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Monotonicity and topologically group on fuzzy topographic topological mappings

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Abstract. The mathematical model Fuzzy Topographic Topological Mapping (FTTM) was developed to solve the neuromagnetic inverse problem during a seizure for determining the location of epileptic foci. The aim of this paper is to investigate various properties for the components and mappings of FTTM. As a result, various kinds of monotonicity have been assured for its mappings. Furthermore, each component of FTTM was demonstrated as a topological group, uncoherent and indecomposable.

1. Introduction

Fuzzy Topographic Topological Mapping (FTTM) is a mathematical model which was introduced in 1999 to solve the neuromagnetic inverse problem for determining the location of epileptic foci in epilepsy disorder patient. Currently, there are three versions of FTTM which are FTTM1 [1], fuzzy topographic topological mapping version 2 (FTTM2) [2], and fuzzy topographic topological mapping digital (FTTM_{dig}) [3]. All these versions are structured based on mathematical concepts of topology and fuzzy set. Furthermore, FTTM1 and FTTM2 were developed to present a 3-dimensional view of unbounded single current source and bounded multi current sources, respectively. In addition, the sequence of n versions of FTTM is defined by Tahir *et al.* [4].

On the other hand, mappings have a mainly influence in preserving and invariant various properties in topology. One of these mappings is a monotone mapping which is introduced in 1942 by G. T. Whyburn [5] Then, J. Charatonik in 1964 initiated a confluent mapping as a generalization notion of a monotone mapping by [6]. Afterward, as mentioned in [7] Maćkowiak in 1979 used the notions of certain types of a monotone mapping that are feebly monotone and almost monotone mappings between continua. In 1989, J. Charatonik introduced the concept of feebly monotone when he studied the invariant of uncoherence property at subcontinua and he related feebly monotone with almost monotone. In addition, he established the invariant of uncoherence and indecomposable properties under these types of monotone mappings [7]. For this reason, many authors had been adopted many studies on this aspect (see [8], [9], [10], [11]).

The aim of this paper is to show that the mappings in FTTM are systematically monotone, almost monotone and feebly monotone mappings. In addition, the components of FTTM have various properties of spaces. Basic definitions and theorems are recalled in Section 2. While, Section 3 covers the previous related works on FTTM. In Section 4, the monotonicity on the sequence of FTTM are verified After that, the topological group properties of the components of FTTM are established in Section 5. The conclusion is drawn in Section 6.



2. Preliminaries

In this section, some basic definitions that are necessary in the paper are reviewed. Firstly, the notion of a continuum and some properties of spaces are given as follows.

Definition 2.1. Let X be a space. Then, X is called:

- (1) continuum if X is connected and compact [12].
- (2) decomposable if it contains two proper continua A and B in X such that $X = A \cup B$. Otherwise, it is indecomposable [13].
- (3) unicoherent if for every two continua A and B in X with $X = A \cup B$, that $A \cap B$ is connected [13].
- (4) homogeneous if for each $x, y \in X$ there is a homeomorphism $f: X \rightarrow X$ such that $f(x) = y$ [13].

Next, we recall some types of confluent and monotone mappings.

Definition 2.2 Let X and Y be two spaces. A mapping (continuous function) $f: X \rightarrow Y$ is called:

- (1) confluent if given a continuum $K \subseteq Y$, then every component C of $f^{-1}(K)$ is mapped by f onto K , that is $f(C) = K$ [6].
- (2) confluent at a point $y \in Y$ if for each continuum K in Y such that $y \in K$, then every component C of $f^{-1}(K)$ is mapped to whole K under f , that is $f(C) = K$ [15].
- (3) confluent relative to a point $x \in X$ if for each continuum K in Y such that $f(x) \in K$, then the component C of $f^{-1}(K)$ containing the point x is satisfied that $f(C) = K$ [15].
- (4) monotone if for each continuum $K \subseteq Y$, then $f^{-1}(K)$ is continuum in X [7].
- (5) almost monotone if for each continuum K in Y with $K^0 \neq \emptyset$, then $f^{-1}(K)$ is connected [13].
- (6) feebly monotone if for every two proper subcontinua A and B of Y with $Y = A \cup B$, then $f^{-1}(A)$ and $f^{-1}(B)$ are connected [13].

