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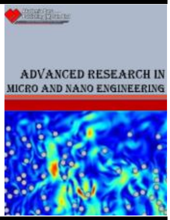

Numerical Solutions of Mixed Convection Flow Past a Horizontal Circular Cylinder with Viscous Dissipation in Viscoelastic Nanofluid

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ABSTRACT

The aim of the present study is to investigate numerically the impact of viscous dissipation on steady two-dimensional mixed convection flow past a horizontal circular cylinder in a viscoelastic nanofluid with convective boundary conditions. The Tiwari and Das model were selected in this study by choosing Carboxymethyl cellulose solution (CMC-water) as the base of fluid and copper (Cu) as the nanoparticle. The transformed boundary layer equations for momentum and energy subject to the appropriate boundary conditions were numerically solved by employing numerical scheme, namely Keller-box method. The accuracy of the present results was validated through comparison with previously published results and revealed an excellent agreement with those results. The results were analysed in detail and presented graphically for the velocity, temperature, skin friction coefficient as well as the heat transfer coefficient. The obtained results indicated that there was no significant effect for velocity and temperature profiles when values of Eckert number increased. However, it is significant for skin friction and heat transfer coefficient profiles. In the meantime, thermal conductivity of the fluid may increase by increasing the concentration of nanofluid.

Keywords:
viscous dissipation; viscoelastic; nanofluid

1. Introduction

Heat transfer system is a crucial part in most of industrial applications such as automotive, manufacturing, maintenance and heat exchangers. Therefore, it is a challenges for the researchers to find and figure out the optimum heat transfer in order to get optimum outcomes for the industries. There are several properties that measure the thermal performance which are thermal conductivity, viscosity, density and specific heat. However, a lower value of thermal conductivity may affect poor in heat transfer systems and become a major issue.

Hence, a new technology is developed to overcome this issue by proposing that nanometer-sized particles that suspended in heat transfer fluids such as water, ethylene glycol, or oil. This new technology is called nanofluid. The earlier attempt who discovered about this nanofluid was Choi [1], who was investigated thermal conductivity enhancement. Later, a series of research by experiment raised such as Das *et al.,* [2], Jang and Choi [3], Murshed *et al.,* [4] and Zhu *et al.,* [5] in order to proof

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the claimed by Choi. For a theoretical part, the first attempt who discovered the nanofluid equation was by Khanafer *et al.,* [6]. Consequence from that, a numerous theoretical study of nanofluid has received considerable attention from other researchers such as Buongiorno [7], Tiwari and Das [8] which leads to Buongiorno and Tiwari and Das model, respectively. Then, the research on nanofluid becomes tremendous among researchers such as [9-12].

In a meantime, non-Newtonian fluid has become more considerable among researchers since most of the fluids in real life are non-Newtonian behaviour such as blood, honey, oil and ketchup. Non-Newtonian fluids can be divided into rheopectic fluids, thixotropic fluids, dilatant fluids, viscoelastic fluids and pseudoplastic fluids. However, only viscoelastic fluids will be focused on in this research. The viscoelastic fluids are a type of non-Newtonian fluid that exhibit both viscous and elastic characteristics. The pioneering research about viscoelastic was proposed by Rivlin [13] which considered the stress deformation relation for isotropic. Later, Min *et al.,* [14] have extended this work by studying the viscoelastic response to small deformations superposed on a large stretch. The purpose of the extended study was to provide the general theoretical framework for the organization of data from such experiments. Besides that, Co and Bird [15] also studied about viscoelastic fluid, and they described that the fluid does not move far or very rapidly from its initial configuration. Later on, an investigation on the flow of elastic-viscous fluids past a circular cylinder was done by Harnoy [16]. After that, the study on viscoelastic was carried out widely by the researchers since viscoelastic fluids have gained considerable importance because of its applications in various branches of science, engineering, and technology such as in chemical and nuclear industries, geophysics, material processing and bio-engineering [17].

In all above-mentioned investigations, the viscous dissipation effect on boundary layer flow was not extensively studied. The viscous dissipation is appreciable when the induced kinetic energy becomes significant as compared to the amount of heat transferred according to Gebhart [18], the first researcher who studied the viscous dissipation in free convection flow. It is known that the viscous dissipation model for a Newtonian fluid is quite different from the non-Newtonian fluid. Previous studies have established the viscous dissipation effect in a viscoelastic fluid with somewhat different mathematical modelling, such as studies conducted by Metri *et al.,* [19], and Abel *et al.,* [20]. In the latest study regarding the non-Newtonian fluids, Dalir [21] considered the numerical study of entropy generation for the forced convection flow and heat transfer of Jeffrey fluid over a stretching sheet. In addition, the viscous dissipation effect generated by the frictional force was comprehensively explored by Zokri *et al.,* [22-23].

