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Frequency Domain Decomposition Method: A Comparative Study on Signal Processing for Unbiased Damping Ratio Estimates

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Abstract. Frequency domain decomposition (FDD) is one of OMA methods in the frequency domain and this method has become well-known among engineering community engaged in the system modal identification due to its capability as a user-friendly and fast processing algorithm. Though, this method has problems in offering an accurate estimation of modal damping ratios, even though natural frequencies and mode shapes can be accurately estimated. The accurate estimation of modal damping is still an open problem and often leads to biased estimates since the errors are stemming from each step in FDD procedures and primarily caused by signal processing. Therefore, the identification of modal damping ratio turns out to be immensely essential in structural dynamics since damping is one of the crucial parameters of resonance. This study is to determine the appropriate signal processing for FDD because signal processing such as the time window, correlation function (CF) and the spectral density (SD) are the main contributors to the bias estimate. The goal of this paper is to provide necessary information on modal damping for reliable estimation.

Keywords: Operational modal analysis; frequency domain decomposition; damping estimation.

1. Introduction

In the early twentieth century, Operational Modal Analysis (OMA) has become well-known among engineering community engaged in the system modal identification by replacing Experimental Modal Analysis (EMA) [1, 2]. The great interest of OMA is powered by its capability to carry out simple, quick and cost-effective tests that depend solely on the responses of the structure without compromising its normal operation [3].

The user-friendly and fast processing algorithm of frequency domain decomposition (FDD) proves that it can yield reliable results but with a good selection of parameters for spectra estimation and well-separated modes [4]. Though, this method has problems in offering an accurate estimation of modal damping ratios, even though natural frequencies and mode shapes can be accurately estimated [3]. The accurate estimation of modal damping is still an open problem and remain unsolved, and often leads to biased estimates since the errors are coming from each step in FDD procedures and primarily caused by signal processing [5]. Even though, this method is capable of dealing with closely spaced modes [6].

For damage detection, modal damping ratio is a very sensitive parameter to damage compared to natural frequency and mode shape. Therefore, the identification of modal damping ratio turns out to be



immensely essential in structural dynamics since damping is one of the crucial parameters of resonance, particularly for the low-damped structure. This is because the dynamic responses of the system are primarily influenced by the structural behavior and input forces [7].

The goal of this paper is to provide necessary information on modal damping for a reliable estimation and to determine the appropriate signal processing for FDD. Signal processing approach that consists of the time window (windowing), correlation function (CF) and the spectral density (SD) are the main contributors to the bias estimate.

2. Numerical Simulation

2.1. Design of simulation

The algorithms are explored using simulated data, implemented autonomously within MATLAB from one simple model of two degrees of freedom (DOF) as shown in Figure 1.

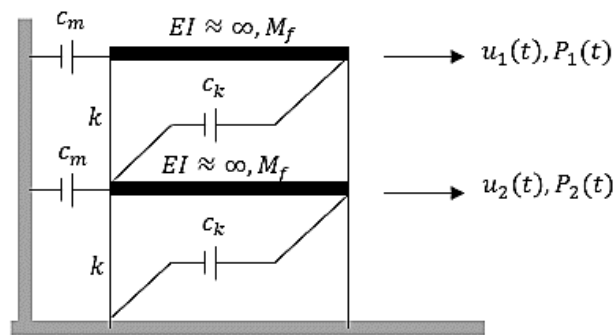


Figure 1. Simulated models of 2 DOF shear frame

The Characteristics of the simulated models of 2 DOF shear frame are characterized by Equation (1) and Equation (2) by the stiffness (K), and mass (M).

$$K = 10^8 \begin{bmatrix} 2.54 & -1.24 \\ -1.24 & 1.24 \end{bmatrix} (N/m) \quad (1)$$

$$M = 70,000 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (kg) \quad (2)$$

Meanwhile, damping (C) matrices of the structure are described by mass and stiffness proportional Rayleigh damping as shown in Equation (3):

$$[C] = [C_m] + [C_k] = a_0[M] + a_1[K] \quad (3)$$

Where $[C_m]$, $[C_k]$, $[M]$ and $[K]$ are the mass proportional damping, stiffness proportional damping, mass and stiffness matrix, respectively and are the coefficient describing the mass and stiffness proportional damping. A complex eigenvalue analysis is carried out on the 2 DOF shear frame. The prescribed modal damping ratios, to determine the coefficients a_0 and a_1 , are set to $\zeta_k = \zeta_j = 0.01$ (1%).