A topological group notion is given as follows:

Definition 2.3. [14] Let $(G, *)$ be a group and G be a space. Then, G is called a topological group (or, a group topology) if, the multiplication function $m: G \times G \rightarrow G$ and the inverse function $inv: G \rightarrow G$ are continuous, where $m(x, y) = x * y$, for all $x, y \in G$, and $inv(x) = x^{-1}$, for all $x \in G$.

In the following definition, group homomorphism and isomorphism between topological groups are recalled.

Definition 2.4. [16] Let G and H be topological groups and let $f: G \rightarrow H$ be function. Then, f is called a morphism of topological groups, if f is a group homomorphism. We call f an isomorphism of topological groups when it is a topological homeomorphism and a group isomorphism.

The concept of manifold is given as follows.

Definition 2.5. [17] A Hausdorff and second countable space is called n -dimensional manifold if it is a locally Euclidean space of dimension n , i.e. for each of its points, there is a neighbourhood which is homeomorphic to an open ball in the Euclidean space \mathbb{R}^n .

3. Fuzzy Topographic Topological Mapping (FTTM)

Fuzzy Topographic Topological Mapping Version 1 (FTTM1) is used to solve the inverse problem for determining single current source. It consists of four components with three algorithms that link between them. These components of FTTM1 are magnetic contour plane (M_1), base magnetic plane (B_1), fuzzy magnetic field (F_1), and topographic magnetic field (T_1) [2].

The component M_1 is a magnetic field on a plane above a current source with $z = 0$, whereas the plane is lowered down to B_1 and it is generated by the single current source, such that

$$M_1 = \{((x, y)_{0, B_Z(x, y)}): x, y \in \mathbb{R}, B_Z(x, y) \in [B_Z \min, B_Z \max]\} \quad (1)$$

Where

$$B_Z(x,y) = \frac{\mu_o I}{2\pi} \left[\frac{(y-y_p)}{(y-y_p)^2 + [h+|x-x_p|\tan(\varphi-90^\circ)]^2} \right] \tag{2}$$

$B_{Z\ min}$ is the smallest magnetic fields reading, $B_{Z\ max}$ is the largest magnetic field reading, μ_o is the permeability of free space and its value is $4\pi \times 10^{-7}$ (meter. Tesla/ ampere), I is the magnitude of current in ampere, $((x_p, y_p)_0, B_Z(x_p, y_p))$ is the element of M_1 which is exactly above the current source, φ is the angle between current source and z -axis, and h is the distance between M_1 and the current source in meter [2].

Base magnetic plane (B_1) is a plane of the current source with $z = -h$. Then the entire B_1 is fuzzified into a fuzzy environment (F_1) where all the magnetic field readings are fuzzified. Finally, a three-dimensional presentation of F_1 is plotted on B_1 . The final process is defuzzification of the fuzzified data to obtain a 3-dimensional view of the current source (T_1). The components of FTTM are defined as follows:

$$B_1 = \{((x, y)_{-h}, B_Z(x, y)): x, y \in \mathbb{R}, B_Z(x, y) \in [B_{Z\ min}, B_{Z\ max}]\} \tag{3}$$

$$F_1 = \{((x, y)_{-h}, \mu_{B_Z(x, y)}): x, y, -h \in \mathbb{R}, \mu_{B_Z(x, y)} \in [0, 1]\} \tag{4}$$

such that:
$$\mu_{B_Z(x, y)} = \frac{|B_Z(x, y) - B_{Z\ min}|}{B_{Z\ max} - B_{Z\ min}} \tag{5}$$

and $B_Z(x, y)$ as in (2). As well,

$$T_1 = \{(x, y, z_{B_Z(x, y)}): x, y \in \mathbb{R}, z_{B_Z(x, y)} \in [-h, 0]\} \tag{6}$$

where

$$z_{B_Z(x, y)} = h(\mu_{B_Z(x, y)} - 1). \tag{7}$$

The components of FTTM1 were proven spaces in 2001 by Yun [1] and homeomorphics in 2005 by Ahmad *et al.* [18] as given in the following:

