MODELING AND CONTROL OF ACTIVE SUSPENSION FOR A FULL CAR MODEL

ROSHEILA BINTI DARUS

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Faculty of Electrical of Engineering
University Teknologi Malaysia

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This thesis dedicated to my dearest family and friends for their encouraging and love.
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ABSTRACT

The objectives of this study are to obtain a mathematical model for the passive and active suspensions systems for full car model. Current automobile suspension systems using passive components only by utilizing spring and damping coefficient with fixed rates. Vehicle suspensions systems typically rated by its ability to provide good road handling and improve passenger comfort. Passive suspensions only offer compromise between these two conflicting criteria. Active suspension poses the ability to reduce the traditional design as a compromise between handling and comfort by directly controlling the suspensions force actuators. In this thesis, the Linear Quadratic Control (LQR) technique implemented to the active suspensions system for a full car model. Comparison between passive and active suspensions system are performed by using different types of road profiles.
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LIST OF SYMBOLS

- M₁ - mass of the wheel /unsprung mass (kg)
- M₂ - mass of the car body/sprung mass (kg)
- r - road disturbance/road profile
- Xₛ - wheel displacement (m)
- Xₜ - car body displacement (m)
- Kₛ - stiffness of car body spring (N/m)
- Kₜ - stiffness of tire (N/m)
- Cₚₐₜ - damper (Ns/m)
- Uₛ - force actuator
- b₁, bᵣ - front and rear damping (Nm/s)
- kₛ₁, kₛₚ - stiffness of car body spring for each corner (N/m)
- kₛ₁, kₛₚ - stiffness tire (N/m)
- mₛ - mass of the car body or sprung mass (kg)
- mₛ₁, mₛₚ - front and rear mass of the wheel or unsprung mass (kg)
- Iₛ₁, Iₛₚ - pitch and roll of moment of inertia (kg m²)
- Zₛ - car body displacement (m)
- Zₛ₁, Zₛ₂, Zₛ₃, Zₛ₄ - car body displacement for each corner (m)
- Xₛ, Xₛₚ - wheel displacement (m)
- Tₛ₁, Tₛₚ - front and rear treat (m)
- a, b - distance from centre of sprung mass to front wheel and rear wheel (m)
- b₁, bᵣ - front and rear damping (Nm/s)
- kₛ₁, kₛₚ - stiffness of car body spring for front and rear(N/m)
- kₛ₁, kₛₚ - stiffness tire (N/m)
- u₁, u₂ - front right and left force actuators
- u₃, u₄ - rear right and left force actuators
- Xₛ - Xₛₚ - suspension travel
\( \dot{X}_s \) - car body velocity
\( \ddot{X}_s \) - car body acceleration
\( X_w - r \) - wheel deflection
\( \dot{X}_w \) - wheel velocity
\( x_1 (Z_u) \) - vertical displacement car body
\( x_2 (\theta_s) \) - pitch angle
\( x_3 (\phi_s) \) - roll angle
\( x_4 (Z_{u1}) \) - vertical displacement front right wheel
\( x_5 (Z_{u2}) \) - vertical displacement front left wheel
\( x_6 (Z_{u3}) \) - vertical displacement rear right wheel
\( x_7 (Z_{u4}) \) - vertical displacement rear left wheel
\( x_8 (\dot{Z}_r) \) - vertical velocity car body
\( x_9 (\dot{\theta}_s) \) - pitch rate
\( x_{10} (\dot{\phi}_s) \) - roll rate
\( x_{11} (\dot{Z}_{u1}) \) - vertical velocity front right wheel
\( x_{12} (\dot{Z}_{u2}) \) - vertical velocity front left wheel
\( x_{13} (\dot{Z}_{u3}) \) - vertical velocity front right wheel
\( x_{14} (\dot{Z}_{u3}) \) - vertical velocity front left wheel
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CHAPTER 1

INTRODUCTION

1.1 Introduction

Traditionally automotive suspension designs have been compromise between the three conflicting criteria’s namely road handling, load carrying, and passenger comfort. The suspension system must support the vehicle, provide directional control using handling maneuvers and provide effective isolation of passengers and load disturbance. Good ride comfort requires a soft suspension, where as insensitivity to apply loads require stiff suspension. Good handling requires a suspension setting somewhere between it. Due to these conflicting demands, suspension design has to be something that can compromise of these two problems.

