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To cite this article: A.M. Wahab *et al* 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **241** 012002

View the article online for updates and enhancements.

Parametric Instability of Static Shafts-Disk System Using Finite Element Method

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Abstract. Parametric instability condition is an important consideration in design process as it can cause failure in machine elements. In this study, parametric instability behaviour was studied for a simple shaft and disk system that was subjected to axial load under pinned-pinned boundary condition. The shaft was modelled based on the Nelson's beam model, which considered translational and rotary inertias, transverse shear deformation and torsional effect. The Floquet's method was used to estimate the solution for Mathieu equation. Finite element codes were developed using MATLAB to establish the instability chart. The effect of additional disk mass on the stability chart was investigated for pinned-pinned boundary conditions. Numerical results and illustrative examples are given. It is found that the additional disk mass decreases the instability region during static condition. The location of the disk as well has significant effect on the instability region of the shaft.

1. Introduction

Work by Bolotin [1] has well presented the general theory of parametric instability and nowadays the subject of parametric instability is going through a period of intensive development. Accurate predictions of parametric instability behaviour become important in design process of high-speed turbo machines, drive train system and any types of rotating machinery as failure to do so may result in fatigue failure of rotating components earlier than expected. In engineering applications, machines may be subjected to periodically changes of stiffness, which can become the source of the parametric excitation.

In order to model and simplified the structure of the rotating machine, beam theory has been used with different geometries and boundary conditions according to their applications. This model can accurately estimate the characteristic and behaviour of the machine. Thus, Chen and Ku [2] has modelled a shaft using Timoshenko beam theory to investigate the effect of gyroscopic towards parametric instability. It was found that rotary inertia and transverse shear deformation contribute to dynamic instability of rotating shaft. Chen and Peng [3] suggested that unstable condition was more at higher mode compare to lower mode. Yim and Yim [4] stated that for shaft and disk system, the size of instability region depended on location of disk. Pei [5] explained that the increase of instability region was due to the width of disk centre orbit of the system growth as axial force fluctuating.

Numerical and analytical methods have been used to estimate the instability condition that could occur in shaft system due to parametric excitation. Finite element method (FEM) is one of numerical method that has been used to estimate the instability region [2], [3], [6], [7]. However, only few studies have

been conducted to show the effect of disk location towards dynamic instability of the shaft system in torsional motion condition.

Thus, the main idea of current work is to estimate and compare the instability region of shaft-disk system under axial load for different locations of disk. In order to model the shaft in torsional motion, Nelson's beam theory has been used. The FEM model is used to establish the Mathieu's equation. Floquet's theory has been applied to estimate the dynamic instability region. Then, the effect of disk location towards dynamic instability region can be investigated.

2. Material and Method

The parametric instability of shaft using FEM is developed based on theory that stiffness changes as load is applied onto the system. Fig. 1 shows a uniform shaft-disk of length L that is exposed to an axial load P(t).
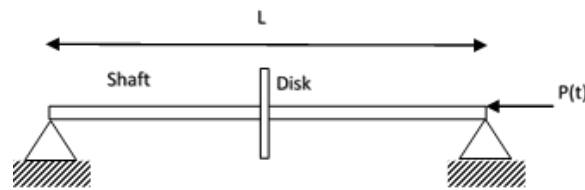

Fig.1. Shaft-disk system subjected to axial load.

Table 1. Geometric of shaft-disk system.

Shaft	Disk	Steel Properties
Length = 300 mm	Diameter, d = 100	Young Modulus, $E = 20 \times 10^{10}$
Diameter, d = 10 mm	Thickness, t = 3 mm	Density, $\rho = 7833$ kg/m^3