It could be observed that limited attention was provided to the flow of viscoelastic nanofluid past a horizontal circular cylinder in previous studies. In this study, the effect of viscous dissipation with convective boundary condition was evaluated. The convective boundary condition is known as the supply of heat through a bounding surface of finite thickness and finite capacity [22]. A comprehensive study on the horizontal circular cylinder with a convective boundary condition has been carried out by El-Amin [24], and Mohamed *et al.,* [25-26]. In the next section, the mathematical formulation has been briefly reviewed.

2. Thermophysical Properties and Preparation of Nanofluid

For an experimental guideline, here are some simple steps about the preparations of nanofluid. A proper mixing sand stabilization are required in order to prepare the nanofluid by dispersing the nanoparticles in a base fluid. The nanoparticles are assumed to have a uniform shape and size as listed in Table 1. The mixture of the based fluid and nanoparticles has an assumption of incompressible and no chemical reaction of heat transfer occurs. The idealized of the mixtures are when the thermal is in equilibrium state and they flow at the same velocity. Nanoparticles volume fraction is the factor that affecting heat transfer in this model. As the nanoparticles volume fraction increases, the effective thermal conductivity of nanofluid is also increase [27]. However, it is worthy to mentioned that by increasing the nanoparticles volume fraction, it may no longer be in a state of suspended between each other. For ensuring the effectiveness, only small quantities of volume fraction are necessary [28]. Therefore, the nanofluid are chosen at volume fractions up to 3% and are well-dispersed in CMC-water as a base fluid. Cu nanoparticles have a spherical shape and their size diameters have a normal distribution in a range from 63 to 100 nm.

Table 1
Thermophysical properties of nanoparticles and base fluid

Physical Properties	$\rho\left(\mathrm{kg}\, m^{-3}\right)$	$C_p\left(\mathrm{J\, kg^{-1}K^{-1}}\right)$	$k\left(\mathrm{Wm^{-1}K^{-1}}\right)$	$\beta\times 10^5\left(\mathrm{K^{-1}}\right)$
Base Fluid (CMC)	997.1	4179	0.613	21
Nanoparticle (Cu)	8933	385	401	1.67

3. Mathematical Formulation

The steady two dimensional mixed convection boundary layer flow past a horizontal circular cylinder of radius *a* placed in a viscoelastic nanofluid has been considered in this study. The Cartesian coordinate (x, y) is chosen and the dimensional gravitational acceleration is defined as $g_x = g\sin\left(\bar{x}/a\right)$, where $\bar{x}$ is the distance from the lower stagnation point. The dimensional velocity outside the boundary layer is $\bar{u}_e\left(\bar{x}\right) = U_\infty \sin\left(\bar{x}/a\right)$ by assuming that the constant free stream velocity is $\left(1/2\right)U_\infty$, which is flowing vertically upwards past the cylinder [29]. The temperature of the ambient nanofluid is T_∞. Figure 1 shows three-dimensional model on the flow of viscoelastic nanofluid past a horizontal circular cylinder with radius a. The surface of the cylinder is considered as convective boundary condition (CBC).

Tiwari and Das model [8] has been chosen in this study and the model is defined as a single-phase model that use Brickman viscosity model. The nanoparticles are assumed to have a uniform shape and size. The mixture of the based fluid and nanoparticles has an assumption of incompressible and no chemical reaction of heat transfer occurs. The idealized of the mixtures are when the thermal is in equilibrium state and they flow at the same velocity. Nanoparticles volume fraction is the factor that affecting heat transfer in this model. As the nanoparticles volume fraction increases, the effective thermal conductivity of nanofluid is also increase [27]. However, it is worthy to mentioned that by increasing the nanoparticles volume fraction, it may no longer be in a state of suspended between each other. For ensuring the effectiveness, only small quantities of volume fraction are necessary [28]. Therefore, the nanofluid are chosen at volume fractions up to 3% to meet the requirement and are well-dispersed in CMC-water as a base fluid.

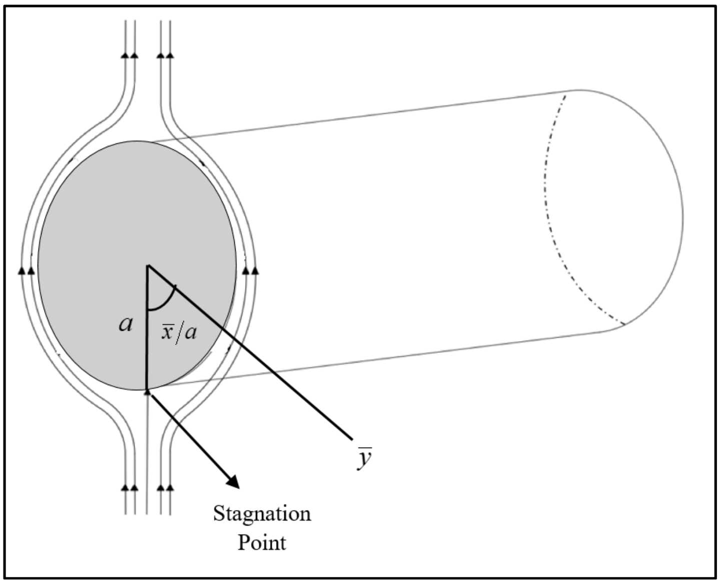


Fig. 1. Three-dimensional model on the flow of viscoelastic nanofluid past a horizontal circular cylinder with radius a