Theorem 3.1. [18] In FTTM1, $M_1 \cong B_1 \cong F_1 \cong T_1$ by the following homeomorphisms:

(1) $b_1: M_1 \rightarrow B_1$ such that:

$$b_1((x, y)_0, B_Z(x, y)) = ((x, y)_{-h}, B_Z(x, y)), \forall ((x, y)_0, B_Z(x, y)) \in M_1 \tag{8}$$

(2) $f_1: B_1 \rightarrow F_1$ such that:

$$f_1((x, y)_{-h}, B_Z(x, y)) = ((x, y)_{-h}, \mu_{B_Z(x, y)}), \forall ((x, y)_{-h}, B_Z(x, y)) \in B_1 \tag{9}$$

(3) $t_1: F_1 \rightarrow T_1$ such that:

$$t_1((x, y)_{-h}, \mu_{B_Z(x, y)}) = (x, y, z), \forall ((x, y)_{-h}, \mu_{B_Z(x, y)}) \in F_1 \tag{10}$$

with $-h < 0$ is a constant, and $B_z \in B \subseteq \mathbb{R}$.

Next, the algebraic structures and some properties for the components of FTTM1 are recalled.

Theorem 3.2. In FTTM1, we have:

- (1) M_1, B_1, F_1 , and T_1 are abelian groups [2].
- (2) M_1, B_1, F_1 , and T_1 are Lie groups [19].
- (3) M_1 is Hausdorff and second countable [19].
- (4) M_1 is path connected and closed [20].
- (5) Each component in FTTM1 is continuum and satisfied the separation axioms T_0, T_1, T_2 , regular, T_3 , normal and T_4 [20].

On the other hand, FTTM2 is the extended version of FTTM1 which is specifically designed to solve the inverse problem of multi-current source. Similar to FTTM1, the model is comprised of four

components. They are Magnetic Image Plane (M_2), Base Magnetic Image Plane (B_2), Fuzzy Magnetic Image Field (F_2) and Topographic Magnetic Image Field (T_2) [2].

As mentioned in [2] that the magnetic fields data which is laid on the component M_1 of FTTM1 are at once analyzed and transformed into image processing data in order to carry out the process in FTTM2. Therefore, the first component in FTTM2 is the component M_2 which has the grey scale image for readings [0,255] of magnetic field. It lies on a plane above a current source with $z = 0$. When the plane is lowered down to the current source with $z = -h$, the component B_2 is obtained. The field of the component F_2 is the fuzzified B_2 plane, i.e., all the grey scale readings are fuzzified into a fuzzy environment. Finally, a three-dimensional presentation of F_2 field is plotted on T_2 field. This final process is the defuzzification of the fuzzified data to obtain a 3-dimensional view of the current source.

FTTM2 is an improvement of FTTM1 because it presents the three-dimensional view of current source in four angles of observation (upper, left, right and back part of a head model). On the contrary, FTTM1 only presents the three-dimensional view of current source in one angle of observation (upper part of a head model). Furthermore, FTTM2 can be applied on single and multiple, bounded and unbounded current source in contrast to FTTM1 which can only be applied to single and unbounded current source.

In addition, Yun [2] verified that the components of FTTM2 are spaces and she confirmed that FTTM2 is also designed to have equivalent topological structures between its components, as follow:

Theorem 3.3. [2] In FTTM2, $M_2 \cong B_2 \cong F_2 \cong T_2$ by following homeomorphisms:

(1) $b_2: M_2 \rightarrow B_2$ such that:

$$b_2((x, y)_0, M_I(x, y)) = ((x, y)_{-h}, M_I(x, y)), \forall ((x, y)_0, M_I(x, y)) \in M_2 \quad (11)$$

(2) $f_2: B_2 \rightarrow F_2$ such that:

$$f_2((x, y)_{-h}, M_I(x, y)) = ((x, y)_{-h}, \mu_{M_I(x, y)}), \forall ((x, y)_{-h}, M_I(x, y)) \in B_2 \quad (12)$$

(3) $t_2: F_2 \rightarrow T_2$ such that:

$$t_2((x, y)_{-h}, \mu_{M_I(x, y)}) = (x, y, z_{M_I(x, y)}), \forall ((x, y)_{-h}, \mu_{M_I(x, y)}) \in F_2 \quad (13)$$

with $-h < 0$ is a constant, $\mu_{M_I(x, y)} \in [0, 1]$ and $z_{M_I(x, y)} \in [-h, 0]$.