In this study, the FEM model of shaft is based on the Nelson's model [8] that consists of five degrees of freedom (DOF) per node i.e. two translations, two rotations about y-axis and z-axis and another torsional rotation about x-axis. The kinetic energy for system in Fig. 1 under bending and torsional mode is shown as[10][9];

$$T = \frac{1}{2}\int_{-a}^{+a}\rho A\left(\left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right)d\hat{x} + \frac{1}{2}\int_{-a}^{+a}\rho I_D\left(\left(\frac{\partial \theta_y}{\partial t}\right)^2 + \left(\frac{\partial \theta_z}{\partial t}\right)^2\right)d\hat{x} - \frac{1}{2}\int_{-a}^{+a}I_P\left(\Omega + \frac{\partial \theta_x}{\partial t}\right)\left(\theta_y\frac{\partial \theta_z}{\partial t} - \theta_z\frac{\partial \theta_y}{\partial t}\right)d\hat{x} + \int_{-a}^{+a}I_p\left(\Omega + \frac{\partial \theta_x^2}{\partial t}\right)d\hat{x}$$

$$(1)$$

I_D and I_P are the diametric and polar mass moments of inertia per unit length of the shaft element respectively. The potential energy including the shear deformation, bending moment, torsional deflection and axial compression load are represented as;

$$U = \frac{1}{2}\int_{-a}^{+a}EI(\theta_z'^2 + \theta_y'^2)d\hat{x} + \frac{1}{2}\int_{-a}^{+a}k'GA[(v'-\theta_z)^2 + (w'+\theta_y)^2]d\hat{x} + \frac{1}{2}\int_{-a}^{+a}GI_P\theta_x'^2 d\hat{x} - \frac{1}{2}\int_{-a}^{+a}P[v'^2 + w'^2]d\hat{x} \qquad (2)$$

where symbol ' denotes the differentiation with respect to axial length, k' is the shear correction factor, A is the cross-sectional area, G is the shear modulus, E is the elastic modulus and I is the cross sectional second moment of inertia. Eq.1 and Eq.2 were transformed into FEM equation. Then Lagrangian theory was applied to establish the equation of motion in FEM that includes the buckling load P;

$$[M]\{\ddot{q}\} - \Omega[G]\{\dot{q}\} + ([K] - P(t)[K_g])\{q\} = 0 \qquad (3)$$

The periodic axial compression load is expressed as fundamental static buckling load P*;

$$P(t) = \alpha P^* + \beta P^* \cos \phi t \tag{4}$$

where ϕ, α and β are the axial compress frequency, static and dynamic load factors respectively. Mathieu type is established by inserting Eq. 4 into Eq. 3.

$$[M]\{\ddot{q}\} - \Omega[G]\{\dot{q}\} + ([K] - \alpha P^*[K_g] - \beta P^* \cos \phi t [K_g])\{q\} = 0 \tag{5}$$

Eq. 6 is used to estimate the solution of Eq. 5 which lead to non-trivial solutions that show by linear homogenous equations as Eq. 7.

$$\{q\} = \{a\}\sin(\phi t / 2) + \{b\}\cos(\phi t / 2) \tag{6}$$

$$\begin{vmatrix} [K] - \left(\alpha + \dfrac{\beta}{2}\right)P^*[K_g] - \dfrac{\phi^2}{4}[M] & \Omega\dfrac{\phi}{2}[G] \\[2ex] -\Omega\dfrac{\phi}{2}[G] & [K] - \left(\alpha - \dfrac{\beta}{2}\right)P^*[K_g] - \dfrac{\phi^2}{4}[M] \end{vmatrix} = 0 \tag{7}$$

In order to estimate the stability boundaries, the eigenvalue for the determinant equation in Eq. 7 is obtained. Reference [11] explained the step for solving the eigenvalue for quadratic matric equation.

3. Results and Discussion

In order to validate current FEM work, the first four natural frequency of the disk-shaft system were compared with Hili et.al [12] as shown in Table. 2. The comparison was for simply supported steel shaft-disk system in static condition as Fig. 1. Geometry and material properties of the system are shown in Table. 1. MATLAB coding was established to obtain the numerical result of the model. Fig. 2 shows relative error between current work and reference [12]. For mode 1, mode 2 and mode 4 the relative error is below 10%, which is reasonable for the current model. However, at mode 3, the relative error is around 22 % that is still in reasonable condition. The models start to converge at 30th elements as number of elements increased. Therefore, the result at 30 elements has been used for FEM analysis in this paper.

