

# SOME NUMERICAL METHODS ON TEMPERATURE AND MASS TRANSFER SIMULATION ON DEHYDRATION OF TROPICAL FRUITS

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## Abstract

Advances in tropical fruit processing and technologies are very significant and efficient in order to reduce post-harvest losses. The dynamic mathematical model of simultaneous heat and moisture transfer is being established for the simulation of temperature and moisture distributions during dehydration potential of foodstuffs in a slab-shaped solid. Mathematical modeling presents the exchange of heat and mass transfer between material and drying air. Finite-difference method (FDM) based on Crank-Nicolson was used to discretise a partial differential equations (PDE) with parabolic type. This paper aims to illustrate the simulation of tropical fruits' dehydration through some numerical methods such as Jacobi, Gauss-Seidel and Red-Black Gauss-Seidel. This study focuses on the implementation of sequential algorithms on the simulation. 3D geometric visualization by COMSOL Multiphysics and graphical numerical results of FDM approximation in mass and heat transfer demonstrates the results of this research. Most thermal and moisture models are empirical rather than theoretical. However, the best sources of thermal and moisture data for the commercial tropical fruits dehydration derived from the mathematical model in order to avoid high prototype costs, improving inaccurate conditions and processing parameters with the advancement of dryer technologies. The contribution of this paper is to derive the mathematical simulation in representing the actual process of dehydration in foodstuff industry. The computational platform supported by Intel@Core™ based on Quadcore processors and configure with MATLAB software. The performance evaluations of numerical methods are presented in terms of execution time, computational complexity, number of iterations, errors, accuracy and convergence rate. As a conclusion, the temperature and mass transfer generates high significant impact to the efficiency of the dehydration process.

**Keywords:** Parabolic Equation, dehydration, tropical fruit, iterative methods

**Scope:** Applied Mathematics Engineering Mathematics

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## 1.0 Introduction

A majority of drying R&D efforts today is devoted to the food and agricultural sectors [1]. Demands for agricultural products like tropical fruits for healthy are on the rise as consumers is becoming more health conscious. Agricultural producers therefore have to be aware of advances in fruit processing and post harvest technologies in order to efficiently lower post-harvest losses. It is very necessary to dry the tropical fruits to a certain level after harvest. Nowadays, the use of technology has been a must in almost all of industries in Malaysia to improve their service and productivity in order to fulfill the customers' demands. Drying processes are widely used in food production especially fruits, but a scientific approach has not so widely been applied, so rather empiric rules are often used to set up industrial production, particularly in small-medium firms.

The main objective of food drying process is water removal up to particular moisture content in order to prevent food from microbial spoilage and deterioration reactions and to increase the product shelf life [2]. Drying is a process involving simultaneous heat and mass transfer phenomena. Formulation of adequate mathematical models to describe the transfer phenomena during dehydration fruit is very important to optimize the process-leading in improvement product quality and reduction process cost. It takes people to share information and make critical decision, all along a value chain of suppliers, partners and customers. Selecting an appropriate drying model to

evaluate the drying distribution is very important to get best prediction of drying kinetics. In the dehydration process, it is important to know the temperature and moisture distribution in the material and its change during dehydrating; the present mathematical model is able to predict this. The governing differential equations describing moisture content and temperature distribution were solved numerically. The mathematical model using PDE will be helpful to estimate the drying rate at any moisture condition of the material and to estimate drying time for particular task during dehydration of tropical fruits products. Computer program was developed to solve the set of finite difference equations.

Computer simulation is a powerful tool for achieving measurement of changes in temperature and moisture during dehydration process. Simulation results and information of drying kinetics of fruit material such as time-temperature-moisture content distributions, as well as theoretical approaches to moisture movement, is very essential for the prevention of quality degradation and for the achievement of fast and effective drying. Such information will be very useful to optimize production processes of tropical fruits dried. The increasing development of computer program had a great impact on the quality evaluation of agricultural products. Invention of the computer also influenced the field of numerical analysis, since now longer and more complicated calculations could be done. Therefore, the objectives of this paper are first, to simulate moisture and temperature versus time in produce dehydration of fruit products based on convection-diffusion model using partial difference equation (PDE). Second, to find solution of parabolic PDE using some methods and analyze the performance of each method presented in terms of temperature and moisture content values.

Most thermal property models are empirical rather than theoretical, i.e. they are based on statistical curve fitting rather than theoretical derivations involving mass and heat transfer analysis. In modeling, water is treated as a single, uniform component of the food product. It could be argued that the thermal properties of water in the food depend on how it is configured or “bound” within the product [3]. In fact of integration technologies in foodstuff production, it is necessary to simulate more often in order to control prototype costs and to simulate better in order to get process parameters with the needed accuracy. Therefore, there is a need for some modeling studies on heat and mass transfer aspects to improve drying conditions and process parameters [4]. The high contribution of this paper is successful modified the mathematical simulation in representing the actual process of dehydration in commercial foodstuff industry in terms of heat and mass transfer inside tropical fruits material.