Under the above assumptions and by considering the nanofluid model, the dimensional governing equations of momentum equation and energy equation can be expressed as

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\bar{u}_e\frac{\partial \bar{u}_e}{\partial \bar{x}}+\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}-\frac{k_o}{\rho_{nf}}\left[\frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)+\bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}-\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right]+g\beta_{nf}\left(T-T_\infty\right)\sin\left(\frac{\bar{x}}{a}\right), \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \bar{y}^2}+\frac{\mu_o}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2-\frac{k_o}{\rho C_p}\left[\bar{u}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right)+\bar{v}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)\right], \tag{3}$$

with the boundary conditions

$$\begin{aligned}&\bar{u}=0,\ \bar{v}=0,\ -k_{nf}\frac{\partial T}{\partial \bar{y}}=h_f\left(T_f-T\right) && \text{at} \quad \bar{y}=0,\ \bar{x}\ge 0,\\ &\bar{u}=\bar{u}_e\left(\bar{x}\right),\quad \frac{\partial \bar{u}}{\partial \bar{y}}=0,\quad T=T_\infty && \text{at} \quad \bar{y}\to\infty,\ \bar{x}\ge 0,\end{aligned} \tag{4}$$

where k_{nf} is the thermal conductivity of nanofluid, *h* is the convection heat transfer coefficient, *T* is the fluid temperature, q_w is the constant heat flux and T_f is the temperature when the bottom surface of the cylinder is heated by convection from a hot fluid. The dimensionless variables are introduced to simplify the complexity of the governing equations. Based on Anwar *et al.,* [30], the dimensionless variables are defined as

$$x=\bar{x}/a,\ y=\mathrm{Re}^{1/2}\left(\bar{y}/a\right),\ u=\bar{u}/U_\infty,\ v=\mathrm{Re}^{1/2}\left(\bar{v}/U_\infty\right),$$
$$u_e\left(x\right)=\bar{u}_e\left(\bar{x}\right)/U_\infty,\ \theta=\left(T-T_\infty\right)/\left(T_f-T_\infty\right), \tag{5}$$

where *Re* is Reynolds number. By substituting Eq. (5) into Eqs. (1) - (3), the dimensionless system below is yielded

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$\left[\left(1-\phi\right)+\phi\frac{\rho_s}{\rho_f}\right]\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=\left[\left(1-\phi\right)+\phi\frac{\rho_s}{\rho_f}\right]\sin x\cos x+\frac{1}{\left(1+\phi\right)^{2.5}}\frac{\partial^2 u}{\partial y^2}$$
$$-K\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right]+\left[\left(1-\phi\right)+\phi\frac{\left(\rho\beta\right)_s}{\left(\rho\beta\right)_f}\right]\lambda\theta\sin\left(x\right), \tag{7}$$

$$\left[\left(1-\phi\right)+\phi\frac{\left(\rho C_p\right)_s}{\left(\rho C_p\right)_f}\right]\left[u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right]=\frac{\left(k_s+2k_f\right)-2\phi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\phi\left(k_f-k_s\right)}\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}$$
$$+Ec\left[\left(\frac{\partial u}{\partial y}\right)^2-K\left(u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right)\right], \tag{8}$$

with the new boundary conditions as

$$u=0,\quad v=0,\quad \frac{\partial\theta}{\partial y}=-\gamma_1\left(1-\theta\right),\quad \text{at}\quad y=0,\ x\geq 0,$$
$$u=u_e\left(x\right),\quad \frac{\partial u}{\partial y}=0,\quad \theta=0,\quad \text{as}\quad y\to\infty,\ x\geq 0, \tag{9}$$

where $\mathrm{Pr}=\mu_f C_p/k_f$ is Prandtl number, $Ec=U_\infty^2\Big/\left(\left(C_p\right)_f\left(T_f-T_\infty\right)\right)$ is Eckert number $K=k_o U_\infty/\mu_f a$ is viscoelastic parameter, γ_1 is Biot number and λ is mixed convection parameter.