Table 2. Comparison of natural frequency (rad/s) between reference [12] and current work (at 30 number of elements)

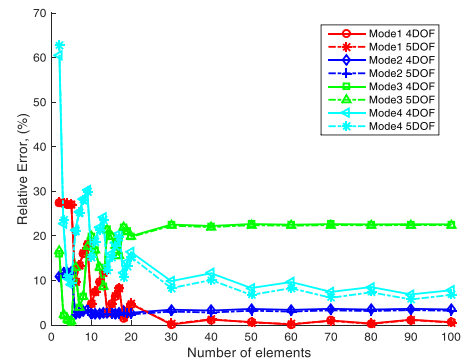| Mode	Current work	Reference [12]
1	951.15	948.76
2	3728.46	3845.30
3	5731.10	7376.45
4	12428.60	13552

Fig. 2. Convergence of relative error with reference [12]

On the other hand, there are slightly different readings of relative error between Timoshenko beam represented by 4 DOF and Nelson's beam element represented by 5 DOF. It indicated that torsional motion has significant effect on natural frequency of the shaft.

Fig. 3 shows the effect of additional rigid disk towards parametric instability region of shaft system under axial load. The dynamic instability region shifted inwards as additional disk was applied. This condition was expected since in general natural frequency decreased as mass increased and instability region location depends on natural frequency of the system. Chen and Peng [3] stated that parametric instability region occur in between of natural frequency for non-rotating shaft. On the other hand, the instability region slightly narrow especially at higher mode as disk was attached. This showed that shaft-disk system was more stable compared to system with shaft only.

In general, the instability region shifted as location of disk changed. Fig. 3 shows that, at 50% of shaft length, the instability region shifted outwards compared to 75%. Furthermore, the instability region narrowed especially at higher mode as the location of disk is getting nearer to end of the shaft. This showed that shaft system was more stable as disk is located nearer to the end of the system. This results is in line with Yim and Yim [4]. In addition, there are very small different between 4 DOF and 5 DOF model in term of instability region. This condition may due to torsional motion has slightly effect especially on simply supported boundary condition.
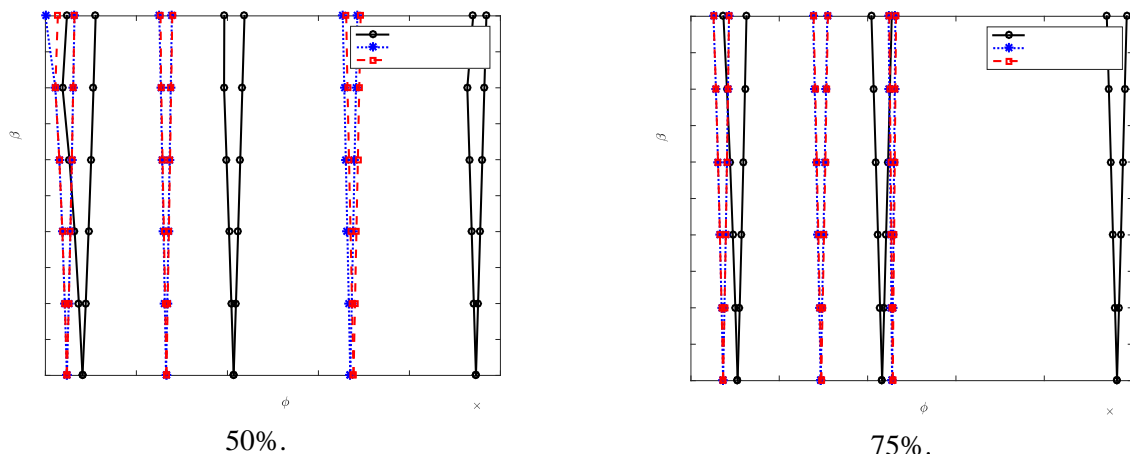

50%.   75%.

Fig.3. Parametric instability region comparison for different disk location from end.

4. Conclusion
The parametric instability region of shaft-disk system with different location of disk has been estimated using FEM and Floquet's theory. This study shows that the location of the disk has significant effect on the parametric instability region of shaft-disk system. The next study will consider the system in rotating condition.

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