The broadband ambient excitation with zero mean, Gaussian white noise is replicated by time series with normally distributed random numbers assuming independent inputs for all DOF of the models. The white noise has a constant PSD and ensures a broad banded input [5]. The white noise input also

simulates the assumption of the excitation system to be linear and time-invariant which characterized by constant parameters if all its fundamental properties are invariant with respect to time. The response of the system is simulated using Newmark's method with constant average acceleration (i.e., $\gamma = \frac{1}{2}$ and $\beta = \frac{1}{4}$) [8]. The adopted parameters for the two-storey shear frame in the processing are shown in Table 1.

Table 1. The adopted parameters for the two-storey shear frame

Parameters	Two-storey frame
Length of time series, t (s)	400
Sampling frequency, f_s (Hz)	200
Adopted Frequency resolution, Δf (Hz)	0.0025

In this numerical simulation, the eigenvalue problem analysis is used to validate the effectiveness of the proposed approach. By taking values of the stiffness K , mass M , and damping C , the modal parameters (modal damping ratio, natural frequency, and mode shape) can be estimated.

2.2. Classical FDD algorithm procedure

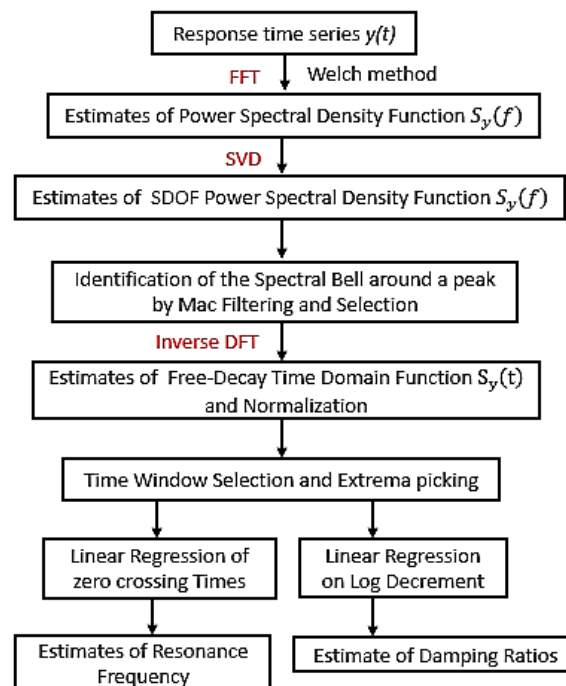


Figure 2. Schematic Illustration of classical FDD Procedure

Normally, signal processing serves as a tool to give a clear illustration of the physical problems we face. A good selection of signal processing techniques can provide all necessary information about the system that obtained from the response signals of the system itself. Signal processing consists of correlation functions (CF) and spectral densities (SD) function. Before estimating SD, the FFT algorithm requires time windows to reduce leakage by forcing the endpoints of each signal sample data to zero. Next, the

Singular Value Decomposition (SVD) is applied to decompose the output SD into auto SD that represents an SDOF system. The singular values that are determined around a resonance peak with an equivalent singular vector that possesses a sufficiently high MAC index value (for example 0.95 [9]) were transformed to SDOF of the time domain (TD) signal by the inverse FFT [10, 11]. All extrema of the free decay that within an appropriate time window (for example 90% to 20% of the maximum amplitude) were used to implement the subsequent linear regression operations for assessing the logarithmic decrement (LogDec), δ :

$$\delta = \frac{2}{k} \ln \left(\frac{r_0}{|r_k|} \right) \quad (4)$$

$$2 \ln(|r_k|) = 2 \ln(r_0) - \delta k \quad (5)$$

where k is an integer counter of the k th extreme of the auto-correlation function, $k = 1, 2, 3, \dots$, while r_0 and r_k are the initial and the k th extreme value of the auto-correlation function, respectively. Then, the modal damping ratio can be obtained.