A passive suspension has the ability to store energy via a spring and to dissipate it via a damper. Its parameters are generally fixed, being chosen to achieve a certain level of compromise between road handling, load carrying and ride comfort. An active suspension system has the ability to store, dissipate and to introduce energy to the system. It may vary its parameters depending upon operating conditions.

Suspension consists of the system of springs, shock absorbers and linkages that connects a vehicle to its wheels. In other meaning, suspension system is a mechanism that physically separates the car body from the car wheel. The main function of vehicle suspension system is to minimize the vertical acceleration transmitted to the passenger which directly provides road comfort. There are three
types of suspension system; passive, semi-active and active suspension system. Traditional suspension consists springs and dampers are referred to as passive suspension, then if the suspension is externally controlled it is known as a semi-active or active suspension.

1.2 Literature Review of Linear Passive Suspension System

Passive suspension system can be found in controlling the dynamics of vertical motion of a vehicle. There is no energy supplied by the suspension element to the system. Even though it doesn’t apply energy to the system, but it controls the relative motion of the body to the wheel by using different types of damping or energy dissipating elements. Passive suspension has significant limitation in structural applications. The characteristic are determined by the designer according to the design goals and the intended application. The disadvantage of passive suspension system is it has fix characteristic, for example if the designer design the suspension heavily damped it will only give good vehicle handling but at the same time it transfer road input (disturbance) to the vehicle body. The result of this action is if the vehicle travel at the low speed on a rough road or at the high speed in a straight line, it will be perceived as a harsh road. Then, if the suspension is design lightly damped, it will give more comfortable ride. Unfortunately this design will reduce the stability of the vehicle in make turn and lane changing. Figure 1.1 shows traditional passive suspension components system that consists of spring and damper.
1.3 Literature Review of Linear Semi-active Suspension System

Semi-active suspension system was first proposed in 1970’s. It’s provides a rapid change in rate of springs damping coefficients. It does not provide any energy into suspension system but the damper is replaced by controllable damper. The controller’s determine the level of damping based on control strategy and automatically adjust the damper to the desired levels. This type of suspension system used external power to operate. Sensors and actuator are added to detect the road profile for control input. The most commonly semi-active suspension system is called skyhook damper. Schematic diagram for semi-active suspension is shown is Figure 1.2.
1.4 **Literature Review of Linear Active Suspension System**

Active suspension system has the ability to response to the vertical changes in the road input. The damper or spring is interceding by the force actuator. This force actuator has it own task which is to add or dissipate energy from the system. The force actuator is control by various types of controller determine by the designer. The correct control strategy will give better compromise between comfort and vehicle stability. Therefore active suspension system offer better riding comfort and vehicle handling to the passengers. Figure 1.3 shows simple block diagram to explain how the active suspension can achieve better performance. Figure 1.4 describe basic component of active suspension. In this type of suspension the controller can modify the system dynamics by activating the actuators.
All these three types of suspension systems have their own advantages and disadvantages. However, researchers are focusing on the active car suspension and it is because the performance obtained is better than the other two types of suspension systems as mentioned before. For example, the passive suspension system's design is fixed depending on the goal of the suspension. The passive suspension is an open loop control system. It doesn’t have any feedback signal to correct the error. It means that the suspension system will not give optimal ride comfort. In other side which is
active suspension, it has that ability to give ride comfort. This is happen by having force actuator control by the controller. The active suspension system is a close loop control system. It will correct the error and gave the output to the desired level. In this project observation will be made at the vertical acceleration of the vehicle body called sprung mass and tire deflection. By using the right control strategy the ride quality and handling performance can be optimize. Therefore, in this project there will be modeling for active and passive suspension only.

1.5 Literature Review of Vehicle Model

Quarter-car model in Figure 1.5 is very often used for suspension analysis; because it simple and can capture important characteristics of full model. The equation for the model motions are found by adding vertical forces on the sprung and unsprung masses. Most of the quarter-car model suspension will represent the M as the sprung mass, while tire and axles are illustrated by the unsprung mass m. The spring, shock absorber and a variable force-generating element placed between the sprung and unsprung masses constitutes suspension.

