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Comparative analysis of different weight matrices in subspace system identification for structural health monitoring

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Abstract. Subspace System Identification (SSI) is considered as one of the most reliable tools for identification of system parameters. Performance of a SSI scheme is considerably affected by the structure of the associated identification algorithm. Weight matrix is a variable in SSI that is used to reduce the dimensionality of the state-space equation. Generally one of the weight matrices of Principle Component (PC), Unweighted Principle Component (UPC) and Canonical Variate Analysis (CVA) are used in the structure of a SSI algorithm. An increasing number of studies in the field of structural health monitoring are using SSI for damage identification. However, studies that evaluate the performance of the weight matrices particularly in association with accuracy, noise resistance, and time complexity properties are very limited. In this study, the accuracy, noise-robustness, and time-efficiency of the weight matrices are compared using different qualitative and quantitative metrics. Three evaluation metrics of pole analysis, fit values and elapsed time are used in the assessment process. A numerical model of a mass-spring-dashpot and operational data is used in this research paper. It is observed that the principal components obtained using PC algorithms are more robust against noise uncertainty and give more stable results for the pole distribution. Furthermore, higher estimation accuracy is achieved using UPC algorithm. CVA had the worst performance for pole analysis and time efficiency analysis. The superior performance of the UPC algorithm in the elapsed time is attributed to using unit weight matrices. The obtained results demonstrated that the process of reducing dimensionality in CVA and PC has not enhanced the time efficiency but yield an improved modal identification in PC.

1. Introduction

Civil engineering structures are generally designed to serve for the lifetime of the occupants or facilities and their failure may lead to catastrophic economic or human losses. Hence, it is important to monitor the health state of these structures during their service life. Structural Health Monitoring (SHM) is an effective solution introduced for the need of a safer and more efficient condition assessment of structures. SHM is widely accepted tool for damage diagnosis in civil engineering communities and has been subject of various studies for the past three decades. SHM systems are used in many civil engineering structures including buildings [1-2], bridges [3-4] or dams [5- 6]. Vibration-based Damage Detection (VDD) is an area of significant interest in SHM and considerable effort have been dedicated toward developing novel approaches and improving existing strategies [7-8]. VDD methods rely on change of dynamic properties as an indicator of damage existence. These methods exploit the observable variation in modal parameters such as resonant frequency, damping and mode shape or their derivative to identify changes in physical properties of a structure. VDD Variation in environmental and operational condition of structures alters the modal parameters of structures that pose negative effect in detectability of the VDD algorithm. The influence of environmental and operational condition and discrimination of their effects are discussed in [9-10]. Time-domain methods are the simplest VDD methods to perform due to direct use of time series in analysis process. Time-domain VDD methods generally use acceleration response due to its higher sensitivity and richer dynamic contents compared to velocity and displacement signals. SSI method is reported as one of the most reliable time-domain methods in VDD and most of the studies are specially concentrated on subspace method in the recent years [11]. The outstanding performance of SSI algorithm is particularly due to its capability in global noise rejection [12]. The most researched subspace methods in the field of system identification are i) Canonical Variate Analysis (CVA), ii) Multivariable Output Error State-sPace (MOESP) and iii) Numerical algorithms for State-Space Subspace System Identification (N4SID) [3]. In order to reduce the complexity of implementing the aforementioned algorithms, Overschee, Moor [12] introduced an unified stochastic, deterministic and combined subspace schemes which put CVA, MOESP and N4SID into a pragmatic approach. Peeters and De Roeck [13] for the first time adapted SSI algorithm to deal with output-only measurements of ambient input. Output-only system identification approaches are the most appropriate methods for damage detection of civil engineering structures where the ambient method is used as excitation source [14-15].