In the following theorem, the homeomorphism between each component of FTTM1 and its corresponding components of FTTM2 are given.

Theorem 3.4. [2] Between FTTM1 and FTTM2, $M_1 \cong M_2$, $B_1 \cong B_2$, $F_1 \cong F_2$ and $T_1 \cong T_2$ by the following homeomorphisms:

(1) $m_{1,2}: M_1 \rightarrow M_2$ such that:

$$m_{1,2}((x, y)_0, B_Z(x, y)) = ((x, y)_0, M_I(x, y)), \forall ((x, y)_0, B_Z(x, y)) \in M_1 \quad (14)$$

(2) $b_{1,2}: B_1 \rightarrow B_2$ such that:

$$b_{1,2}((x, y)_{-h}, B_Z(x, y)) = ((x, y)_{-h}, M_I(x, y)), \forall ((x, y)_{-h}, B_Z(x, y)) \in B_1 \quad (15)$$

(3) $f_{1,2}: F_1 \rightarrow F_2$ such that:

$$f_{1,2}((x, y)_{-h}, \mu_{B_Z(x, y)}) = ((x, y)_{-h}, \mu_{M_I(x, y)}), \forall ((x, y)_{-h}, \mu_{B_Z(x, y)}) \in F_1 \quad (16)$$

(4) $t_{1,2}: T_1 \rightarrow T_2$ such that:

$$t_{1,2}(x, y, z_{B_Z(x, y)}) = (x, y, z_{M_I(x, y)}), \forall (x, y, z_{B_Z(x, y)}) \in T_1 \quad (17)$$

The algebraic structures and some facts for the components of FTTM1 are given as follows:

Theorem 3.5. [2] In FTTM2 we have:

- (1) M_2 , B_2 , F_2 , and T_2 are abelian groups [2].
- (2) M_2 , B_2 , F_2 , and T_2 are Lie groups [19].

- (3) Each component in FTTM2 is continuum and satisfied the separation axioms T_0, T_1, T_2 , regular, T_3 , normal and T_4 [20].

In 2010, Tahir *et al.* [4] introduced a sequence of n versions of FTTM which is defined as FTTM1, FTTM2, FTTM3, ..., FTTM n where $n \in \mathbb{Z}^+$. The relations between the components in that sequence and the properties of these components are given in the following theorem.

Theorem 3.6. [4]. In the sequence of n versions of FTTM that:

- (1) In any version v of FTTM, $M_v \cong B_v \cong F_v \cong T_v$, where $v=1, 2, \dots, n \in \mathbb{Z}^+$.
- (2) Between any consecutive versions v and $(v+1)$, that: $M_v \cong M_{v+1}, B_v \cong B_{v+1}, F_v \cong F_{v+1}, T_v \cong T_{v+1}$, where $v=1, 2, \dots, (n-1) \in \mathbb{Z}^+$.
- (3) Each component in the sequence of n versions of FTTM is continuum and satisfied the separation axioms T_0, T_1, T_2 , regular, T_3 , normal and T_4 [19].

4. The Monotonicity on the sequence of FTTM

In this section, new results on the sequence of FTTM are established. Firstly, the confluence is proven as follows:

Theorem 4.1. Each mapping in the sequence of FTTM is confluent.

Proof. This theorem can be established by a mathematical induction. Each mapping in any version v of FTTM symbolically as FTTM v is confluent for $v=1, 2, 3, \dots, n \in \mathbb{Z}^+$ and can be proven as follows:

- (1) The mappings b_1, f_1 and t_1 between the components of FTTM1 are homeomorphisms by Theorem 3.1.

