NATURAL CONVECTION IN SEMI-CYLINDRICAL CAVITY WITH HEAT FLUX BOUNDARY CONDITIONS INTO THE SYSTEM

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A project report submitted in partial fulfillment
of the requirement for the award of the
Degree of Master of Engineering (Mechanical)

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MAY, 2006

To my Parents,

My beloved wife, Siti Aminah,

My childrens, Muhammad Affiq Firdaus & Muhammad Arrif Firdaus.

And my siblings.

ACKNOWLEDGEMENT

I would like to forward my greatest appreciation to Prof Amer Nordin that have been very professionally guiding me throughout the project preparation and throwing challenges as well as trust until the completion of this thesis.

My wife, the late night studies has finally over and I owe you those quality times supposedly spent with you and kids. My parents and siblings your understanding were highly appreciated.

To Farouk that have spent his precious time in order to guide us struggling with the programming, your deed will be return in a greater manner that you definitely deserve. Hassanazhari, your spirit of striving, should have been enlightening my will to complete the project. Redhwan, your prayers for our success have been fulfilled and I could only thank you for that.

Finally to whomever that has also indirectly ensured the success of this numerical project.

-eddie-
2003-2006

ABSTRACT

Numerical analysis approach for natural convection in semi-cylindrical cavity is investigated by heat flux effect ($\dot{q}$) *to the outer wall as the boundary condition. Buoyancy consideration of Boussinesq approximation noted valid for this incompressible problem of ideal gas with low (Pr) Prandtl number and laminar steady state condition. Taking a physical geometry of top half of a circle* (r,ϕ) *the base behaves as an insulated boundary* $(\Delta q = 0)$ *and z-direction to be infinite (2 - dimensional).*

Method of solution acquired will deal with discretizing the governing equation of non-dimensional, using staggered grid procedure of primitive variables (u,v,p) and applying ghost nodes. and handle velocity (u,v) & pressure (p) using SIMPLE algorithm. Correlation between Rayleigh and Nusselt number, Nu=f (Ra) and parametric study on the behavior of the isotherms and streamlines in the cavity would then be elaborated upon the final results.

ABSTRAK

Analisis kaedah berangka untuk perolakan bebas di dalam ruangan separuh bulatan dikaji dengan haba (flux) terikan ($\dot{q}$) *pada diding luaran atas sebagai syarat sempadan. Anggapan pengapungan dengan penghampiran 'Boussinesq' dinyatakan benar untuk gas unggul dan tak boleh mampat dengan nombor (Pr) Prandtl rendah serta keadaan stabil laminar. Dengan mengambil bentuk fizikal sebagai separuh bulatan pada bahagian atas* (r, ϕ) *diding dasarnya bertindak sebagai sempadan tebatan* $(\Delta q = 0)$ *dan arah-z menjadi berterusan (2 - dimensi).*

Kaedah penyelesaian yang digunakan akan dihalusi (discretised) daripada persamaan utama (Governing Equation) dalam bentuk tanpa dimensi menggunakan prosedur anjakan grid pada pemalar primitif (u,v,p) dan nodal maya serta mengurus kelajuan (u,v) & tekanan (p) menggunakan algoritma SIMPLE. Kaitan di antara nombor Rayleigh dan Nusselt,, Nu=f (Ra) dan Kajian parameter terhadap perilaku aliran haba malar dan aliran kelajuan malar di dalam ruangan akan diolah pada keputusan akhir nanti.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

$\dot{q}$	Heat flux, W m $^{-2}$
ρ	Density, kg m^{-3}
μ	Dynamic viscosity, kg m^{-1} s $^{-1}$
ν	Kinetic viscosity, m^{2} s $^{-1}$
k	Thermal conductivity, W m $^{-1}$ K $^{-1}$
T	Temperature, K
Re	Reynolds number $(=\frac{\overline{w}}{\nu}D_h)$
u, v	Dimensional velocity
ϕ	$angular\ \ coordinates$
θ	Dimensionless temperature
u, v	$U=\frac{V_r}{(\alpha/D)(Ra\,\mathrm{Pr})^{2/5}}$ $\quad V=\frac{V_\theta}{(\alpha/D)(Ra\,\mathrm{Pr})^{2/5}}$
	$P=\frac{pD}{\rho\alpha^2(Ra\,\mathrm{Pr})^{4/5}}$
	$R=\frac{r}{D}$ $\quad \theta=\frac{T-T_w}{\Delta T}$
D_w	Diffusion coefficient on west face
D_e	Diffusion coefficient on east face
D_s	Diffusion coefficient on south face
D_n	Diffusion coefficient on north face
F_w	Convection coefficient on west face
F_e	Convection coefficient on east face
F_s	Convection coefficient on south face
F_n	Convection coefficient on north face
nb	Neighboring

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

There are two modes of **heat transfer** which are diffusion and radiation. A diffusion process occurs due to the presence of a gradient (say, of temperature, density, pressure, concentration, electric potential, etc.) and requires a material medium. Both conduction and convection are diffusion processes. A radiative process, on the other hand, does not require a material medium [1].