In order to solve Eqs. (6) to (8), subject to the boundary conditions in Eq. (9), the following variables have been considered

$$\psi=xF\left(x,y\right),\qquad \theta=\theta\left(x,y\right), \tag{10}$$

are introduced where ψ is the stream function defined as

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x}. \tag{11}$$

By substituting Eqs. (10) and (11) into Eqs. (6) to (8), obtained

$$\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[\left(\frac{\partial F}{\partial y}\right)^2+x\frac{\partial F}{\partial y}\left(\frac{\partial^2 F}{\partial x\partial y}\right)-x\frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial y^2}-F\frac{\partial^2 F}{\partial y^2}\right]$$
$$=\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\frac{\sin x\cos x}{x}+\frac{1}{(1+\phi)^{2.5}}\frac{\partial^3 F}{\partial y^3}+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta\frac{\sin x}{x} \quad (12)$$
$$+K\left[2\frac{\partial F}{\partial y}\frac{\partial^3 F}{\partial y^3}-F\frac{\partial^4 F}{\partial y^4}-\left(\frac{\partial^2 F}{\partial y^2}\right)^2+x\left(\frac{\partial^2 F}{\partial x\partial y}\frac{\partial^3 F}{\partial y^3}-\frac{\partial F}{\partial x}\frac{\partial^4 F}{\partial y^4}+\frac{\partial F}{\partial y}\frac{\partial^4 F}{\partial x\partial y^3}-\frac{\partial^2 F}{\partial y^2}\frac{\partial^3 F}{\partial x\partial y^2}\right)\right],$$

$$\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]F\frac{\partial\theta}{\partial y}$$
$$=x\left[\begin{array}{l}(1-\phi)+\phi\dfrac{(\rho C_p)_s}{(\rho C_p)_f}\left(\dfrac{\partial F}{\partial y}\dfrac{\partial\theta}{\partial x}-\dfrac{\partial F}{\partial x}\dfrac{\partial\theta}{\partial y}\right)\\ -Ecx\left[\left(\dfrac{\partial^2 F}{\partial y^2}\right)^2+K\left(\begin{array}{l}x\dfrac{\partial F}{\partial y}\dfrac{\partial^2 F}{\partial y^2}\dfrac{\partial^3 F}{\partial x\partial y^2}+\dfrac{\partial F}{\partial y}\left(\dfrac{\partial^2 F}{\partial y^2}\right)^2\\ -x\dfrac{\partial F}{\partial x}\dfrac{\partial^2 F}{\partial y^2}\dfrac{\partial^3 F}{\partial y^3}-F\dfrac{\partial^2 F}{\partial y^2}\dfrac{\partial^3 F}{\partial y^3}\end{array}\right)\right]\end{array}\right], \quad (13)$$

which are subject to the following boundary conditions

$$F=0,\quad \frac{\partial F}{\partial y}=0,\quad \frac{\partial\theta}{\partial y}=-\gamma_1(1-\theta),\qquad \text{at } y=0,\ x\ge 0,$$

$$\frac{\partial F}{\partial y}=\frac{\sin x}{x},\quad \frac{\partial^2 F}{\partial y^2}=0,\quad \theta=0,\qquad \text{as } y\to\infty,\ x\ge 0, \quad (14)$$

When $x\approx 0$, Eqs. (12) and (13) reduce to the following ordinary differential equations:

$$\frac{1}{(1+\phi)^{2.5}}f'''-\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right]\left[f'^2-ff''\right]+K\left(2ff'''-ff^{iv}-f''^2\right)$$
$$+\left[(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\lambda\theta=0, \quad (15)$$

$$\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)}\frac{1}{\Pr}\theta''+\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]f\theta'=0, \quad (16)$$

with the boundary conditions

$$\begin{aligned} &f(0)=0, \qquad f'(0)=0, \qquad \theta'(0)=-\gamma_1\left(1-\theta(0)\right), \\ &f'(\infty)=1, \qquad f''(\infty)=0, \qquad \theta(\infty)=0. \end{aligned} \tag{17}$$

The physical quantities of principal interest in this problem are the skin friction coefficient C_f and heat transfer coefficient $\theta_w(x)$. We define these coefficients in non-dimensional form as

$$C_f = \mathrm{Re}^{1/2}\frac{\tau_w}{\rho U_\infty^2}, \qquad \theta_w(x) = \mathrm{Re}^{-1/2}\frac{a q_w}{k\left(T_w - T_\infty\right)}, \tag{18}$$

where *k* is the thermal conductivity of the viscoelastic fluid. From Jaluria [31], the skin friction τ_w and the heat flux from the surface q_w in *x*-direction are defined as

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} + k_o\left(u\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^2 u}{\partial^2 y} + 2\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right)\right)_{y=0}, \; q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{19}$$

Using Eqs. (18) and (19), we obtain

$$C_f(x) = \frac{1}{(1-\phi)^{2.5}}\, x\left(\frac{\partial^2 F}{\partial y^2}\right)_{y=0}, \qquad \theta_w(x) = -\frac{k_{nf}}{k_f}\left(\frac{\partial \theta}{\partial y}\right). \tag{20}$$

4. Results and Discussion

The system Eqs. (12) -(13) and (15) -(16), together with the boundary conditions in Eqs. (14) and (17) respectively, are solved numerically using an implicit finite-difference method known as the Keller-box method. The method has been discussed by Cebeci and Bradshaw [32] and has been found to be particularly accurate for nonlinear problems. The model has been solved in two types of equations where in Eqs. (12) -(13) are in the form of PDE (full equation) and Eqs. (15) -(16) are in the form of ODE (stagnation point) corresponding to boundary conditions from Eqs. (14) and (17), respectively. The present results for the heat transfer coefficients of the cylinder for CBC is compared with those in Merkin [29] and Rashad *et al.,* [33], as shown in Table 2 when the viscous dissipation effect and viscoelastic is neglected. The results are found to be in excellent agreement. This supports the validity of the other graphical results for dimensionless velocity and temperature profiles, as well as skin friction and heat transfer coefficients.