$$\zeta_q = \frac{\delta_q}{\sqrt{4\pi^2 + \delta_q^2}} \quad (6)$$

Meanwhile, the damped natural frequency, ω_d was estimated by linear regression on the zero crossing times of the equivalent SDOF correlation function. Then, the undamped natural frequency, ω_n was computed by using the following equation:

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} \quad (7)$$

Further information on all alternative FDD method can be found in [9, 12, 13].

2.3. Signal processing

The step of denoted signal processing contains the tapering (windowing), correlation functions (CF) and the spectral densities (SD).

2.3.1. Tapering (windowing)

The various type of tapering on correlation function for 2 DOF models shear frame were used in this study with 10 datasets. In literature, there is a lot of tapering (windowing) functions introduced by researchers, but in this paper only concern the normally used tapering (windowing) function in OMA such as Hanning window with 50 % overlap, flat-triangular window with a value of $\alpha = 0.5$ and classical exponential window. The correlation function matrix was computed by using the direct estimation method via Equation (8) with a maximum time lag equivalent to 1024 times the sampling time step.

$$\hat{R}(k) = \frac{1}{N - k} Y_{(1:N-k)} Y_{(k+1:N)}^T \quad (8)$$

where the measured responses that arrange as a column in data series, N is the total number of data points in the time series, k corresponds to the time lag $\tau = k\Delta t$ and $(N - k)\Delta t$ corresponds to $T - \tau$, T is the total length of the time series.

2.3.2. Correlation function and spectral density estimate

There are various approaches to estimate correlation function (CF) and spectral density (SD). One of them is by direct estimates of the correlation function matrix via Equation (8) and then convert to spectral

density (SD) by the FFT. The flat-triangular window with $\alpha = 0.5$ was applied in correlation function before estimating SD in order to suppress sidelobe noise. Besides that, the conventional Welch approach with a Hanning window and 50% overlap, involves signal sectioning, windowing, and overlapping. Moreover, the random decrement (RD) was computed by using band triggering with an asymmetric band around zero and a width of $\pm 2\sigma$ where σ is the standard deviation of the triggering signal. This approach neglects the need of windowing due to the natural decay of the RD functions. Finally, the half-spectrum was obtained based on the estimated zero-padded direct correlation function with a flat-triangular window and then converted by FFT.

2.4. Statistical analysis

The 10-data set from the 2 DOF models shear frame are analyzed to estimate spectral densities. All spectral plots were in dB relative to the measurement unit in order to provide a clearer picture of the spectral density. Thus, all necessary information of singular values of spectral density can easily be captured in one single plot. In addition, all the analysis was done by applying a maximum time lag of the correlation functions equivalent to 1024 times the sampling time step.

The damping estimation is evaluated by examining the percentage error of the difference between the estimated and the assigned damping as shown in Equation (9),

$$\Delta\zeta_{k,j} = \frac{|\hat{\zeta}_i - \zeta_{k,j}|}{\zeta_{k,j}} \times 100(\%) \quad (9)$$

The difference $\Delta\zeta_{k,j}$ is referred to the error, where $\hat{\zeta}_i$ and $\zeta_{k,j}$ are the estimated damping mode i and the assigned damping respectively.

3. Results and Discussion

3.1. Tapering (windowing)

The results of the tapered correlation function and spectral density with the various application of tapering (windowing) are shown in Figure 3.