Heat convection itself is a term applied to the process involved when energy is transferred from a surface to a fluid flowing over it as a result of a difference between the temperatures of the surface and the fluid [2]. Heat transfer occurs due to actual material transport, unlike in the case of conduction where one part of a solid body is at a higher temperature than the rest of it. In many engineering applications the heat transfer due to convection may be calculated by using **Newton's law of cooling** (the relation that determines the heat flux due to convection between a surface and moving fluid)[1]. It states that if the heat transfer coefficient is h, the heat flux q due to convection from a surface at temperature T_w into a fluid at temperature T_f is given by,

$$q = h(T_w - T_f) \qquad (1.1)$$

There are three kinds of convection: forced, natural, and mixed. *Forced convection* takes place when the motion of the fluid that causes convection is sustained by an externally imposed pressure gradient. Forced convection typically occurs in systems such as blowers and air conditioners. Sometimes, even in the absence of external forces, pressure gradients are created due to differences in density that are caused by local heating in the fluid. This heat transfer is known as *free* or *natural* convection. *Mixed* convection, as the name implies, is the situation in which both forced and free convection are present [1].

Free Convection or the natural convection arises when heat transfer between a surface and a fluid moving over it with the fluid motion caused entirely by the buoyancy forces that arise due to the density changes that result from the temperature variations in the flow[2]. Behavior of Nusselt number in terms of function Rayleigh number (GrPr) is commonly described for such natural convection problems. Table 1.1 specified the interpretation of related dimensionless numbers in the studies. This work will eventually determine the correlation towards the better understanding of such radiation effect.

Table 1.1: Dimensionless Numbers table of definition

Dimensionless number			
Name	**Symbol**	**Definition**	**Interpretation**
Eckert Number	*Ec*	$V^2 \mu / k(T_w - T_f)$	Dissipation / heat transfer rate
Grashof Number	*Gr*	$\beta g(T_w - T_f)L^3 / v^2$	Bouyancy force / viscous force
Nusselt Number	*Nu*	hL/k	Convective heat transfer rate / conduction heat transfer rate
Prandtl Number	*Pr*	v/α	Rate of diffusion of viscous effects / rate of diffusion of heat
Rayleigh Number	*Ra*	$\beta g(T_w - T_f)L^3 / v\alpha$	GrPr

1.1 Literature Review

Natural convection phenomenon in cavity has been an interesting yet unlimited discovery to be made by researchers. Several attempt on various geometry, boundary conditions, field property and flow regime considered for very wide application in the actual field of problems. Application such as aircraft-brake housing system, pipes connecting reservoirs of fluids with different temperatures, refrigerators, fire research, electronic cooling, energy saving household refrigerators, waste heat disposal, building, insulation and micro-electronic equipment [4].

Reviews of papers have enlighten the research studies as various geometrical, boundary conditions & methods solving the natural convection in cavities and it is all being arranged as the base reference and guidance. Methods involving experimental or measurements have been kept as direct pre-examined for results gain later in this numerical approach.

Walid *et al* [4] presents the effect of roughness on heat transfer for semi-cylindrical cavity which is performed on smooth and rough surfaces for different tilt angles. Roughness shows a large effect on heat transfer for the semi-cylindrical cavities. Heat transfer for the cylindrical cavity is noted higher than the rectangular cavity for all tilt angles (both cavities have the same surface area and the same heat flux). This increase in heat transfer for the cylindrical cavity is due to the absence of sharp corners that can slow the buoyancy driven convection mechanism. Two competing effects are present with the existence of roughness where, roughness may increase the blockage effect on the flow that can cause the buoyancy force to decrease, but on the other hand it increases the turbulence intensity resulting in a higher heat transfer. Both effects are function of tilt angles. The data was collected for 13 different inclination angles ranging from 90° (when the cavity is facing down) to -90° (when the cavity is facing up).