Figure 2 shows the comparison of heat transfer coefficient with different values of nanoparticle volume fractions ϕ. From that figure, it can be seen that the heat transfer coefficient of the fluid is higher at stagnation point, $x = 0^o$, and slowly reduced as the fluid past the circular cylinder at $x = 50^o$. This is because of the source of heat at the boundary conditions. The heat transfer coefficient increase slowly from $\phi = 0$ to $\phi = 0.02$ and rapidly from $\phi = 0.02$ to $\phi = 0.03$. This is probably because of the high thermal conductivity of the fluid when the concentration of nanoparticles volume fraction increase.

Table 2
Comparison values of heat transfer coefficient when $K=0$, $\Pr=1$, $\phi=0$, $Ec=0$, $\gamma_1=1000$ and different values of λ

λ	Merkin (1977) (Newton-Raphson Method)	Rashad *et al.,* (2013) (Tri-diagonal Method)	Present (Keller-box Method)
-1.75	0.4199	0.4202	0.419804
-1.5	0.4576	0.4579	0.457196
-1.0	0.5067	0.5068	0.506451
-0.5	0.5420	0.5421	0.541784
0.0	0.5705	0.5706	0.570141
0.5	0.5943	0.5947	0.594164
0.88	0.6096	0.6111	0.610363
0.89	0.6110	0.6114	0.610770
1.0	0.6158	0.6160	0.615180
2.0	0.6497	0.6518	0.651019

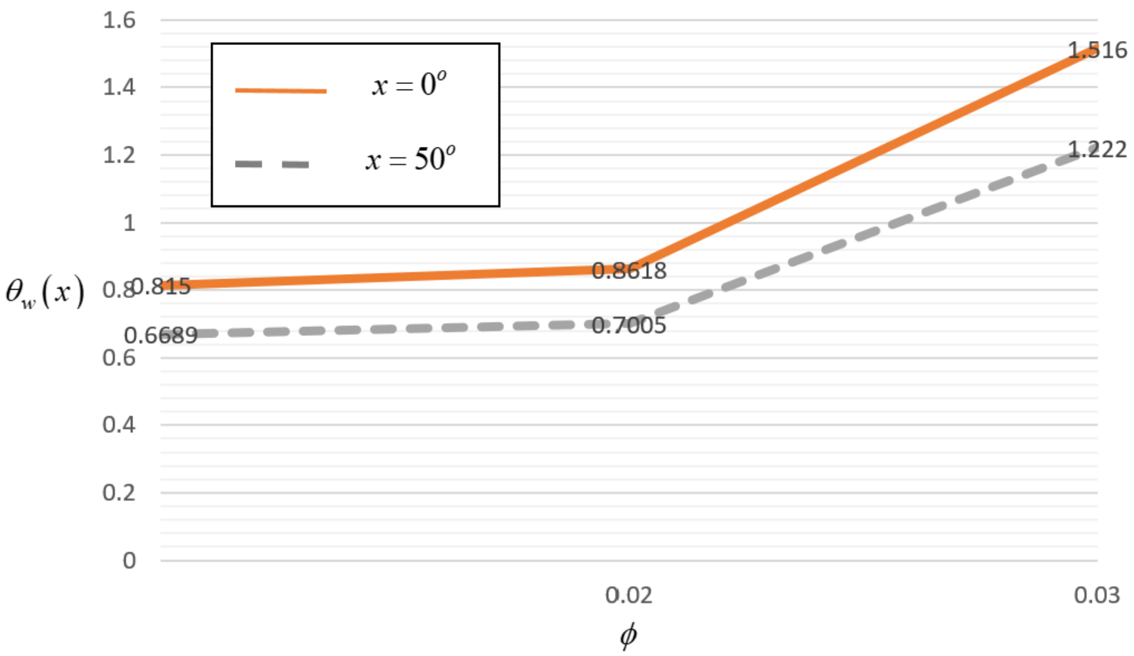
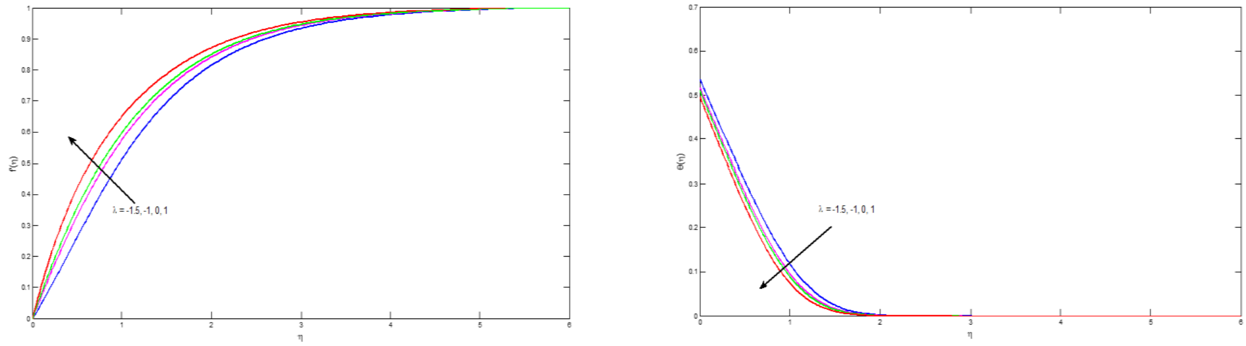