From the quarter car model, the design can be expend into full car model by adding the link between the sprung mass to the four unsprung masses (body - front left and right, rear left and right). Generally the link between sprung and unsprung masses will gave roll and pitch angle. The basic modeling is still the same but there is additional consideration about the rolling, pitching and bouncing need to be count. The rolling, pitching and bouncing can be represented in the X, Y and Z axis. Figure 1.6 ~ Figure 1.7 shows the quarter car and full car models.
Figure 1.5: Quarter Car Model

Figure 1.6: Half Car Model
1.6 Problem Statement

The passive suspension system is an open loop control system. It only designs to achieve certain condition only. The characteristic of passive suspension fix and cannot be adjusted by any mechanical part. The problem of passive suspension is if it designs heavily damped or too hard suspension it will transfer a lot of road input or throwing the car on unevenness of the road. Then, if it lightly damped or soft suspension it will give reduce the stability of vehicle in turns or change lane or it will swing the car. Therefore, the performance of the passive suspension depends on the road profile. In other way, active suspension can gave better performance of suspension by having force actuator, which is a close loop control system. The force actuator is a mechanical part that added inside the system that control by the controller. Controller will calculate either add or dissipate energy from the system, from the help of sensors as an input. Sensors will give the data of road profile to the controller. Therefore, an active suspension system shown is
Figure 1.8 is needed where there is an active element inside the system to give both conditions so that it can improve the performance of the suspension system. In this project the main objective is to observe the performance of active by using LQR controller and passive suspension only.

![Active Suspension System](image)

**Figure 1.8: Active Suspension System**

### 1.7 Objective

The objectives of this project are:

1. To establish the active and passive suspension system models.
2. To establish the mathematical model for active and passive suspension system for quarter and full car models.
3. To observe the performance of active suspension system with LQR controller through the computer simulation work.
1.8 Scope of project

The scopes of work for this project are:

i) To derive and establish the mathematical equation for passive and active suspension for quarter car model for literature purpose.

ii) To implement LQR controller to the active suspension system for full car model.

iii) Computer simulation study by using MATLAB/Simulink

1.9 Research Methodology

a) To understand active and passive suspension component
   (i) Literature research on active and passive suspension
   (ii) Identify type of active and passive suspension component.
   (iii) Literature research about control strategy

b) To derive and establish mathematical model for active and passive suspension system for full car model.
   (i) By using physical laws from the suspension components get the state space equation for quarter car model, continue expand the equation to a full car model.
   (ii) By using the matrix equation given, get the state space equation.
   (iii) Continue to get the state space equation for full car model.

c) Implementation of LQR controller into the system.
   (i) Literature review on the control technique.
   (ii) Use LQR to compare output performance compare with passive suspension.
d) Computer simulation
   (i) Involves learning how to transform the state space equation into
       SIMULINK diagram.
   (ii) Simulation of propose controller using MATLAB/Simulink.

e) Result analysis
   (i) Involves observation of the preliminary result.
   (ii) Simulation to investigate dynamics of active suspension.
   (iii) Obtain suitable matrix feedback gain.
   (iv) Observe force generated from the simulation.
Flow chart in Figure 1.9 shows overall process for the project research.

Figure 1.9: Flow Chart for Computer Simulation
1.10 Thesis Outlines

This project is organized into 5 chapters. Chapter 1 discusses literature review on passive, semi-active and active suspension system. Objectives, scope of project and research methodology are explained in this chapter.

Further explanations on the mathematical modeling for a quarter car and a full car model for active and passive suspension system are included in chapter 2. Mathematical model can described behavior of overall system. This chapter explains method used in this research in order to obtain mathematical model for passive and active suspension system for a full car model.

Chapter 3 reviews relevant literatures and previous works regarding controller design. Controller design for the project is also included which is LQR controller.

In chapter 4, computer simulation between passive and active suspension system will be carried out. There are two types of input disturbance that will be used to test the system. Simulation based on the mathematical model of a full car model is done by using MATLAB/SIMULINK software.

At the end of this project report, conclusion and future works will be discussed furthermore in chapter 5. In addition to that, some recommendations to improve the outcomes for this project are discussed in this chapter.
CHAPTER 2

MATHEMATICAL MODELING

2.1 Introduction

The main focus of this chapter is to provide background for mathematical model of a full car model. The dynamic model, which can describes the relationship between the input and output, enables ones to understand the behaviour of the system. Firstly, this chapter will discuss how to obtain mathematical model passive and active suspension for quarter car model. Then, continue with for full car model. Figure 2.1 shows general principles of mathematical modeling. The purpose of mathematical modeling in this project is to obtain a state space representation of the full car model. In this project the suspension system is modeled as a linear suspension system. The state variable can be represented as a vertical movement of the car body and a vertical movement of the wheels. In order to obtain linear model, roll and pitch angles are assume to be small.
2.2 Mathematical Modeling of Passive Suspension for Quarter Car Model.