Several research studies have been conducted to improve the performance of the SSI to deal with low quality of the input data including short length of measurement data [16], gluing the extracted data from multiple sets of sensors [17], noise inclusion [18], bias errors [19], complexity of structures [20], non-structural elements [21], Hammerstein systems [22], unexcited modes [23] and superiors modes [15]. Selecting of the appropriate weight matrix is a factor that must be considered in implementation of the SSI algorithm. Weight matrix was first introduced by Overschee, Moor [12] to reduce the dimensionality of data space through using principle parameters in PC, UPC and CVA methods. PC analysis is a multivariate statistics technique for reducing dimensionality of a data space. CVA is linear regression method for data analysis that is applied to quantify the relation between expectation and the extracted normalized variables. UPC algorithm is simpler than PC and CVA algorithms and use matrices of unit weight in the approximation process [24]. Some studies are conducted to compare the performance of PC, UPC and CVA as below. Cismaşiu, Narciso [25] studied optimization routine for implementation of FE updating techniques based on identified dynamic response of a real footbridge structure. It was stated that the SSI–UPC was chosen due to its simplicity and superior performance dealing with modes having comparable energy levels. Nguyen [26] proposed a unified sensing configuration for VDD of complex civil engineering structures. It was demonstrated that the SSI-UPC algorithm was selected due to powerful estimation capabilities and its application in most of modal analysis used in civil structures. Pioldi, Pansieri [27] investigated modal dynamic properties of buildings under earthquake base-excitations. It is stated that the CVA was found to be the most stable weighting option to achieve a reliable estimation at seismic excitation. It is claimed that CVA returns less noise or mathematical poles and higher capabilities to separate true physical modes from spurious earthquake harmonics. However, no direct comparison between these three subspace algorithms has been reported in the aforementioned studies and no evidence was given to prove the advantage of the CVA approach. Kompalka, Reese [28] presented a monitoring framework to deal with progressive damage using SSI-DATA algorithm together with model updating. In the paper it is demonstrated that the use of different weighting matrices of PC, CVA or UPC yield similar results. Miguel, Lopez [29] presented a hybrid stochastic/deterministic optimization algorithm to provide a starting point for optimizer. It is stated that the performance of three different variants of CVA, PC and UPC is quite similar, thus the variant PC was chosen for system identification. Herlufsen, Andersen [30] presented a damage detection technique including SSI technique together with a recently developed projection channel technique. It was mentioned that the analysis was performed on UPC, CVA and PC algorithms for different channels scenarios and all three algorithms gave almost identical results. However just the evaluation result of the SSI-PC was presented in the paper. As mentioned above, several researches have reported the effect of selecting weight matrices in SSI algorithm. However, there are limited studies, which comprehensively investigate on the influence of weight factor in efficiency of the SSI algorithm.

The aim in the present paper is to evaluate the influence of weight matrix on estimation accuracy, noise robustness and time efficiency of the SSI-PC, SSI-UPC and SSI-CVA algorithm. Three different metrics of fit values, poles analysis and the elapsed computation time are introduced to compare the performance of the SSI algorithms. The obtained results show that UPC algorithm had the best record in computation time analysis. The superior performance of the UPC algorithm in the elapsed time is attributed to using unit weight matrices. The use of unit weight matrix reduces the computational burden significantly and improves the time efficiency of the algorithm. The obtained results demonstrated that the process of reducing dimensionality in CVA and PC has not enhanced the time efficiency but yield an improved modal identification in PC. It is observed that the principal components obtained using PC algorithms are more robust against noise uncertainty and give more stable results for the pole distribution. Furthermore, higher estimation accuracy is achieved using UPC algorithm. CVA had the worst performance for pole analysis and time efficiency analysis.

2. Subspace algorithm and implementation of weight matrices

SSI is a time-domain identification method to extract the parameter of dynamic system. The subspace algorithm is divided into four steps of i) QR decomposition, ii) state sequence determination, iii) least square estimation and iv) Kalman filter. In the first step, the extracted response signal is cast into the form of block Hankel matrix. Then, the Hankel matrix is decomposed into triangular matrix and block matrix where the extracted triangular matrix is subspace representation of the Hankel matrix. The oblique projection of the past and future output data are used to determine the weighting matrices. The extracted oblique projection is pre and post multiplied by appropriate weight matrices to infer the system order and state sequence. In the second step, a geometrical projection is adapted to eliminate dependence of the SSI algorithm onto future output. The oblique projection of the past and future output data are used to determine the state-sequence of the system. In the third step, the LS is deployed to drive system matrices $(A \text{ and } C)$. Finally, the Kalman predictor is used to estimate the system model by inferring the Kalman gain K of the state-space model. High resistance against noise in SSI algorithm to a great extent is achieved by adopting Singular Value Decomposition (SVD). SVD provide a simple way to improve the performance of SSI algorithm and reduce dimensionality with minimal loss of information. The principle angle and direction between subspaces can be determined using singular values of the obtained oblique projection. The cosines of the principle angles $(U \text{ and } V)$ are denoted by the singular values (S).

$$W_1 \mathrm{O}_i W_2 = USV^T \tag{1}$$

Where W_1 and W_2 are the weighting matrices of the oblique projection. Weighting matrices of W_1 and W_2 allows to draw the most proper state-space basis of an identified model. Three weighting algorithm are defined for implementation of the SSI algorithm including PC, UPC and CVA. The PC method incorporates right weight matrix to determine singular values. The output covariance matrix of the past $\Phi_{[Y_p,Y_p]}$ is used to determine the block Toeplitz matrices in PC algorithm whereas the output covariance matrix of the future data $(\Phi_{[Y_p,Y_p]})$ is used for CVA algorithm. UPC method is special case of PC analysis that gives the first principle component index of a system with equal weight factors to each set of data. The CVA algorithm selects equal weights for the all incorporated system. The weighting matrices in CVA are obtained from singular values.