Figure 2. Comparison of heat transfer coefficients with different values of nanoparticles volume fraction when $x=0^o$ and $x=50^o$

Velocity profiles for viscoelastic nanofluid as well as temperature profiles for the variation values of λ, Ec, ϕ, γ_1 and *K* are presented graphically in Figures 3 to 7. Figure 3 depicts the effect of mixed convection parameter λ on the velocity and temperature profiles, respectively for the fixed values of $K=1$, $\phi=0.03$ and $Ec=0.2$. From this figure, it is predicted that the velocity profile tends to increase, and the boundary layer becomes thinner. This results of λ shows that the buoyancy effects help the fluid accelerates and consequently leads to an increase in velocity profiles. The temperature profile decreases as λ increases, where λ represents the buoyancy effect. This is because as λ increases, the convection cooling effect increases and reducing the temperature of the fluid. Besides that, the buoyancy force is more effective than the viscous force. Therefore, the temperature profile is reduced.

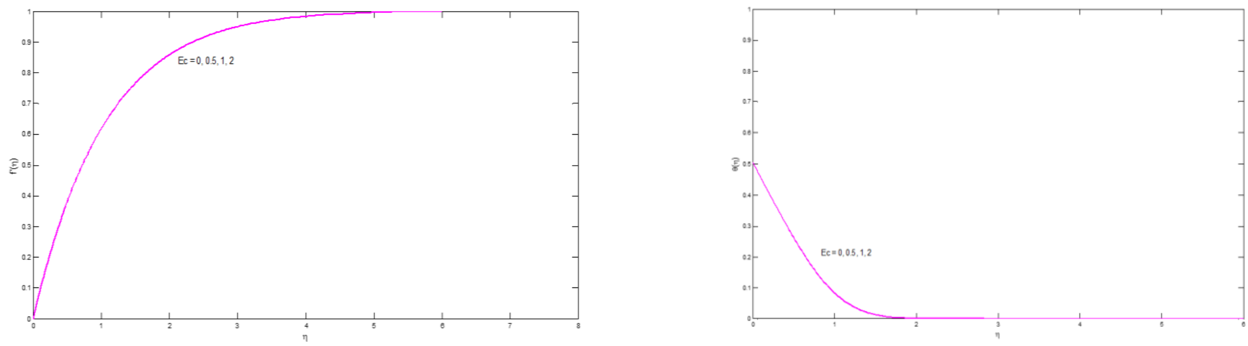


(a) (b)

Fig. 3. Effect of λ on (a) velocity and (b) temperature profiles when $K=\gamma_1=1,\ Ec=0.2,\ \phi=0.03,\ \Pr=6.2$

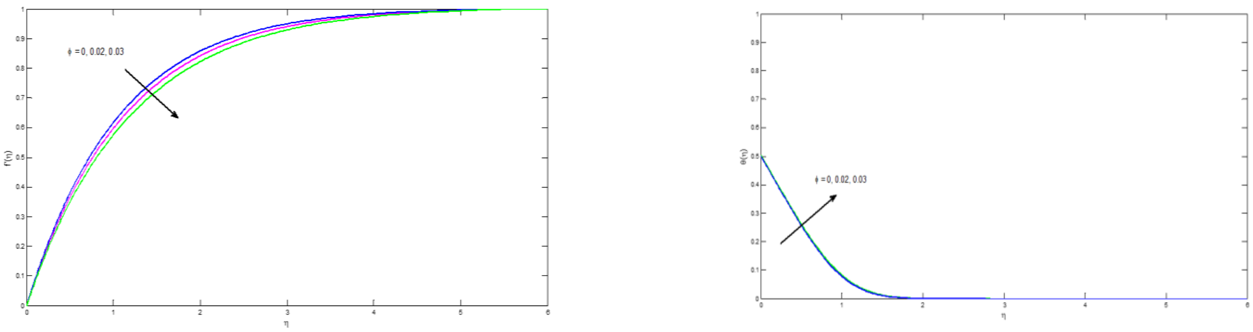
Figure 4 depicts the variation in velocity and temperature profiles due to increment in Ec. It is observed that there is no effect in the increment of Ec for both distributions. Referring to the energy Eq. (8), it is interesting to remark that both velocity and temperature profiles do not pronounce any effect on the Ec at the lower stagnation point of the cylinder because at this point, the velocity of the fluid is zero. Basically, Eckert number represent the kinetic energy of the flow and this effect is significant for high acceleration of the fluid flow. A similar result is shown in Zokri *et al.,* [34].