From the Figure 2.1 obtain the mathematical equation

For $M_1$

\[
F = Ma \\
K_w(X_w - r) - K_a(X_w - X_s) - C_a(\dot{X}_w - \dot{X}_s) = M_1\ddot{X}_w \tag{2.1}
\]

\[
\ddot{X}_w = \frac{K_w(X_w - r) - K_a(X_w - X_s) - C_a(\dot{X}_w - \dot{X}_s)}{M_1}
\]

For $M_2$

\[
F = Ma \\
- K_a(X_s - X_w) - C_a(\dot{X}_s - \dot{X}_w) = M_2\ddot{X}_s \tag{2.2}
\]

\[
\ddot{X}_s = \frac{-K_a(X_s - X_w) - C_a(\dot{X}_s - \dot{X}_w)}{M_2}
\]

where;

- $M_1$ = mass of the wheel /unsprung mass (kg)
- $M_2$ = mass of the car body/sprung mass (kg)
- $r$ = road disturbance/road profile
- $X_w$ = wheel displacement (m)
- $X_s$ = car body displacement (m)
- $K_a$ = stiffness of car body spring (N/m)
- $K_t$ = stiffness of tire (N/m)
- $C_a$ = damper (Ns/m)

Let the state variables are

\[
X_1 = X_s - X_w \\
X_2 = \dot{X}_s \\
X_3 = X_w - r \\
X_4 = \dot{X}_w \tag{2.3}
\]
where

\[ X_s - X_w = \text{suspension travel} \]
\[ \dot{X}_s = \text{car body velocity} \]
\[ \ddot{X}_s = \text{car body acceleration} \]
\[ X_w - r = \text{wheel deflection} \]
\[ \dot{X}_w = \text{wheel velocity} \]

Therefore in state space equation, equation (2.3) can be written as;

\[ \dot{X}(t) = Ax(t) + f(t) \]  

(2.4)

where

\[ \dot{X}_1 = \dot{X}_s - \dot{X}_w \approx X_2 - X_4 \]
\[ \dot{X}_2 = \ddot{X}_s \]
\[ \dot{X}_3 = \dot{X}_w - \dot{r} \approx X_4 - \dot{r} \]
\[ \dot{X}_4 = \dot{X}_w \]  

(2.5)

Rewrite equation (2.4) into the matrix form

\[
\begin{bmatrix}
\dot{X}_1 \\
\dot{X}_2 \\
\dot{X}_3 \\
\dot{X}_4 
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 & 0 & -1 \\
-K_a & -C_a & 0 & M_2 \\
0 & 0 & 0 & M_1 \\
-K_a & C_a & K_t & -C_a \\
M_1 & M_1 & M_1 & M_1
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4 
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0 \\
0 \\
-1 
\end{bmatrix} \dot{r} 
\]

(2.6)
Parameters of the system are taken from [2] and as shows in Table 2.1

<table>
<thead>
<tr>
<th>Parameter for Quarter Car</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1 = 59kg$</td>
</tr>
<tr>
<td>$M_2 = 290kg$</td>
</tr>
<tr>
<td>$K_a = 16812(N/m)$</td>
</tr>
<tr>
<td>$K_t = 190000(N/m)$</td>
</tr>
<tr>
<td>$C_a = 1000(Ns/m)$</td>
</tr>
</tbody>
</table>

### 2.3 Mathematical Modeling of Passive Suspension for a Full Car Model

In the full car model the car body or sprung mass is free to heave roll and pitch. The suspension system connects the sprung mass to the four unsprung masses which are front-left, front-right, rear-left, and rear-right wheels. They are free to bounce vertically with respect to the sprung mass. Matrix equation for full car model cannot be displayed due to large matrix size. It directly converts inside MATLAB/SIMULINK and the general principle can refer in section 2.2. Mathematical model for full car suspension is established from [3].