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Table 1. The W_1 and W_2 weighting matrices in PC, UPC and CVA algorithms.

	W_1	W_2
PC	I_{li}	$Y_p^T \Phi_{[Y_p, Y_p]}^{-1/2} Y_p$
UPC	I_{li}	I_j
CVA	$\Phi_{[Y_f, Y_f]}^{-1/2}$	I_j

Table 1 presented the weighting matrices for PC, UPC and CVA algorithms. The weights matrices of PC, UPC and CVA algorithms are used to determine the singular values. PC analysis is a left side weighting obtained from the covariance matrix of the past and future data. UPC method use identity matrix for weighting and CVA incorporates right-hand weight matrix.

3. Numerical simulation case study

Mass-spring-damper (MSD) system is the most common reduced-order engineering model in structural dynamics. A 6-DOF, Mass-Spring-Damper (MSD) simulation model is used in this study to compare the performance of PC-SSI, UPC-SSI and CVA-SSI algorithms.

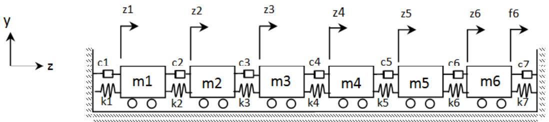

Figure 1. The schematic of the simulation system.

Figure 2 schematically shows the simulation model used in this case study. The proposed numerical model includes six mass elements with seven massless components, which are excited in z direction at node 6. The dynamic simulation example was defined by mass $C\,(M)$, stiffness (K) and damping (C) matrices. The mass matrix M is an identity matrix of order six. The stiffness of each mass to the adjacent elements equals $Ak = 2000N/m$ and the Rayleigh damping was given by a combination of damping matrix proportion to stiffness and mass matrices as: A, equal to:

$$C = \begin{bmatrix} 1.377 & -0.3486 & 0 & 0 & 0 & 0 \\ -0.3486 & 1.377 & -0.3486 & 0 & 0 & 0 \\ 0 & -0.3486 & 1.377 & -0.3486 & 0 & 0 \\ 0 & 0 & -0.3486 & 1.377 & -0.3486 & 0 \\ 0 & 0 & 0 & -0.3486 & 1.377 & -0.3486 \\ 0 & 0 & 0 & 0 & -0.3486 & 1.377 \end{bmatrix}$$

An impulsive load, with duration of 0.01 second and excitation force of 1 kN is applied to the MSD structure in node 3. The corresponding acceleration time-history is calculated with sampling frequency of 500 Hz and 1000 Hz. The imposed noise uncertainty in a real structure is simulated by adding random values into the extracted response signal.

3.1. Fit analysis

Fit-value is a similarity measure between two signals, which is described with Variance Accounted For (VAF). VAF is an evaluation metric associated to the fit quality of the estimation. Higher VAF values suggest a better fit of the estimation result into a particular signal and thus better identification performance[31]. The VAF value is used in this study to appraise the performance of the CVA, PC and UPC algorithms. VAF criterion is defined as:

$$VAF = \left(1 - \frac{Variance(y - y_1)}{Variance(y)}\right) \times 100 \tag{14}$$

Where (y) is response signal of the simulation model and (y_1) is the predicted values of the dynamic system.

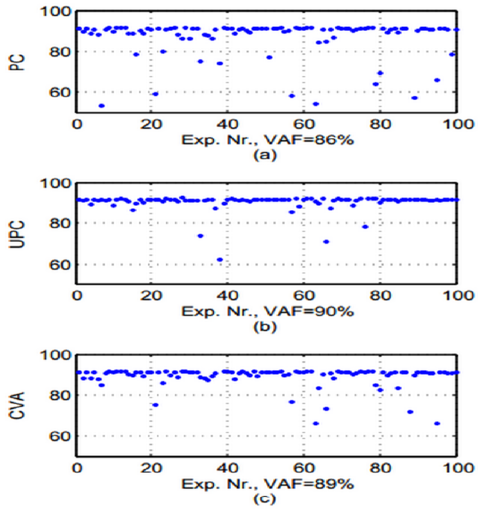

Figure 2. Distribution of the VAF values for the subspace algorithms using (a) PC algorithm (b) UPC algorithm and (c) CVA.