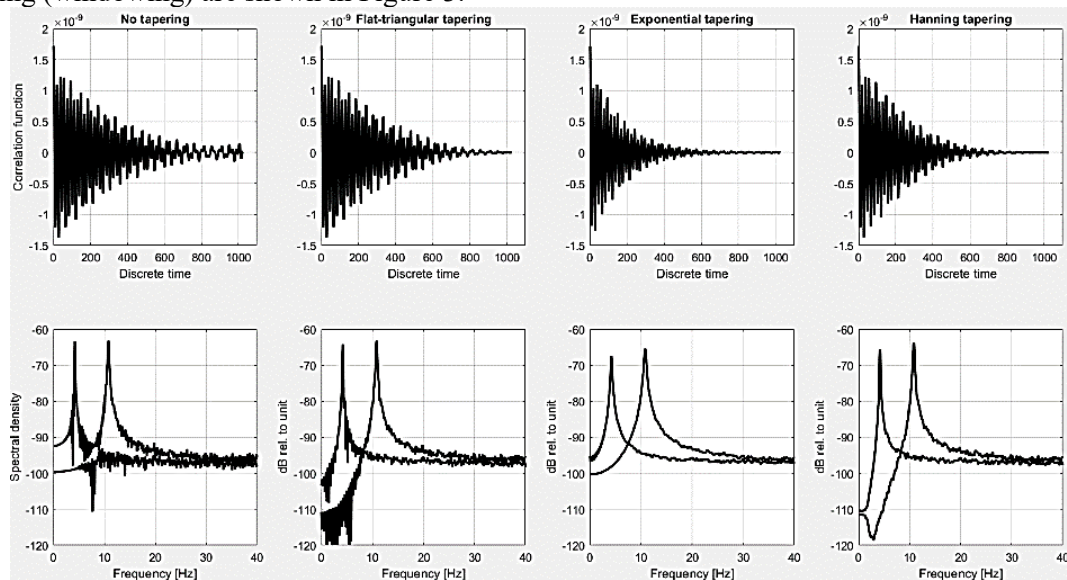


Figure 3. Result of various tapering (windowing) application. Top plots indicate the autocorrelation function with different cases of tapering. From the left: The first plot is without any tapering, the second plot is tapered by the flat-triangular window, following

with an exponential window, and lastly by Hanning window. Bottom plots illustrate the corresponding singular values of spectral density plots