Therefore, b_1^{-1}, f_1^{-1} , and t_1^{-1} are homeomorphisms, hence $b_1, f_1, t_1, b_1^{-1}, f_1^{-1}$, and t_1^{-1} are confluent mappings [21]. Since the composition of confluent mappings is confluent [6], $(t_1 \circ f_1 \circ b_1)$ and $(b_1^{-1} \circ f_1^{-1} \circ t_1^{-1})$ are confluent mappings. Consequently, all the mappings between the components of FTTM1 are confluent.

- (2) Suppose that each mapping in FTTM n is confluent.

- (3) Observe that in FTTM $n+1$, we have $M_{n+1} \cong B_{n+1} \cong F_{n+1} \cong T_{n+1}$ by Theorem 3.6 part (1) which imply that these homeomorphisms are confluent mappings [21].

From parts (i), (ii), and (iii), the mappings in FTTM v are confluent for $v=1, 2, \dots, n \in \mathbb{Z}^+$.

The following results are an immediate consequence of the fact that any confluent is confluent at each point in its range and confluent relative to each point in its domain [22] and Theorem 4.1, is the following:

Corollary 4.1. Each mapping in the sequence of FTTM is confluent at each point in its range.

Corollary 4.2. Each mapping in the sequence of FTTM is confluent relative to each point in its domain.

Example 4.1: In FTTM1, the mapping $b_1: M_1 \rightarrow B_1$, which is defined as in equation (8), is confluent due to Theorem 4.1. To apply Corollary 4.1 on b_1 , pick $y \in B_1$ and K be any continuum in B_1 such that $y \in K$. Define, $(b_1^{-1} | K): K \rightarrow (b_1^{-1} | K)(K)$, such that $(b_1^{-1} | K)((x, y)_{-h}, B_Z(x, y)) = ((x, y)_0, B_Z(x, y)), \forall ((x, y)_{-h}, B_Z(x, y)) \in K$.

The function $(b_1^{-1} | K)$ is homeomorphism since b_1 is homeomorphism. Therefore, $(b_1^{-1} | K)(K)$ is continuum. That is, $(b_1^{-1} | K)(K)$ is connected due to Definition 2.1 part (1). Since the connected set is a component for each of its points [14], $(b_1^{-1} | K)(K)$ is the unique component in it. Thus, $(b_1^{-1} | K)^{-1}((b_1^{-1} | K)(K)) = (b_1 | K)(b_1^{-1} | K)(K) = K$.

Since K is an arbitrary continuum in B_1 , it is therefore true for every continuum containing y in B_1 . Thus, b_1 is a confluent mapping at y . Similarly, for each point in B_1 . Hence, b_1 is a confluent mapping relative to each point in B_1 .

The strongly connectedness is required for establishing the monotonicity for all the mappings in FTTM, as follows:

Theorem 4.2. Each component in the sequence of FTTM is strongly connected.

Proof. Initially, the component M_1 of FTTM1 is a Hausdorff space by Theorem 3.2 part (3) and it is a continuum by Theorem 3.2 part (5). Thusly, M_1 will be strongly connected due to it is continuum and Hausdorff [23]. Follows in the same manner as M_1 , the other components in FTTM will be also strongly connected by Theorem 3.6 part (3).

Next, FTTM and feebly monotone mapping are intimately linked.

Theorem 4.3: Each mapping in the sequence of FTTM is feebly monotone.

Proof. This theorem can be established by a mathematical induction.

