuals or prediction errors $\varepsilon(t)$ should be unpredictable from all linear and nonlinear combinations of past inputs and outputs. The derivation of simple tests that can detect these conditions is complex, but it can be shown that the following conditions should hold [7]:

$$\phi_{\varepsilon\varepsilon}(\tau) = E[\varepsilon(t-\tau)\varepsilon(t)] = \delta(\tau) \qquad (5)$$

$$\phi_{u\varepsilon}(\tau) = E[u(t-\tau)\varepsilon(t)] = 0 \quad \forall\tau \qquad (6)$$

$$\phi_{u^2\varepsilon}(\tau) = E[(u^2(t-\tau) - \bar{u}^2(t))\varepsilon(t)] = 0 \quad \forall\tau \quad (7)$$

$$\phi_{u^2\varepsilon^2}(\tau) = E[(u^2(t-\tau) - \bar{u}^2(t))\varepsilon^2(t)] = 0 \quad \forall\tau \quad (8)$$

$$\phi_{\varepsilon(\varepsilon u)}(\tau) = E[\varepsilon(t)\varepsilon(t-1-\tau)u(t-1-\tau)] = 0 \quad \tau \ge 0 \quad (9)$$

where, $\phi_{u\varepsilon}(\tau)$ indicates the cross-correlation function between $u(t)$ and $\varepsilon(t)$, $\varepsilon u(t) = \varepsilon(t+1)u(t+1)$, $\delta(\tau)$ is an impulse function. Ideally the model validity tests should detect all the deficiencies in network performance including bias due to internal noise. The cause of the bias will however be different for different assignments of network input nodes. Consequently the full five tests defined by equations (5)-(9) should be satisfied if $u(.)$'s and $y(.)$'s are used as network input nodes. Correlation function between two sequences $\psi_1(t)$ and $\psi_2(t)$ is given by:

$$\hat{\phi}_{\psi_1\psi_2}(\tau) = \frac{\sum_{t=1}^{N-\tau} \psi_1(t)\psi_2(t+\tau)}{\sqrt{\sum_{t=1}^{N} \psi_1^2(t) \sum_{t=1}^{N} \psi_2^2(t)}}$$

In practice normalised correlations are computed. Normalisation ensures that all the correlation functions lie in the range $-1 \le \hat{\phi}_{\psi_1\psi_2}(\tau) \le 1$ irrespective of the signal strengths The correlation's will never be exactly zero for all lags and the 95% confidence bands defined as $\frac{1.96}{\sqrt{N}}$ are used to indicate if the estimated correlations are significant or not, where N is the data length. Therefore, if the correlation functions are within the confidence intervals the model is regarded as adequate.

4. Implementation and Results

An aluminium type fixed-free beam of 0.635m length, 0.019230m width, and 0.00091863m thickness was simulated for 4 seconds. The beam was divided into 20 sections and a sampling time of 0.2ms that satisfies the stability requirements of the FD simulation algorithm and is sufficient to cover all the resonance modes of vibration, was utilised. A PRBS signal covering the dynamic range of interest of the system was used to train the network. The primary force was applied at grid point 13. The input and output sensors were placed at grid points 12 and 19 respectively.

4.2 Modelling with BPMLP NNs

Using data gathered at grid point 12 as the input and 19 as the output, the network was trained using the scheme shown in Figure 2. From the simulation carried out, different structures such as number of nodes and layers were investigated. Using the mean square errors (MSE) and correlation tests as part of the guidelines, it was found that good result was achieved using an MLP network with two hidden layers having 6 tansigmoid neurons each, one output layer with one linear neuron and $n_u = n_y = 6$. The MSE obtained for the neuro-model was 0.00035827. Figures 6, 7,8 and 9 show the performance of the trained network. The corresponding correlation tests were found to be within 95% confidence interval indicating an adequate model fit.

4.2 Modelling with RLS

RLS algorithm, based on the well known least squares (LS) method, uses an iterative refinement technique to continuously tune estimated parameters using knowledge of some existing parameters as well as information obtained from the continuous operation of the system [8]. While the LS provides a best-fit estimate for a set of recorded data, the RLS algorithm creates a continuous estimate for a set of unknown system parameters. For comparison purposes, the system was also modelled with RLS algorithm using the same input-output data gathered where a 10-model order was used. Results of the simulation are shown in Figures 10, 11 and 12. The mean-square error for RLS-based model was 0.0208. Comparing these with the corresponding results of neuro modeling reveals that the identification using NN has performed better than the linear model. The correlation tests for the RLS-based model were also largely found to be within the 95% confidence interval.