Acceleration response of the simulation FE model with 100 sets of noise patterns was used for evaluation of the SSI weight algorithms. The noise ratio of 30% is used in fit analysis experiment. Figure 6 displays the fit analysis for three SSI algorithms. In Figure 6 (a,b and c) the oscillation pattern of PC, UPC and CVA algorithm are plotted. UPC algorithm has the best prediction capability among all, and PC is only slightly worse, while the CVA performs the worst in obtained results.

3.2. Analysis of the system poles

Poles are dynamic parameters of a system and are depend on distribution of mass, stiffness and damping within a system. Complex poles are a common phenomenon in modal identification of damped structures however, there is not any unified procedure to quantify the poles complexity [32- 33]. Poles can be plotted in a complex plane of real and imaginary components. In an undamped structure, the poles lie on the imaginary axis whereas the real portions of the complex values are zero.

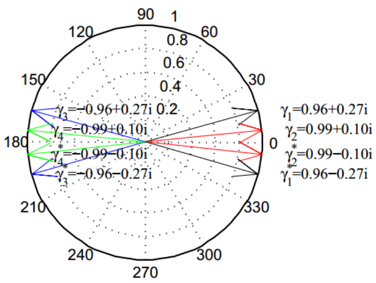

Figure 3. Complex poles nomenclature of the simulation numerical system in complex plane.

Figure 7 illustrates the extracted poles of the four first orders in the noise-free simulation model. The poles correspond to each of the four orders are complex conjugate of each other. Hence, $\lambda_1, \lambda_2, \lambda_3$ $\lambda_1^*, \lambda_2^*, \lambda_3^*$ and λ_4^* are the first order of the numerical model and $\lambda_1^*, \lambda_2^*, \lambda_3^*$ and, λ_4^* $\lambda_1^*, \lambda_2^*, \lambda_3^*$ are the conjugates. The poles are plotted on real and imaginary axes of a complex plane. In a separate numerical experiment, various noise ratios ranged from zero to 30% was used to determine the optimum ratio, which provides the best resolution for assessing the inherent oscillatory patterns of system poles.

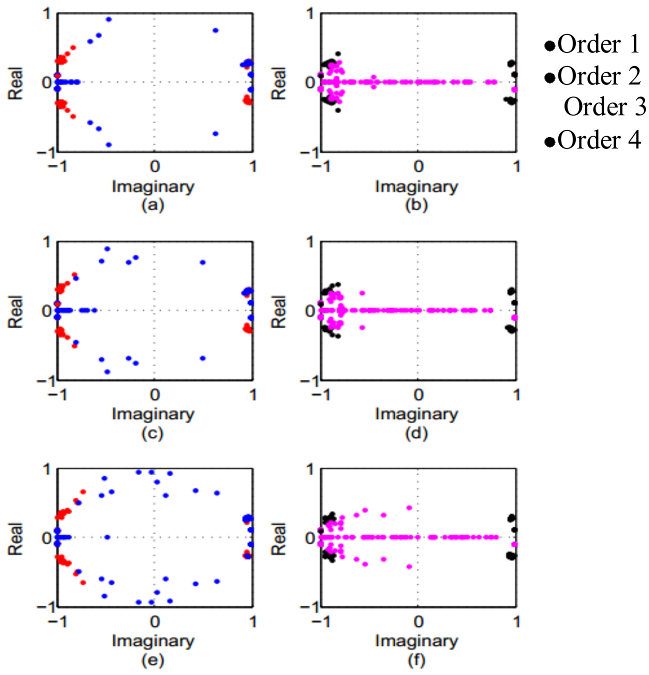

Figure 4. Plot of the estimated poles of the simulation case study obtained from: (a) PC algorithm for order 1 and 2, (b) PC algorithm for order 3 and 4, (c) UPC algorithm for order 1 and 2, (d) UPC algorithm for order 3 and 4, (e), CVA for order 1 and 2, and (f) CVA for order 3 and 4.