![Figure 2.3: Full Car Model](image-url)
For rolling motion of the sprung mass

\[
I_r \ddot{\phi}_s = -b_j T_f (\dot{Z}_{s1} - \dot{Z}_{u1}) + b_j T_f (\dot{Z}_{s2} - \dot{Z}_{u2}) - b_r T_r (\dot{Z}_{s3} - \dot{Z}_{u3}) \\
+ b_i T_i (\dot{Z}_{s4} - \dot{Z}_{u4}) - k_j T_f (Z_{s1} - Z_{u1}) + k_j T_f (Z_{s2} - Z_{u2}) \\
- k_r T_r (Z_{s3} - Z_{u3}) + k_r T_r (Z_{s4} - Z_{u4})
\]  

(2.7)

For pitching motion of the sprung mass

\[
I_p \ddot{\theta}_s = -b_j a (\dot{Z}_{s1} - \dot{Z}_{u1}) - b_j a (\dot{Z}_{s2} - \dot{Z}_{u2}) + b_i b (\dot{Z}_{s3} - \dot{Z}_{u3}) \\
+ b_i b (\dot{Z}_{s4} - \dot{Z}_{u4}) - k_j a (Z_{s1} - Z_{u1}) - k_j a (Z_{s2} - Z_{u2}) \\
+ k_i b (Z_{s3} - Z_{u3}) + k_i b (Z_{s4} - Z_{u4})
\]  

(2.8)

For bouncing of the sprung mass

\[
m_j \ddot{Z}_s = -b_j (\dot{Z}_{s1} - \dot{Z}_{u1}) - b_j (\dot{Z}_{s2} - \dot{Z}_{u2}) - b_r (\dot{Z}_{s3} - \dot{Z}_{u3}) \\
- b_i (\dot{Z}_{s4} - \dot{Z}_{u4}) - k_j (Z_{s1} - Z_{u1}) - k_j (Z_{s2} - Z_{u2}) \\
- k_r (Z_{s3} - Z_{u3}) - k_i (Z_{s4} - Z_{u4})
\]  

(2.9)

And also for each side of wheel motion (vertical direction)

\[
m_{uj} \ddot{Z}_{u1} = b_j (\dot{Z}_{s1} - \dot{Z}_{u1}) + k_j (Z_{s1} - Z_{u1}) - k_i Z_{u1} + k_i Z_{u1}
\]  

(2.10)

\[
m_{uj} \ddot{Z}_{u2} = b_j (\dot{Z}_{s2} - \dot{Z}_{u2}) + k_j (Z_{s2} - Z_{u2}) - k_i Z_{u2} + k_i Z_{u2}
\]  

(2.11)

\[
m_{uj} \ddot{Z}_{u3} = b_r (\dot{Z}_{s3} - \dot{Z}_{u3}) + k_r (Z_{s3} - Z_{u3}) - k_i Z_{u3} + k_i Z_{u3}
\]  

(2.12)

\[
m_{uj} \ddot{Z}_{u4} = b_r (\dot{Z}_{s4} - \dot{Z}_{u4}) + k_r (Z_{s4} - Z_{u4}) - k_i Z_{u4} + k_i Z_{u4}
\]  

(2.13)

where

\[
Z_{s1} = T_f \phi_s + a \theta_s + Z_s \\
\dot{Z}_{s1} = T_f \dot{\phi}_s + a \dot{\theta}_s + \dot{Z}_s
\]  

(2.14)
\[ Z_{s2} = -T_f \varphi_s + a \theta_s + Z_s \]
\[ \dot{Z}_{s2} = -T_f \dot{\varphi}_s + a \dot{\theta}_s + \dot{Z}_s \]  
\[ Z_{s3} = T_f \varphi_s - b \theta_s + Z_s \]
\[ \dot{Z}_{s3} = T_f \dot{\varphi}_s - b \dot{\theta}_s + \dot{Z}_s \]  
\[ Z_{s4} = -T_r \varphi_s - b \theta_s + Z_s \]
\[ \dot{Z}_{s4} = -T_r \dot{\varphi}_s - b \dot{\theta}_s + \dot{Z}_s \]

(2.15)

(2.16)

(2.17)

where

\[ m_s = \text{mass of the car body or sprung mass (kg)} \]
\[ m_{uf} \text{ and } m_{ur} = \text{front and rear mass of the wheel or unsprung mass (kg)} \]
\[ I_p \text{ and } I_r = \text{pitch and roll of moment of inertia (kg m}^2) \]
\[ Z_s = \text{car body displacement (m)} \]
\[ Z_{s1}, Z_{s2}, Z_{s3} \text{ and } Z_{s4} = \text{car body displacement for each corner (m)} \]
\[ Z_{u1}, Z_{u2}, Z_{u3} \text{ and } Z_{u4} = \text{wheel displacement (m)} \]
\[ T_f \text{ and } T_r = \text{front and rear treat (m)} \]
\[ a = \text{distance from centre of sprung mass to front wheel (m)} \]
\[ b = \text{distance from centre of sprung mass to rear wheel (m)} \]
\[ b_f \text{ and } b_r = \text{front and rear damping (Nm/s)} \]
\[ k_f \text{ and } k_r = \text{stiffness of car body spring for front and rear (N/m)} \]
\[ k_{tf} \text{ and } k_{tr} = \text{stiffness tire (N/m)} \]
The state variables of the system are shown in Table 2.2 and the definition of each state variable is given in Table 2.3.