5. Conclusion

The development of a neuro-modelling strategy based on OSA prediction has been presented and verified in the identification of the flexible beam system in comparison to a linear modeling approach. BPMLP NN has been introduced and the capability of the network in characterising highly nonlinear dynamic systems has been investigated. OSA predictions have been used as training method and model validity tests using correlation tests have been carried out. It has been shown that with suitable choice of the input data structure the system data can be predicted with a minimal prediction error. The significance of the neuro-modelling strategy has been clearly demonstrated through the level of performance achieved in the identification of the dynamics of flexible beam system. The developed neuro-modelling approach will further be utilized in the design and implementation of suitable controllers, for vibration suppression in flexible structures.

6. References

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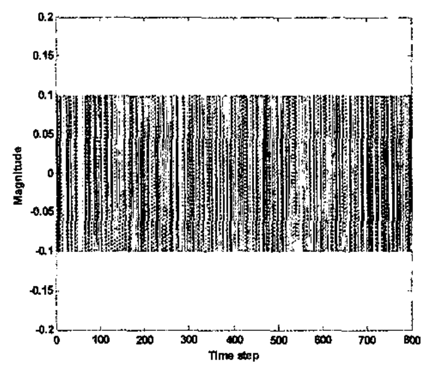

Figure 5: PRBS signal

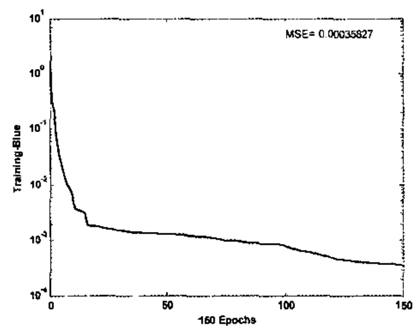

Figure 6: Number of epochs used in training the neuro model

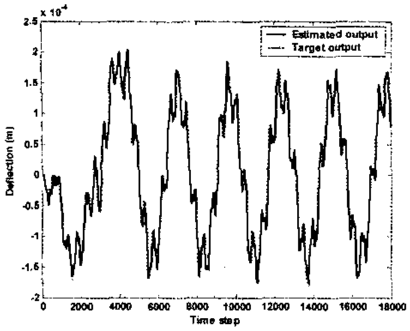

Figure 7: Estimated and target output in time domain for neuro modelling

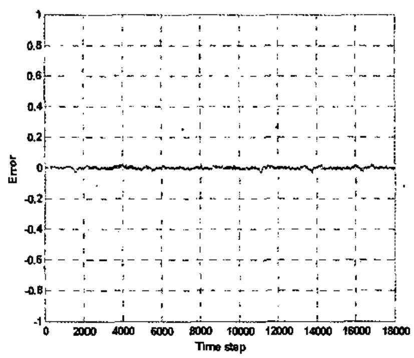

Figure 8: Error between estimated and target outputs for neuro modelling

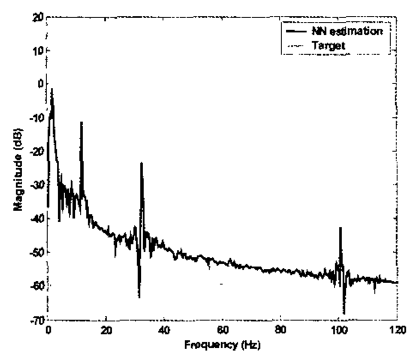

Figure 9: Estimated and target output in frequency domain for neuro modelling

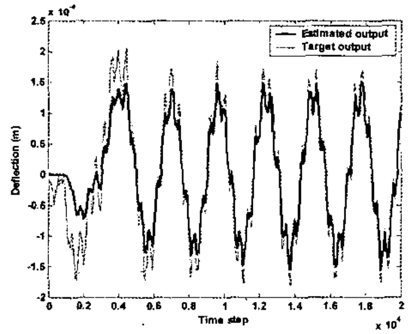

Figure 10: Estimated and target output in time domain for RLS modelling

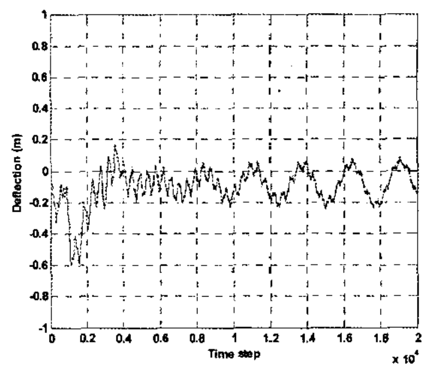

Figure 11: Error between estimated and target outputs for RLS modelling

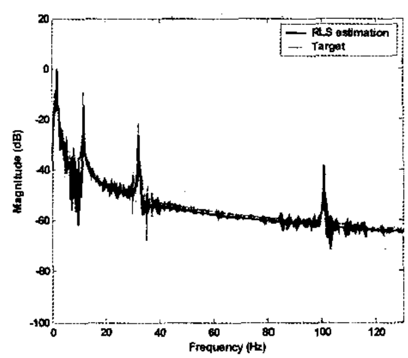

Figure 12: Estimated and target output in frequency domain for RLS modelling