- (1) The mappings between the components of FTTM1 are $b_1, f_1, t_1, b_1^{-1}, f_1^{-1}, t_1^{-1}, (t_1 \circ f_1 \circ b_1)$ and $(b_1^{-1} \circ f_1^{-1} \circ t_1^{-1})$ which are homeomorphisms by Theorem 3.1. The mapping $b_1: M_1 \rightarrow B_1$ is confluent by Theorem 4.1. Since any confluent mapping of a strongly connected compact space onto a T_1 -space is a monotone mapping [21], b_1 is monotone due to M_1 is a strongly connected space by Theorem 4.2 and B_1 is a T_1 -space by part (5) of Theorem 3.2. Then, b_1 is almost monotone because any monotone mapping is almost monotone [13]. Moreover, every almost monotone mapping is feebly monotone [13] which imply b_1 is feebly monotone. By applying the similar argument used to b_1 , the mappings $f_1, t_1, b_1^{-1}, f_1^{-1}$, and t_1^{-1} are feebly monotone. The other mappings in FTTM1 which are $(t_1 \circ f_1 \circ b_1)$ and $(b_1^{-1} \circ f_1^{-1} \circ t_1^{-1})$ are feebly monotone mappings as feebly monotone mapping has the composition property [13]. Therefore, all the mappings between the components of FTTM1 are feebly monotone.
- (2) Suppose that each mapping in FTTM n is feebly monotone, where $n \in \mathbb{Z}^+$.
- (3) Observe that in FTTM $n+1$ the components are $M_{n+1}, B_{n+1}, F_{n+1}$, and T_{n+1} such that $M_{n+1} \cong B_{n+1} \cong F_{n+1} \cong T_{n+1}$ by Theorem 3.6 part (1). These homeomorphisms are confluent mappings by Theorem 4.1. Since each component in the sequence of FTTM is strongly connected and T_1 -space by Theorem 4.2 and Theorem 3.6 part (3), respectively. Then, these mappings are monotone. Consequently, they are almost monotone and then feebly monotone.

From parts (1), (2), and (3), the mappings in in FTTM v are monotone for $v=1, 2, 3, \dots, n \in \mathbb{Z}^+$.

As a direct consequence of using similar arguments as in Theorem 4.3, the following results are established:

Corollary 4.3. Each mapping in the sequence of FTTM is almost monotone.

Corollary 4.4. Each mapping in the sequence of FTTM is monotone.

5. Topological group properties of the components of FTTM

In this section, the components of FTTM will have many properties that utilize by the previous properties for them, as they will be topological groups in the following theorem:

Theorem 5.1. Each component in the sequence of FTTM is a topological group.

Proof. First, the component M_1 is a Lie group by Theorem 3.2 part (2). This implies M_1 is a topological group [24]. The other components in the sequence of FTTM will be a topological group by applying the similar argument used on the component M_1 .

Corollary 5.1: Every component in the sequence of FTTM is a homogeneous space.

Proof. Starting with the component M_1 of FTTM1, which is a topological group by Theorem 5.1. Since every topological group is a homogeneous space [16], M_1 is homogeneous. The other components in the sequence of FTTM will be also homogeneous by applying the similar argument used on the component M_1 .

The properties locally compact, paracompact and Lindelöf are held for all the components in the sequence of FTTM as follows:

Theorem 5.2. Each component in the sequence of FTTM is locally compact, para compact, and Lindelöf.

Proof. Initial, the component M_1 of FTTM1 is continuum by Theorem 3.2 part(5). Consequently, M_1 is compact from Definition 2.1 part (1). Note that every compact space is locally compact [14], paracompact and Lindelöf [24]. Therefore, M_1 has these properties. The other components in the sequence of FTTM will have these properties by Theorem 3.6 part (3) with using the same reason and applying the similar argument used on the component M_1 .

Next, certain types of connectedness for FTTM are explored as follows:

Theorem 5.3. Each component in the sequence of FTTM is path connected.

Proof. This theorem can be proven by mathematical induction to show that each component in FTTM $_v$ is path connected, for $v=1, 2, 3, \dots, n \in \mathbb{Z}^+$ as follows:

- (1) The components of FTTM1 are $M_1, B_1, F_1,$ and T_1 . The functions b_1, f_1 and t_1 are homeomorphisms by Theorem 3.1 which implies at once b_1 is a mapping from M_1 onto B_1 . From Theorem 3.2 part (4), M_1 is path connected. Based on the fact that path connectedness is invariant under mapping [14], B_1 is path connected. Using the same reasoning, F_1 and T_1 is also path connected. Thus, each FTTM1 component is path connected.
- (2) Suppose that each component in FTTM $_n$ is path connected, namely $M_n, B_n, F_n,$ and $T_n,$ for $n \in \mathbb{Z}^+.$
- (3) Observe that $M_v \cong M_{v+1}, B_v \cong B_{v+1}, F_v \cong F_{v+1},$ and $T_v \cong T_{v+1}$ by Theorem 3.6 part (2), for all $v=1,2,3, \dots, (n-1)$ and $n \in \mathbb{Z}^+.$ This entails that $M_n \cong M_{n+1}, B_n \cong B_{n+1}, F_n \cong F_{n+1},$ and $T_n \cong T_{n+1}.$ By the homeomorphisms between the corresponding components, the components $M_{n+1}, B_{n+1}, F_{n+1},$ and T_{n+1} are path connected.