The semi-cylindrical cavity also shows higher values of Nusselt numbers than that of rectangular cavity at the same inclination angle. This increase ranges from 50 to 200% depending on the inclination angle. Minimum Nusselt number occurs when the cavities are facing down. The buoyancy driven flow gets stronger when the inclined angle decreases resulting in higher Nusselt number. The initial inclination of the cavity from 90° to 0° causes a significant increase in average heat transfer rate, but a further increase in inclination angle appears to cause a small increase in average heat transfer rate. It is observed that the presence of roughness delays the onset of convection motion at small deviation from the 90° angle. Therefore the heat transfer is more for the smooth cavity in this range. Beyond this range, the roughness element increases the heat transfer rate. At low tilt angle molecular conduction is the dominant mechanism of heat transfer; roughness seems to slow down the heat transfer mechanism by adding more form drag to the momentum of the flow. When the cavity is placed in an upward position, high convection current occurs. Roughness helps to trip the boundary layer and adds more turbulence to the flow resulting in increase in heat transfer.

Asfia *et al* [5] In this paper, experiments were conducted to examine natural convection heat transfer in internally and volumetrically heated hemispherical pools with external cooling. Heat transfer coefficient is lowest at the stagnation point and increases along the spherical segment except at the location near the pool surface. Maximum heat transfer coefficient occurs slightly below the pool surface. Different boundary conditions in the pool surface (free surface, insulated rigid surface or cooled rigid surface) make only a slight difference in the average heat transfer coefficients. Average heat transfer coefficients on the y cold rigid wall bounding the pool surface, appears to compare favorably with the correlation of Kulacki *et al* [38].

A study carried out at UCLA showed that the flooding of cavity could indeed be a viable option. The vessel inner wall temperature was not held constant and varied from the stagnation point to the equator. Experiments were performed for pools with nearly insulated and cold rigid walls at the top. The depth of the pool was varied parametrically. Nusselt numbers data obtained in this work compare favorably with the prediction from the correlation of Asfia *et al*. [39].

Liaqat *et al*. [6] presented a numerical investigation of a detailed comparison of the conjugate and non-conjugate natural convection within a semi-cylindrical cavity. The cavity is assumed to be filled with a fluid containing uniformly distributed internal heating sources. The bottom circular wall of the cavity is taken to be thick with finite conductive properties, while the top wall is considered to be isothermal. The Navier-Stokes and energy equations are solved numerically by using the SIMPLER algorithm. A Rayleigh number range from $3.2\times10^{6}\ to\ 3.2\times10^{11}$ has been investigated and the effects of solid-to-fluid conductivity ratios of 1.0, 5.0 and 23.0 have been analyzed by this paper.

The average Nusselt number for the solid-fluid interface shows a decrease while the top wall average Nusselt number has increased. These effects increase for a system with a low solid-to-fluid conductivity ratio. It is evident from the present conjugate results that the assumption of isothermal enclosing walls gives somewhat different result when the walls are thick and the solid-to-fluid conductivity ratio small. It's thick finite conducting wall enclosing the cavity from the bottom, the heat flow is redirected toward the top wall, which is prominent for low solid to fluid conductivity ratios. Numerical approach in this paper has also provided a base reference equation related to natural convection in semi-cylindrical which is later being used for formulation.

Marcelo *et al* [7] deals with the use of the Conjugate Gradient Method of function estimation with Adjoin Problem for the simultaneous identification of two boundary conditions in natural convection inverse problems in two-dimensional irregular cavities. The paper proposed that formulation can be applied to the solution of inverse problems in different geometries. The methodology is applied to cases involving an annular cavity, where the position- and time-dependent heat fluxes are unknown at the inner and outer surfaces.

Direct and inverse problems were formulated in terms of generalized coordinates. Therefore, the present solution procedure can readily be applied to cavities with different geometries. The results obtained with simulated temperature measurements reveal that quite accurate estimates can be obtained for the unknown functions with the present inverse problem approach.

In summary of all 4 most relevant papers from above review, and taking focus on the problem issue, Table 1.2 is presented for it's similar geometric shapes which is semi-cylindrical. Supporting and referring to Table 1.3, is the explanation of boundary conditions and the Figure 1.1 is the possible boundary for the problem stated as semi cylindrical shapes.