The 5% noise ratio was found to be the optimum ratio, which shows a clear insight into the transition pattern of the system's complex poles from the concentrated into the oscillated phase. The oscillation pattern of the numerical model under various noise distributions is used to evaluate the noise-robustness of the identification algorithms [34]. The results obtained from different SSI algorithms of the simulation model are illustrated in complex plane considering the four order system. 100 sets of output signal with different noise patterns are used as the input of the SSI algorithms. Sampling frequency of 500Hz for 2048 data point was used for poles analysis. Figure 8 shows the comparisons of the oscillation patterns for complex poles correspond to PC, UPC and CVA algorithms. As it could be seen the poles of a dynamic system lie inside a circle centered at zero with radius of one. The obtained values are in the form of conjugate pair symmetric about the real axes. The poles values are heavily distributed near the real axes than imaginary one. For the purpose of brevity and informativeness the zeros are not plotted in the figures. In Figure 8(a) and (b), the oscillation pattern of the four-order dynamic parameters of the numerical system obtained from the PC algorithm is plotted. The extracted results represent a relatively lower oscillation of the pole values. The plots show that the poles in the 4^{th} order are heavily distributed about the real axes. The complex pole values of the Figure 7 still could be traced through the distribution density. Figure 8 (c) and (d) illustrates the oscillation pattern of the poles associated with UPC algorithm for the conducted experiments. The scattering increases to approach the imaginary axis. Figure 8 (e) and (f) depicted the oscillation pattern for the CVA algorithm. The CVA algorithm provides high scattering compared to other SSI algorithms. The PC algorithm has the best prediction capability among all, and UPC is only slightly worse, while the CVA performs the worst.

3.3. Analysis of the elapsed time

In a continuous monitoring process, a large repetition cycle is carried out to obtain the system parameters. The large number of repetition time executed in a highly populated dataset may have a prohibitive computational cost in structural systems. Accordingly, it is important to reduce the elapsed analysis time by striking a reasonable balance within the required accuracy level. Therefore the best SSI algorithm is the one that achieves higher accuracy in an optimal time.

In this subsection, the elapsed time for the repetitive running of the subspace algorithms of PC, UPC and CVA is analyzed to verify the effectiveness of each algorithm. The comparison scheme of the time-efficiency was implemented in a desktop computer of a *msi* Dual Core CPU with two 3GHz cores and 2 GB RAM running windows 7 having Matlab 2014a installed. 100 different noise patterns was extracted and used as input data for the three subspace algorithms. The elapsed computation time for each algorithm is measured and recorded in a separate data-base.

Table 2. Elapsed computation time for subspace algorithms.

Algorithm	Computation time (s)
PC	10.8
UPC	5.2
CVA	11.3

In Table 2, the elapsed computation time for SSI algorithms is presented. The mean values of the computation time for PC, UPC and CVA algorithms are 10.8, 5.2 and 11.3, respectively. The UPC algorithm has the best performance in these three techniques with nearly two times better in term of computation efficiency. CVA algorithm has the worst performance while PC algorithm has resulted slightly lower time for the same set of estimation data.

4. Remarks

This study present comparison of three different subspace implementation of PC, UPC and CVA. Numerical simulation was used for evaluation process. The subspace algorithms were evaluated based on the fit values, poles variances and the elapsed time. Table 3 outlines brief remarks of this study. The best VAF value among three algorithms was observed in UPC algorithm whereas the lowest performance was for PC algorithm. The highest efficiency was in UPC algorithm while the recorded time for the PC and CVA algorithms were nearly two times of UPC's. The least oscillation of poles for the first four orders were achieved in PC algorithm for both numerical and field test data and the smallest variances was mapped in CVA.

Table 3. Evaluation of the performance in PC, UPC and CVA subspace algorithms.

		PC	UPC	CVA
Case study I	VAF analysis	+*	+++	++
	Pole analysis	+++	++	+
	Elapsed time analysis	++	+++	+

* The notion '+', '++' and '+++' represent the 'poor', 'medium' and 'strong' identification results in the incorporated analysis technique, respectively.

5. Summary
The PC and CVA algorithms are designed to reduce the dimensionality of the identification result using orthogonal transformation and linear regression, respectively. Since the UPC algorithm doesn't use any weighting matrix (unit weight is used), the computation time is significantly low compared to the other counterparts. As it can be seen by the computation time analysis, the process of reducing dimensionality has not enhanced the time efficiency of the PC and CVA algorithms but yield an improved modal identification in PC. The principal components obtained using PC algorithms are more robust against noise uncertainty and give more stable results for the pole distribution. Higher estimation accuracy is achieved using UPC algorithm however it is not as good as PC for discrimination of the poles in the first four orders. CVA had the worst performance for pole analysis and time efficiency analysis.

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