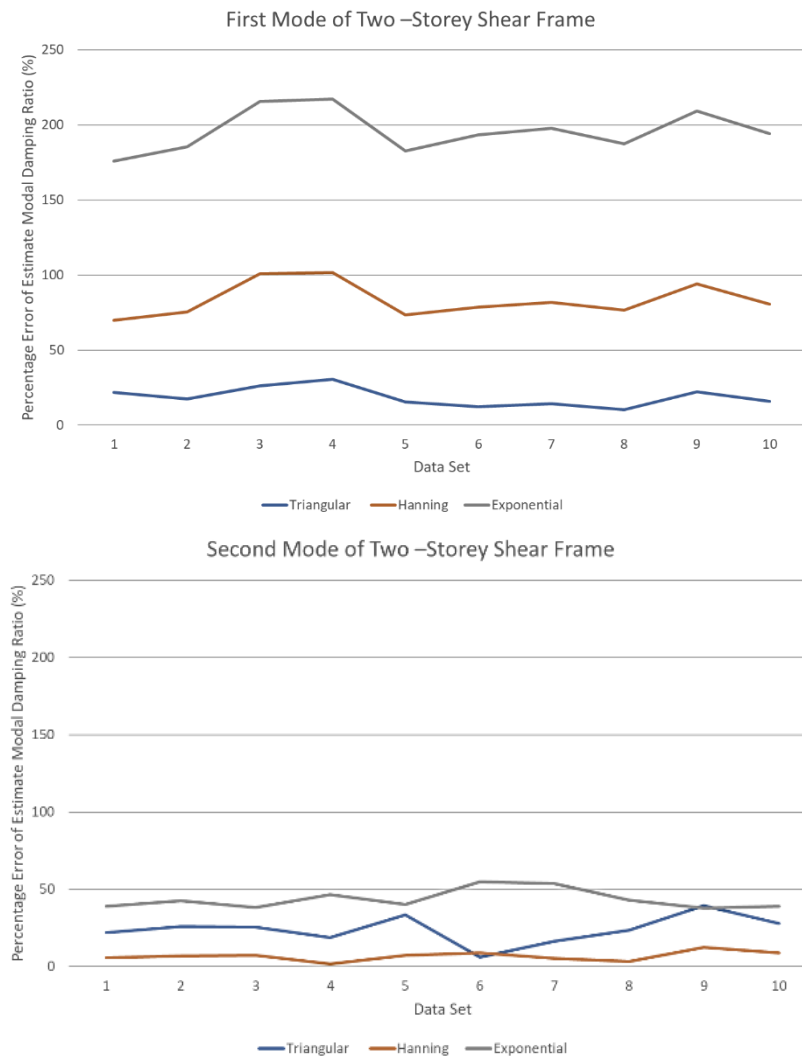


Figure 4. The percentage error (%) of modal damping ratio for the first and second mode of the two-story shear frame with the various application of tapering (windowing).

In the case of without any tapering (left side of Figure 3), there was significant side lobe noise on PSD which caused by the corrupted CF with noise tail.

Meanwhile, for application of flat-triangular window with $\alpha = 0.5$ as shown in the second bottom plot in Figure 3, it can slightly minimize the side lobe noise of the SD. Instead, this windowing function able to consistently reduce the percentage error of modal damping ratio for both modes of the two-story shear frame with the mean percentage errors is 18.6% mode and 23.8% for the first mode and second mode respectively compared to other application of windowing.

Additionally, the adoption of the classic exponential window can minimize the side noise significantly by 5% of the initial value at the boundaries, but it will lead to a clear appearance of bias at the spectrum peaks. This can clearly see on the first and the second mode with the mean percentage error is 195.8 % and 43.3 % respectively. It was neither accurate nor precise.

Even though Hanning window with 50% overlap is a well-known among analysts especially when doing signal processing which can minimize leakage effects in spectrum estimation, but it will lead to bias error with respect to the actual damping value. This can be seen in the last plot in Figure 3. Besides that, the results of modal damping ratio were not consistent or precise, although the result for the second mode was quite accurate with 6.6% of the mean percentage error and 2.96% for standard deviation error, however for the first mode produced higher error with the mean percentage error up to 83.2 % and 11.4% for standard deviation error.

4. Correlation Function and Spectral Density Estimate

The computation of modal damping ratios relies heavily on the singular value plot, obtained from singular value decomposition (SVD) of the estimated SD. Figure 5 below depicted the various approaches for of singular values estimation.

The top plot of Figure 5 showed the SD which estimated using a direct estimation of the CF matrix with the application of the flat-triangular window ($\alpha = 0.5$) provided the significant reduction of mean percentage error for the first and second mode which were 18.6% and 23.8% respectively. The results of this approach showed a quite consistent and close to the true value of assigned modal damping ratio compared to other approaches.

The second plot from the top of Figure 5 is the traditional Welch approach with a Hanning window and 50% overlap, that involved signal sectioning, windowing, and overlapping. The results showed inconsistent for both modes. The first mode yielded higher error which is 63.1%, meanwhile, for the second mode, the result was opposite with the mean percentage error only 10.7% and with 4.9 % of standard deviation error. It proved that the application of signal sectioning, windowing, and overlapping occasionally may produce noisier SVs. Thus, it needs some additional process in order to enhance the quality of the results. Even though, it has a low computational effort.

Besides that, the third plot from the top of Figure 5 is the RD estimate. This approach has shown it neglects the need for windowing due to the natural decay of the RD functions. Thus, there was significant side lobe noise on PSD which was due to the corrupted CF with noise tail and yields 72.4% and 62.7% of mean percentage errors for the first and second mode respectively. It should be noted that the triggering condition that used in RD function had influenced the initial value and affected the scale of the RD spectral estimate.