Table 1.2: Summary of Semi cylindrical problems

Review of previous work on semi-cylindrical cavities							
Reference	Geometry	B.C.	α	*Pr*	*Gr or Ra*	Numerical Method	Measurement
Walid et al [4]	Semi-cylindrical cavity	D	$-90^0 \le \alpha \le 90^0$	0.7	$Gr_L = 5.5 \times 10^8$		√
Asfia et al [5]	Semi spherical cavity	A	-90^0	2.75 - 6.85	$Ra = 2.8 \times 10^4 - 10^7$		√
Liaqat et al. [6]	Semi-cylindrical cavity	C	-90^0	7.0	$Ra = 3.2 \times 10^6 - 10^{11}$	√	
Marcelo et al [7]	Irregular cylindrical cavity	B	0^0 *to* 360^0	0.7	$Ra = 5.0 \times 10^4$	√	

Table 1.3: Boundary conditions on the wall

Explanation for the types of boundary conditions given in Table 1.2				
B. C. type	Boundary conditions on walls			
	1	2	3	4
A	*N/A*	T_h	T_∞	*N/A*
B	T_c	$\Delta q'' = 0$	T_h	$\Delta q'' = 0$
C	T_h	T_h	T_h	T_h
D	*q”*	*q”*	*q”*	*q”*

1) T_h, T_c *- constant wall temperature (hot , cold)*

2) $T_c < T_\infty < T_h$

3)q” - constant heat flux on the wall,

4) $\Delta q'' = 0$ *(insulated)*

5) N/A - not applicable.

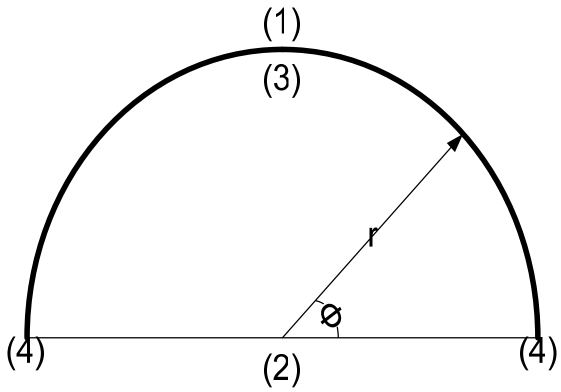

Figure 1.1 Semi cylindrical geometry and boundary numbering

Zhu *et al.* [8] In this paper, natural convective heat transfer between two horizontal, elliptic cylinders is numerically studied using the differential quadrature (DQ) method. The DQ method is employed to discretize the derivatives in the governing equations and boundary conditions. The governing equations are taken to be in the vorticity-stream function formulation. To apply the DQ method, the coordinate transformation is performed. An elliptic function is used, which makes the coordinate transformation from the physical domain to the computational domain be set up by an analytical expression. A systematic study is conducted for the analysis of flow and thermal fields at different eccentricities and angular positions. It was found that the position of the major axis of the inner ellipse takes effect on the streamlines, and very little effect on the average Nusselt number.

Chiu *et al* [9] the transient analysis has been investigated numerically to determine heat transfer by natural convection between concentric and vertically eccentric spheres with constant heat flux on the inner wall and a specified isothermal temperature on the outer wall. The governing equations, in terms of vorticity, stream function and temperature are expressed in a spherical polar coordinate system. The alternating direction implicit method and the successive over-relaxation techniques are applied to solve the finite difference form of governing equations. A physical model is introduced which accounts for the effects of fluid buoyancy as well as eccentricity of the outer sphere. Transient solutions of the entire flow field are obtained for a range of modified Rayleigh number ($10^3<Ra^*<5x10^5$), for a Prandtl number of 0.7 and a radius ratio of 2.0, with the outer sphere near the top and bottom of the inner sphere ($\varepsilon \pm 0.625$).

Ruixian *et al* [10] suggest that analytical solutions are meaningful in both theoretical investigation and practical applications as there are many natural convection processes in various fields, and it is still a hot topic to investigate the fluid dynamics and heat transfer of natural convection. These phenomena are very useful to computational fluid dynamics and heat transfer as the benchmark solutions to check the numerical solutions and to develop numerical differencing schemes, grid generation methods and so forth.

Grundmann *et al* [11] deals analytical study of free convection in a cubic cavity of fluid-saturated porous material. The analytical algorithm integrates the Darcy-Boussinesq equations formulated in terms of pressure and temperature. The dependent variables (P,T) are expressed using an asymptotic development in the parameter $\varepsilon = (Ra - Ra_c)^{1/2} / \pi$ where ($Ra_{c1} = 4\pi^2$ for the 2D flow and $Ra_{c2} = 4.5\pi^2$ for 3D flow). Terms up to order 48 for the 2D flow and up to order 16 for the 3D flow are computed. The analytical value of the Nusselt number is in good agreement with a numerical one obtained using a Legendre spectral collocation method.