From parts (1), (2), and (3), each component in the sequence of FTTM is path connected.

Theorem 5.4. Each component in the sequence of FTTM is locally connected.

Proof. At once, the component M_1 of FTTM1 is strongly connected by Theorem 4.2 which imply that it is locally connected [23]. The other components in the sequence of FTTM will be locally connected by applying the similar argument used on the component $M_1.$

Next, another property for the components of FTTM is realized.

Theorem 5.5. Each component in the sequence of FTTM is clopen.

Proof. The component M_1 is connected by Theorem 3.2 part (5) and Definition 2.1 part (1). Therefore, it is a component for each of its points [14]. Moreover, M_1 is locally connected by Theorem 5.4 which imply that M_1 is open caused by every connected component in a locally connected space is open [14]. In addition, M_1 is closed by Theorem 3.2 part (4). Thus, M_1 is clopen. The other components in the sequence of FTTM are clopen by using the same argument as in Theorem 5.3.

Other properties will be established as follows:

Theorem 5.6. Each component in the sequence of FTTM has the separation properties $T_2, T_{2\frac{1}{2}},$ Uryshon, completely regular, $T_{3\frac{1}{2}},$ completely normal, $T_5,$ perfectly normal, $T_6,$ metrizable and first countable.

Proof. The component M_1 of FTTM1 is T_3 -space by Theorem 3.2 part (5). Thus, M_1 is a $T_{2\frac{1}{2}}$ - space [25]. In addition, since M_1 is a T_4 - space by Theorem 3.2 part (5), it is a $T_{3\frac{1}{2}}$ - space. The component M_1 is completely regular [14] and then it is satisfying the property of a Uryshon space [25] owing to its normality from Theorem 3.2 part (5). The normality and second countability for M_1 implies its

metrizable [25]. Consequently, it is a perfectly normal [14] that entails first with its satisfying the property of T_1 -space by Theorem 3.2 part (5) to establish the axiom of T_6 -space for it and after that it is a T_5 -space [25]. Secondly, M_1 is completely normal based on its perfectly normal property [26]. Furthermore, first countability property for M_1 is due to its holding second countability property by Theorem 3.2 part (3) [26]. The other components in the sequence of FTTM will have these properties based on their invariant under a homeomorphism and by applying the same argument used to satisfy the path connectedness in Theorem 5. 3.

Next, the unicoherent property is established for the components of FTTM.

Theorem 5.7. Each component in the sequence of FTTM is unicoherent.

Proof. Initially, pick A and B as arbitrary proper continua in M_1 . Hence, A and B are compact by Definition 2.1 which implies that they are closed in M_1 [14] because M_1 is Hausdorff by Theorem 3.2 part (3). Therefore, $A \cap B$ is closed in M_1 [14]. Note that M_1 is strongly connected by Theorem 4.2, $A \cap B$ is connected because of every closed set in a strongly connected space is connected [23]. Since A and B are arbitrary proper subcontinua of M_1 , then M_1 is unicoherent by Definition 2.1 part (3). The other components are unicoherent by following the same manner of M_1 .

Another property of continua holds for all the components in the sequence of FTTM, as follows:

Theorem 5.8. Each component in the sequence of FTTM is indecomposable.

Proof. Firstly, the component M_1 is related with other components of FTTM1 by the mappings bm , bm^{-1} , $(tm \circ fm \circ bm)$ and $(bm^{-1} \circ fm^{-1} \circ tm^{-1})$ which are feebly monotone mappings by Theorem 4.3. Therefore, each mapping from a continuum onto M_1 is feebly monotone which implies that M_1 is indecomposable [13]. By the homeomorphisms among all the components in the sequence of FTTM and by applying the similar argument used to prove Theorem 5. 3, the other components in that sequence are indecomposable.