Figure 5 shows the effect of nanoparticles volume fraction, ϕ on velocity and temperature profiles with $K=\lambda=1$ and $Ec=0.2$. As presented in Figure 5(a), it is noticed that when the nanoparticles volume fraction increases from 0 to 0.03, the velocity profiles decrease while temperature profiles increases. This is due to the addition of the nanoparticles or concentration in the base fluid that makes the fluid more viscous, thus slowing down the fluid flow. Figure 5(b) also shows that, the thermal boundary layer gradually increases with ϕ. This behavior agrees with the physical expectation, by which the increase of ϕ leads to the enhancement of thermal conductivity of the fluid, thus causing an increase in the fluid temperature.



(a) (b)

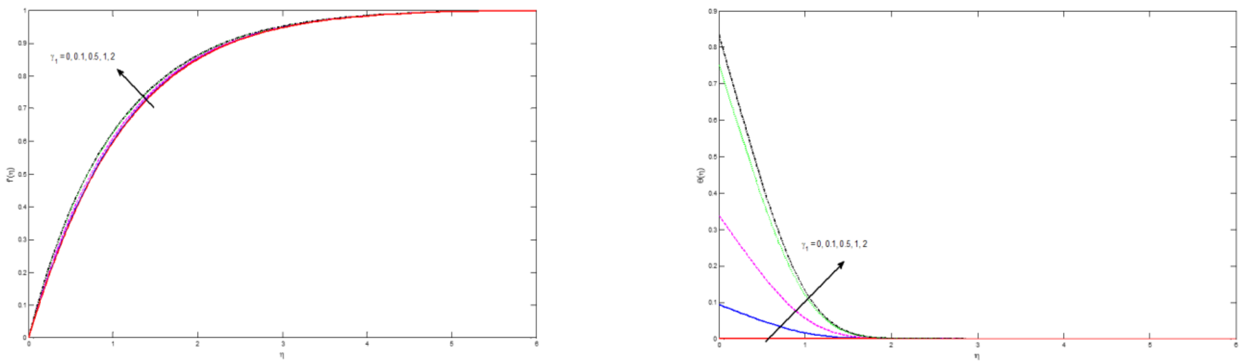
Fig. 4. Effect of ϕ on (a) velocity and (b) temperature profiles when $K=\lambda=\gamma_1=1,\ Ec=0.2,\ \Pr=6.2$



(a) (b)

Fig. 5. Effect of ϕ on (a) velocity and (b) temperature profiles when $K = \lambda = \gamma_1 = 1,\ Ec = 0.2,\ \Pr = 6.2$

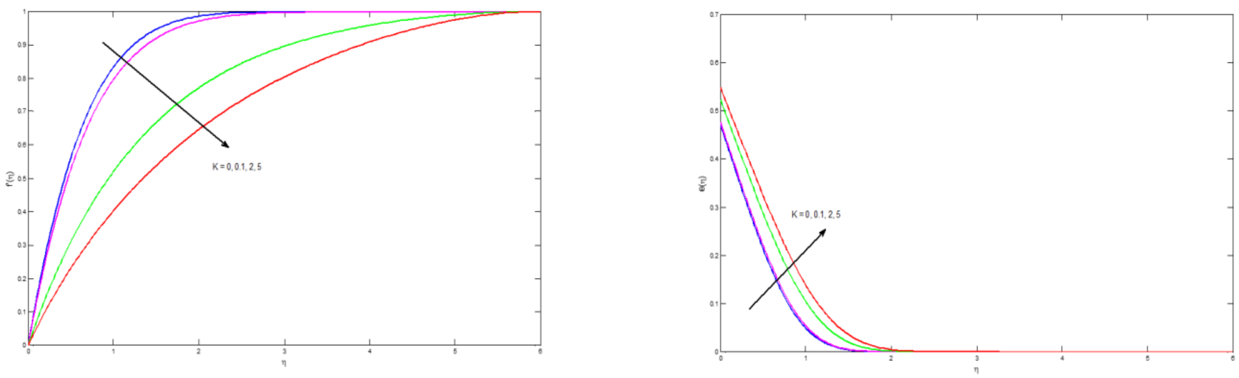
Figure 6 presents the effect of Biot number, γ_1 on velocity and temperature profiles, respectively. This figure illustrates that the increase in the value of Biot number γ_1 causes the increase in both the velocity and temperature profiles. This is supported by the fact that a high value of Biot number produces strong surface convection which in turn supplies more heat to the cylinder surface, which stimulated the isothermal surface and increases the temperature to the maximum. However, the temperature in uniform state when $\gamma_1 = 0$. Therefore, there will be a very less time for heat to transfer as shown in Figure 6(b) since there is no temperature gradient occurred.