Minerva, Abderrahmane and Chen [12],[13],[14] deal with natural convection in vertical cylinder environment where, from Minerva *et al* [12] present a numerical and experimental study of steady natural convection in a small aspect ratio cylindrical cavity with circular cross section. The main objective of the present communication is to highlight some difficulties encountered when a comparison of theoretical and experimental velocity results is attempted. He kept the aspect ratio (radius/height), the Prandtl number and the Rayleigh numbers fixed to 0.28, 6 and 2.25 x 10' respectively and make observations in a vertical plane that contains the axis of symmetry of the cylinder with a particle image velocimetry (PIV) technique. From Abderrahmane *et al* [13], studies based on a numerical calculation involving the finite volume methods. The transient 2D natural convection in vertical cylindrical cavities is addressed. The aim of this study is to determine the heat transfer in a long autonomy isothermal cavity designed for the conservation of insulin cartridges or any other product. In this case of the vertical cylinder filled with air and cooled by the high face following an exponential law that the ascending flow concerns the essential of the heart of the cavity. This ascent is accompanied by an acceleration in the 90% of the cavity followed by a slowing, a stop on the higher face, then, finally, a descent along the lateral face of the cylinder.

Further papers from Hasnoui, Angirasa, Chen [15], [16], [17] have investigated natural convection in rectangular open cavity problems where else Armfield, Langerman, Federico, Matsuda, Young, Fusegi, Ranganathan, Kanchan, Ramaswamy [18], [19], [20], [21], [22], [23], [24], [25], [26] solving various problems in rectangular enclosure.

Numerical experiments from Lee T.S. [40] were performed on an incompressible fluid contained in a tilted non rectangular enclosure. Rayleigh numbers of $10^2 – 10^5$and Prandtl numbers of 0.001-100 are considered. The walls angles are 22.5 ^{o}, 45 ^{o} and 77.5 ^{o} with aspect ratios of 3 and 6. results indicate that heat transfer and fluid motion within the enclosure are strong functions of Rayleigh number, Prandtl numbers, and orientation angle of the enclosure. For Rayleigh number greater than 10^4 and Prandtl numbers greater than 0.1, a minimum and a maximum mean Nusselt Number occurred as the angle of orientation was increased from 0^o to 360^o. A transition in the mode of circulation occurred as the angles corresponding to the minimum or maximum rate of heat transfer.

The results presented here show that heat transfer, in a trapezoidal enclosure with two symmetrical, inclined sidewalls of moderate aspects ratios, is a strong function of the orientation angle of the cavity for $Ra > 10^4$ and $Pr > 0.1$. A maximum Nu occurs around $\theta = 180^o$ and minimum Nu around $\theta = 270^o$. a transition in the mode of circulation within the enclosure occurred at the tilted angle corresponding to the maximum or minimum Nu.

1.2 Current Problem

According to the various papers referred during literature review, problem as such the semi-cylindrical cavity was noted very limited and yet several unsolved boundary conditions, geometrical or solving method still open for exploration. This numerical approach will attempt the natural convection in semi-cylindrical cavity according the following figure and boundary condition.

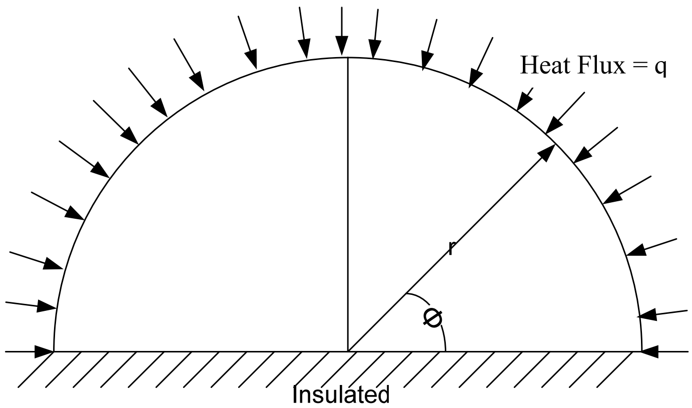

Figure 1.2 Nature problem of Semi cylindrical cavity

1.3 Scope & Objective

Scope of this work will consider the laminar regime with steady state solution and selecting low Prandtl number which the air assumed as an ideal gas. Only numerical approach will be explored throughout the total work objective. The main objective is to:-

a. Determine the temperature distribution in such cavity
b. Determine the heat transfer performance in terms of better correlation between Nusselt & Rayleigh No.
    - $Nu = f\,(Ra)$
c. Perform a parametric study on the behavior of the isotherms and streamlines in the cavity.