6. Conclusion

To recapitulate, several properties are satisfied for the components of FTTM as well as for the mappings in FTTM. Certain types of compactness, connectedness, separation properties countability, unicoherent, indecomposable, topological group and homogeneity hold for the components of FTTM. In addition, confluences with certain types of monotonicity are established for the mappings in FTTM.

References

- [1] Yun L L 2001 *Homeomorfisma elipsoid dengan sfera melalui struktur permukaan Riemann serta deduksi pembuktiannya bagi homeomorfisma FTTM*: thesis, university technology Malaysia (Malaysia).
- [2] Yun L L 2006 *Group-like algebraic structures of fuzzy topographic topological mapping for solving neuromagnetic inverse problem* : diss., university technology Malaysia (Malaysia).
- [3] Ahmad N 2009 *Theoretical foundation for digital space of flat electroencephalography*: diss., university technology Malaysia (Malaysia).
- [4] Ahmad T, Jamian S S and Talib J 2010 Generalized finite sequence of fuzzy topographic topological mapping *J. Math. and Stat.* **6**(2)151-6.
- [5] Whyburn G T 1942 *Analytic Topology* (Amer. Math. Soc. Colloq. Publ. **28**, Providence).
- [6] Charatonik J J 1964 Confluent mappings and unicoherence of continua *Fundamenta Mathematicae* **56** 213-20.
- [7] Charatonik J J 1989 Monotone mappings at subcontinua *Topology and its Applications* **33** 209–15.

- [8] Charatonik J J 1991 Absolutely terminal continua and confluent mappings, *Comment. Math. Univ. Carolin.* **2(32)** 377–82.
- [9] Charatonik J J 2004 Mappings confluent over locally connected continua *Georgian Mathematical J.* **11(4)** 671–80.
- [10] Fernández-Bayort T and Quintero A 2009 Homotopic properties of confluent maps in the proper category *Topology and its Applications* **156(18)** 2960-70.
- [11] Charatonik W J, Fernández-Bayort T and Quintero 2012 On the freudenthal extensions of confluent proper maps *Houston J. Math.* **38(3)** 963–89.
- [12] Anaya J G, Maya Dand Orozco-Zitli F 2016 Making holes in the second symmetric product of unicoherent locally connected continua *Topological Proceedings* **48** 251-9.
- [13] Charatonik J J 2000 On feebly monotone and related classes of mappings *Topology and its Applications* **105** 15–29.
- [14] Munkers J R 2000 *Topology* 2nd edition (USA: Prentice-Hall).
- [15] Charatonik J J 1983 The property of Kelly and confluent mapping *Bulletin of the Polish Academy of Sciences* **31(9-12)** 375–80.
- [16] Higgins P J 1974 *An Introduction to Topological Groups* (London: Cambridge University Press).
- [17] Csikós B 2014 *Differential Geometry* (Budapest: Typotex Publishing House).
- [18] Ahmad T, Ahmad R S, Yun L L, Zakaria F and Wan Abdul Rahman W E Z 2005 Homeomorphism of fuzzy topographic topological mapping *Matematika* **21(1)** 35-42.
- [19] Basri A M 2009 *Lie group of fuzzy topographic topological mapping*: thesis, university technology Malaysia (Malaysia).
- [20] Kadhem H H and Ahmad T 2013 Continua and separation of the components of fuzzy topographic topological mapping *Australian J. Basic and Applied Sciences* **7(7)** 563-9.
- [21] Kadhem H H 2001 *On confluent mappings*: thesis, university of Babylon (Iraq).
- [22] Charatonik J J and Charatonik W J 1983 On projections and limit mappings of inverse systems of compact spaces *Topology and its Applications* **16(1)** 1–9.
- [23] Levine N 1965 Strongly connected set in topology *The American Mathematical Monthly*, **72 (10)** 1098-101.
- [24] De U C and Shaikh A A 2007 *Differential Geometry of Manifolds* (Oxford U. K.: Alpha Science International Ltd).
- [25] Steen L A and Seebach J A 1970 *Counterexamples in Topology* (New York: Holt-Rinehart and Winston).
- [26] Sakai K 2013 *Geometric Aspects of General Topology* (Japan: Springer).