### Table 2.2: State Variable for Full Car Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = \phi_s$</td>
<td>Roll angle</td>
</tr>
<tr>
<td>$X_2 = \theta_s$</td>
<td>Roll rate</td>
</tr>
<tr>
<td>$X_3 = Z_s$</td>
<td>Pitch angle</td>
</tr>
<tr>
<td>$X_4 = Z_{u1}$</td>
<td>Pitch rate</td>
</tr>
<tr>
<td>$X_5 = Z_{u2}$</td>
<td>Vertical displacement</td>
</tr>
<tr>
<td>$X_6 = Z_{u3}$</td>
<td>Vertical velocity</td>
</tr>
<tr>
<td>$X_7 = Z_{u4}$</td>
<td>Vertical displacement of front right wheel</td>
</tr>
<tr>
<td>$X_8 = \dot{\phi}_s$</td>
<td>Roll rate</td>
</tr>
<tr>
<td>$X_9 = \dot{\theta}_s$</td>
<td>Pitch rate</td>
</tr>
<tr>
<td>$X_{10} = \dot{Z}_s$</td>
<td>Vertical velocity</td>
</tr>
<tr>
<td>$X_{11} = \dot{Z}_{u1}$</td>
<td>Vertical displacement of front right wheel</td>
</tr>
<tr>
<td>$X_{12} = \dot{Z}_{u2}$</td>
<td>Vertical velocity of front left wheel</td>
</tr>
<tr>
<td>$X_{13} = \dot{Z}_{u3}$</td>
<td>Vertical displacement of rear right wheel</td>
</tr>
<tr>
<td>$X_{14} = \dot{Z}_{u4}$</td>
<td>Vertical velocity of rear left wheel</td>
</tr>
</tbody>
</table>

### Table 2.3: Definitions of Variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_s$</td>
<td>Roll angle</td>
</tr>
<tr>
<td>$\dot{\phi}_s$</td>
<td>Roll rate</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>Pitch angle</td>
</tr>
<tr>
<td>$\dot{\theta}_s$</td>
<td>Pitch rate</td>
</tr>
<tr>
<td>$Z_s$</td>
<td>Vertical displacement</td>
</tr>
<tr>
<td>$\dot{Z}_s$</td>
<td>Vertical velocity</td>
</tr>
<tr>
<td>$Z_{u1}$</td>
<td>Vertical displacement of front right wheel</td>
</tr>
<tr>
<td>$\dot{Z}_{u1}$</td>
<td>Vertical velocity of front right wheel</td>
</tr>
<tr>
<td>$Z_{u2}$</td>
<td>Vertical displacement of front left wheel</td>
</tr>
<tr>
<td>$\dot{Z}_{u2}$</td>
<td>Vertical velocity of front left wheel</td>
</tr>
<tr>
<td>$Z_{u3}$</td>
<td>Vertical displacement of rear right wheel</td>
</tr>
<tr>
<td>$\dot{Z}_{u3}$</td>
<td>Vertical velocity of rear right wheel</td>
</tr>
<tr>
<td>$Z_{u4}$</td>
<td>Vertical displacement of rear left wheel</td>
</tr>
<tr>
<td>$\dot{Z}_{u4}$</td>
<td>Vertical velocity of rear left wheel</td>
</tr>
</tbody>
</table>
State space form is shown in equation (2.4). Therefore equations (2.7) ~ (2.13) can be written as below

\[ \dot{X}_1 = \dot{\varphi}_1 \approx X_8 \]

\[ \dot{X}_2 = \dot{\theta}_3 \approx X_9 \]

\[ \dot{X}_3 = \dot{Z}_3 \approx X_{10} \]

\[ \dot{X}_4 = \dot{Z}_{u1} \approx X_{11} \]

\[ \dot{X}_5 = \dot{Z}_{u2} \approx X_{12} \]