(a) (b)

Fig. 6. Effect of γ_1 on (a) velocity and (b) temperature profiles when $K = \lambda = 1,\ Ec = 0.2,\ \phi = 0.03,$ $\Pr = 6.2$

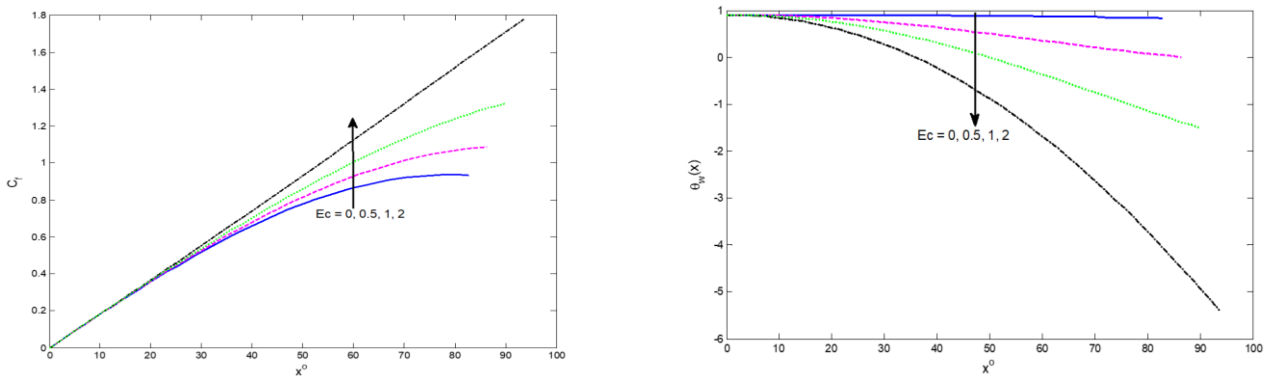
The effects of viscoelastic parameter, *K* that acts on the fluid, where the graphs of velocity and temperature profiles are plotted in Figure 7. The profiles of velocity decreases while the temperature increases with the increase in viscoelastic parameter. From the temperature profiles, an increase in the value of viscoelastic parameter leads to the increment in temperature distribution. This happens because of the properties of viscoelasticity that show both viscous and elastic characteristics. The velocity is decreases when *K* increase because of the viscosity property where fluid with higher viscosity resists motion. Therefore, the temperature profile increases as *K* increase as shown in Figure 7(b).



(a) (b)

Fig. 7. Effect of k on (a) velocity and (b) temperature profiles when $\lambda = \gamma_1 = 1,\ Ec = 0.2,\ \phi = 0.03,$ $\Pr = 6.2$

Typical variations of skin friction and heat transfer coefficients for various values of Eckert number, *Ec* are depicted in Figure 8. Skin friction coefficient increases with the increase of Eckert number. From Figures 8(a) and 8(b), the graphs show that there is a unique solution at the lower stagnation point of the cylinder and starting from $\left(x \geq 10^0\right)$, the graphs increase gradually for skin friction coefficients. This is because when $\left(x \approx 0^0\right)$, the velocity of the fluid is zero. Basically, Eckert number represent the kinetic energy of the flow and this effect is significant for high acceleration of the fluid flow. Figure 8(b) depicts the heat transfer behavior with the Eckert number, and it is observed with an increase in the Eckert number, it leads to decrease in the rate of heat transfer. This happens because of the convection process in convective boundary condition case, which implies the temperature slowly decrease to the surrounding temperature.



(a) (b)

Fig. 8. Effect of Ec on (a) skin friction and (b) heat transfer coefficients when $k = \lambda = \gamma_1 = 1,\ \phi = 0.03,$ $\Pr = 6.2$

5. Conclusions

The study on the problem of mixed convection flow of viscoelastic nanofluid with the additional effect which is viscous dissipation has been carried out and investigated in paper. Convective boundary condition has been considered as well with the effects of Tiwari and Das for the nanofluid. The obtained transformed equations were solved numerically by Keller-box method. Validation with the previously published data has been done and come out with an excellent agreement. Graphical results for velocity, temperature and nanoparticles volume fraction has been obtained. It was found that there is no changes in the increment of Ec for both velocity and temperature profiles at the lower stagnation point of the cylinder because at this point, the velocity of the fluid is zero and there is no such a kinetic energy. However, as the fluid past the circular cylinder, the effect of Eckert number is more significant since there is an increment for skin friction and decrease for heat transfer coefficient. This happens because of the high acceleration of the fluid flow and convection process in convective boundary condition, which implies the temperature slowly decrease to the surrounding temperature and decrease the heat transfer rate. Besides that, the heat transfer coefficient is more efficient and significant at the lower stagnation point because of the main source of heat at the boundary conditions.

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