Finally, the half-spectrum was obtained based on the estimated zero-padded direct correlation function with a flat-triangular window and then converted by FFT. Meanwhile, for results of half spectrum as shown at the bottom plot of Figure 5 which based on the estimated zero-padded direct correlation function with a flat-triangular window ($\alpha = 0.5$) and then converted by FFT to estimate SD. It yielded a smoother and clearer result of the desired physical information compared to other approaches. However, the results also produced a large error relative to actual values of assigned modal damping with the mean percentage error up to 176.7% and 21.5% of standard deviation error for the first mode, following with 72.3% and 15% of mean percentage error and standard deviation error respectively for the second mode.

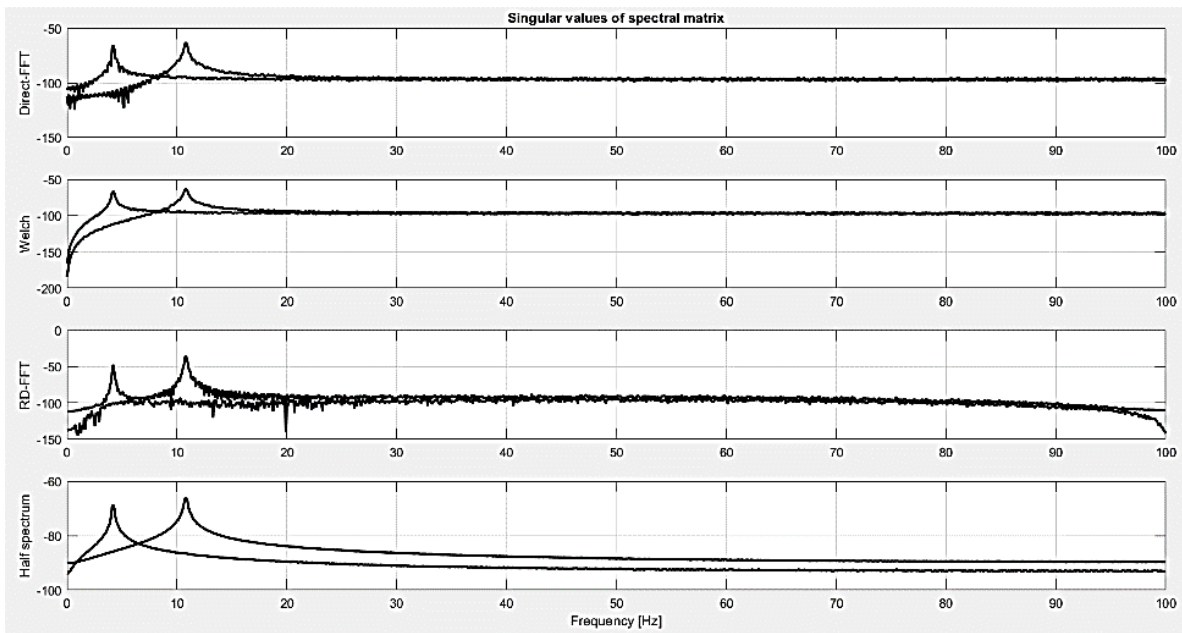
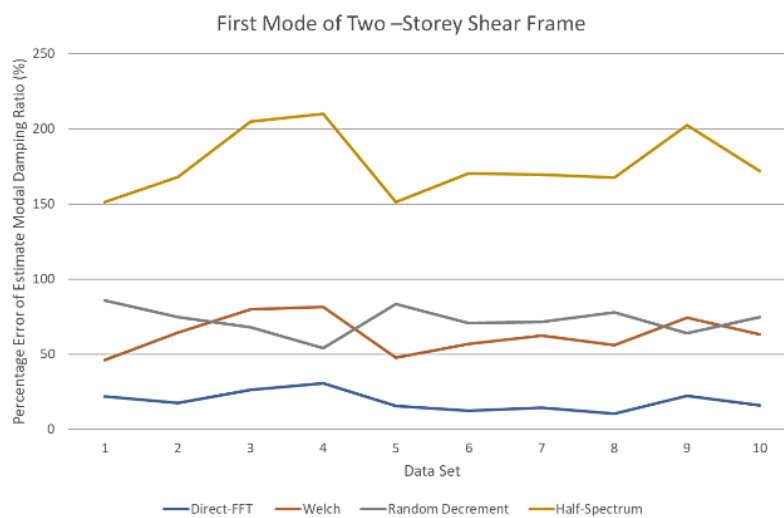