\[ \dot{X}_6 = \dot{Z}_{u3} \approx X_{13} \]

\[ \dot{X}_7 = \dot{Z}_{u4} \approx X_{14} \]

\[ \dot{X}_8 = \dot{\varphi}_3 \approx [-b_f T_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + b_f T_f ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) - b_f T_r ((T_f X_8 - bX_9 + X_{10}) - X_{13}) + b_f T_r ((-T_f X_8 - bX_9 + X_{10}) - X_{14}) - k_f T_f ((T_f X_1 + aX_2 + X_3) - X_4) + k_f T_r ((-T_f X_1 + aX_2 + X_3) - X_5) - k_f T_r ((T_f X_1 - bX_2 + X_3) - X_6) + k_f T_r ((-T_f X_1 - bX_2 + X_3) - X_7)]/I_f \]

\[ \dot{X}_9 = \dot{\theta}_4 \approx [-b_f a((T_f X_8 + aX_9 + X_{10}) - X_{11}) - b_f a((-T_f X_8 + aX_9 + X_{10}) - X_{12}) + b_f b(T_f X_8 - bX_9 + X_{10}) - X_{13}) + b_f b((-T_f X_8 - bX_9 + X_{10}) - X_{14}) - k_f a((T_f X_1 + aX_2 + X_3) - X_4) - k_f b((-T_f X_1 - bX_2 + X_3) - X_5)]/I_p \]

\[ \dot{X}_{10} = \dot{Z}_4 \approx [-b_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) - b_f ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) - b_f ((T_f X_8 - bX_9 + X_{10}) - X_{13}) - b_f ((-T_f X_8 - bX_9 + X_{10}) - X_{14}) - k_f ((T_f X_1 + aX_2 + X_3) - X_4) - k_f ((-T_f X_1 - bX_2 + X_3) - X_5)]/m \]

\[ \dot{X}_{11} = \dot{Z}_{u1} \approx [b_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + k_f ((T_f X_1 + aX_2 + X_3) - X_4) - k_f X_4 + k_f Z_{r1}]/m_f \]

\[ \dot{X}_{12} = \dot{Z}_{u2} \approx [b_f ((-T_f X_8 + aX_9 + X_{10}) - X_{11}) + k_f ((-T_f X_1 + aX_2 + X_3) - X_5) - k_f X_5 + k_f Z_{r2}]/m_f \]
\[ \dot{X}_{13} = \ddot{Z}_{u2} \approx \left\{ b_T ((T_r X_8 - bX_9 + X_{10}) - X_{13}) + k_r ((T_r X_4 - bX_2 + X_3) - X_6) ight\} \]

\[-k_r X_6 + k_r \ddot{Z}_{r4} / m_w \]

\[ \dot{X}_{14} = \ddot{Z}_{u2} \approx \left\{ b_T ((-T_r X_8 - bX_9 + X_{10}) - X_{14}) + k_r ((-T_r X_4 - bX_2 + X_3) - X_7) ight\} \]

\[-k_r X_7 + k_r \ddot{Z}_{r4} / m_w \]

Followed by, covert equation (2.4) into the matrix yield

\[
\dot{X}(t) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} [X(t)] + f(t) \tag{2.18}
\]

where:

\[
\dot{X}(t) = \begin{bmatrix} \dot{X}_1 & \dot{X}_2 & \dot{X}_3 & \dot{X}_4 & \dot{X}_5 & \dot{X}_6 & \dot{X}_7 & \dot{X}_8 & \dot{X}_9 & \dot{X}_{10} & \dot{X}_{11} & \dot{X}_{12} & \dot{X}_{13} & \dot{X}_{14} \end{bmatrix}^T
\]

\[
\dot{X}(t) \approx \begin{bmatrix} \dot{\phi}_s & \dot{\theta}_s & \dot{\phi}_u_1 & \dot{\phi}_u_2 & \dot{\phi}_u_3 & \dot{\phi}_u_4 & \dot{\theta}_s & \dot{\theta}_u_1 & \dot{\theta}_u_2 & \dot{\theta}_u_3 & \dot{\theta}_u_4 \end{bmatrix}^T
\]

\[
X(t) = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 & X_{10} & X_{11} & X_{12} & X_{13} & X_{14} \end{bmatrix}^T
\]

\[
X(t) \approx \begin{bmatrix} \phi_s & \theta_s & Z_s & Z_{u1} & Z_{u2} & Z_{u3} & Z_{u4} & \phi_s & \theta_s & Z_s & Z_{u1} & Z_{u2} & Z_{u3} & Z_{u4} \end{bmatrix}^T
\]

The complete elements of matrix A and f(t), are shown in Appendix A.