For most engineering heat transfer calculations performed in commercial food dehydration, accuracy greater than 2-5 % is seldom needed [3]. This is because errors due to varying or inaccurately measured boundary conditions such as air temperature and velocity would overshadow errors caused by inaccurate thermal properties [3]. One way to get the sources of thermal property data are prediction and simulation equations based on temperature and physical structure (density, size and configuration of void spaces). The novelty of this paper is the thermal and moisture data properties from mathematical model can be the best sources for commercial tropical fruits dehydration. It can control prototype costs and improve drying conditions and process parameters as well as evolution of dryer technologies with needed accuracy.

## 2.0 Mathematical Modeling

Drying kinetics can be modeled mathematically based on Fick’s law of diffusion. The simulation of various product drying systems involves solving a set of heat and mass transfer equations. [5] described transport phenomena as follows:

- i. Heat transfer :
  - Convective heat (energy) transfer from the air to the food’s surface (external heat transfer)
  - Conductive heat transfer within the food (internal heat transfer) and air,
- ii. Moisture transfer :
  - Moisture transport within the food towards its external surface (internal mass transfer)

The first drying process is heat transfer to the product from the heating source and the second is mass transfer from the interior of the material to its surface and from the surface to the surrounding air. The heat and mass transfer equations are presented in the model below are governed by the following PDE. These models had been presented by [6]. To simplify the model, [6] were made the following assumptions:

- i. Moisture movement and heat transfer are one-dimensional.

- ii. No chemical reaction takes place during drying, i.e. thermal and chemical properties of material, air and moisture are constant within the range of temperatures considered.
- iii. The material undergoes shrinkage as drying progresses.
- iv. Uniform distribution of air through the dryer.

Mass transfer equation can be written as:

$$\frac{\partial M}{\partial t} + u \frac{\partial M}{\partial x} = D \frac{\partial^2 M}{\partial x^2} \quad (1)$$

and similarly, the heat transfer equation can be written as:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (2)$$

With the following initial (IC) and boundary conditions (BC) of equation (1) and (2) are given below:

IC:  $M|_{x=0} = M_0$ ,  $T|_{x=0} = T_0$

BC: At  $x=0$ ;  $\frac{\partial M}{\partial x}|_{x=0} = 0$  and at  $x=b$ , moisture balance is  $-D_{eff} \frac{\partial M}{\partial x}|_{x=b} + uM|_{x=b} = h_m(M - M_e)|_{x=b}$  (3)

and  $\frac{\partial T}{\partial x}|_{x=0} = 0$  and at  $x=b$ , heat balance is  $(k \frac{\partial T}{\partial x} - \rho_c p u T)|_{x=b} = h(T_a - T)|_{x=b} - h_m \rho(M - M_e) hfg|_{x=b}$  (4)

Notation			
$b$	half thickness of drying specimen (m)	$u$	shrinkage velocity (m/s)
$c_p$	specific heat (J/kgK)	$x$	distance from the centre of the drying specimen(m)
$D$	diffusion coefficient (m <sup>2</sup> /s)		
$h$	convective heat transfer coefficient (W/m <sup>2</sup> K)		
$hfg$	enthalpy of evaporation (J/kg)	<b>Subscripts</b>	
$h_m$	mass transfer coefficient (m/s)	0	initial, reference
$k$	thermal conductivity (W/m K)	a	air
$L$	thickness of the drying specimen (m)	e	equilibrium
$M$	moisture content of specimen (g/g dry)	eff	effective
$t$	time (s)	m	material (fruit sample)
$T$	temperature (°C)		

Table 1: Parameters of mass and heat transfer equation

### 3.0 Discretization of Mathematical Modeling

Discretization is transformation of differential equations into discrete difference equations for numerical computing. A discrete form can be found by using FDM. [7-12] were solved their dehydration model using FDM.

In this study, Crank-Nicolson scheme is used to describe the diffusion and convection terms for the equation (1) and (2). Subsequently, some iterative methods were applied to solve both the equations. In order to simulate the mass and heat transfer numerically, the Crank-Nicolson FDM method was applied to discretise the equation (1) and (2). [7,10,13] utilized implicit Crank-Nicolson scheme with the numerical procedure in their experiments. The drying material is considered as a thin slab of thickness,  $L(2b)$  at a uniform initial temperature,  $T_0$  and moisture content,  $M_0$  according to [10].

For this method, the simulated material (total thickness,  $L$ ) is sliced up into  $N_i$  samples and the times (total duration  $T$ ) are split into  $N_j$  samples:  $h=\Delta x = b/N_i$  and  $k=\Delta t = T/N_j$ . According to Crank-Nicolson schemes (where  $i$ ,