Figure 5. Singular values of the spectral density matrix with various signal processing approaches. From top plot is the Fourier transformed direct estimate of the correlation functions; the conventional Welch approach with a Hanning window and 50% overlap; following with the RD estimate; and finally, the half spectrum based on the zero-padded direct correlation function matrix estimate.



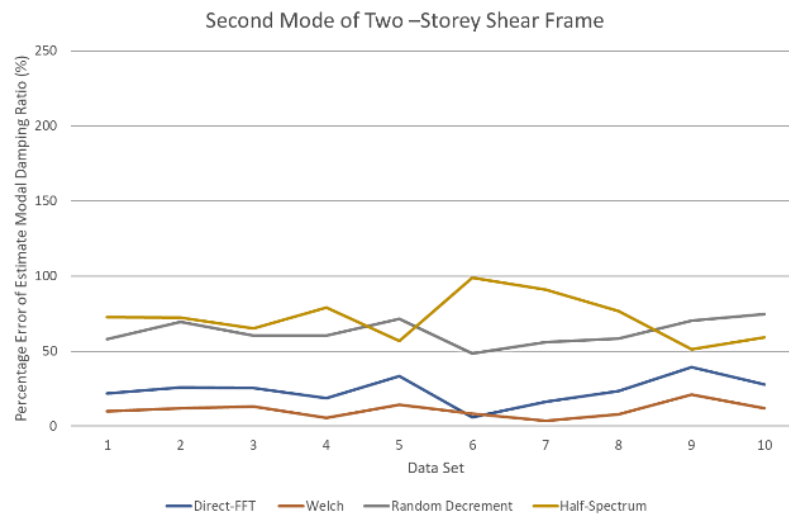


Figure 6. The percentage error (%) of modal damping ratio for the first and second mode of the two-story shear frame with a different application of signal processing.

5. Conclusion

The accurate estimation of modal damping ratio using frequency domain decomposition (FDD) is still an open problem, and often leading to bias estimates since the errors are primarily caused by signal processing [5]. The rapid development of FDD method proves the need for comparative studies, in order to find the best criteria for appropriate signal processing approaches such as the time window function, correlation function (CF) and the spectral density (SD) which are the main contributor to bias estimate. From this study, all necessary information about modal damping for a reliable estimation, reduce uncertainties and define error bounds also can be determined. Based on the results of the different application of tapering (windowing), the time window can illustrate the spectral peaks more clearly, instead, produces a clear bias of the spectral peaks. However, among this different application of time window, the flat-triangular window ($\alpha = 0.5$) depicted the significant reduction of mean percentage error for the first and second mode which are 18.6% and 23.8% respectively and showed a quite consistent and close to the true value of assigned modal damping ratio compare to other approaches. Meanwhile for different ways of estimating the SD, all approaches provided almost the similar information on spectral representations of the modes, however, the high suppression of the side lobe noise as in the half spectrum, will lead to the appearance of a clear bias at the spectral peaks since each modal peak corresponds to the damping. Thus, the application of direct FFT which involved direct estimation of the CF matrix with a flat-triangular window ($\alpha = 0.5$) turn out to be the best approach for signal processing with significantly reduce the mean percentage error for the first and second mode into 18.6% and 23.8% respectively and depicted quite consistent results. Further studies are required to enhance the exact computation of modal damping in FDD using necessary information provided by the current study.

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