Furthermore, values of such parameters are shows in Table 2.4

<table>
<thead>
<tr>
<th>Table 2.4: Parameter for a Full Car Model [3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_s = 1136 kg</td>
</tr>
<tr>
<td>m_{uf} and m_{ur} = 63 kg and 60 kg</td>
</tr>
<tr>
<td>k_f and k_r = 182470 N/m</td>
</tr>
<tr>
<td>I_p and I_r = 2400 kg m^2 and 400 kg m^2</td>
</tr>
<tr>
<td>k_f and k_r = 36297 N/m and 19620 N/m</td>
</tr>
<tr>
<td>a and b = 1.15m and 1.65 m</td>
</tr>
</tbody>
</table>
2.4 Mathematical Modeling of Active Suspension for Quarter Car Model

Mathematical modeling for active suspension is derived from Figure 2.1 and Figure 2.3. There is slightly difference in the derivation of the mathematical modeling for active suspension from the passive suspension system. Derivation for $M_1$ (unsprung mass) and $M_2$ (sprung mass) is shown below:

Figure 2.4: Active Suspension for Quarter Car Model
For $M_1$

$$F = Ma$$

$$K(X_w - r) - K_a(X_w - X_s) - C_a(\ddot{X}_w - \ddot{X}_s) - U_a = M_1\ddot{X}_w$$  \hspace{1cm} (2.19)

$$\ddot{X}_w = \frac{K(X_w - r) - K_a(X_w - X_s) - C_a(\ddot{X}_w - \ddot{X}_s) - U_a}{M_1}$$

For $M_2$

$$F = Ma$$

$$- K_a(X_s - X_w) - C_a(\ddot{X}_s - \ddot{X}_w) + U_a = M_2\ddot{X}_s$$  \hspace{1cm} (2.20)

$$\ddot{X}_s = \frac{- K_a(X_s - X_w) - C_a(\ddot{X}_s - \ddot{X}_w) + U_a}{M_2}$$

where:

$M_1$ = mass of the wheel /unsprung mass (kg)

$M_2$ = mass of the car body/sprung mass (kg)

$r$ = road disturbance/road profile

$X_w$ = wheel displacement (m)

$X_s$ = car body displacement (m)

$K_a$ = stiffness of car body spring (N/m)

$K_t$ = stiffness of tire (N/m)

$C_a$ = damper (Ns/m)

$U_a$ = force actuator

The state variables are established in equation (2.3). Therefore, equations (2.19) and (2.20) can be written as below

$$\dot{X}(t) = Ax(t) + Bu(t) + f(t)$$  \hspace{1cm} (2.21)
Rewrite equation (2.20) into the matrix form yield

\[
\begin{bmatrix}
\ddot{X}_1 \\
\ddot{X}_2 \\
\ddot{X}_3 \\
\ddot{X}_4
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & -1 \\
-K_a & -C_a & 0 & C_a \\
M_2 & M_2 & 0 & M_3 \\
0 & 0 & C_a & K_r \\
M_1 & M_1 & 0 & M_1
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
X_4
\end{bmatrix} + \begin{bmatrix}
0 \\
1 \\
0 \\
-1
\end{bmatrix} U_a + \begin{bmatrix}
0 \\
-1 \\
0 \\
0
\end{bmatrix}
\]  
(2.22)

2.5 Mathematical Modeling of Active Suspension for a Full Car Model

Mathematical modeling of active suspension for full car model is derived from Figures 2.2 and 2.3. Similar approach to a quarter car modeling for active suspension is used to model the full car system. The equations of motion for full car model can be derived as follows.

Equation of rolling, pitching, bouncing motion of the sprung mass and wheel motion is presented in equation (2.23) ~ equation (2.29)

\[
I_p \ddot{\phi}_j = -b_j T_j (\dot{Z}_{s1} - \dot{Z}_{a1}) + b_j T_j (\dot{Z}_{s2} - \dot{Z}_{a2}) - b_j T_j (\dot{Z}_{s3} - \dot{Z}_{a3}) \\
+b_j T_j (\dot{Z}_{s4} - \dot{Z}_{u4}) - k_j T_j (Z_{s1} - Z_{a1}) + k_j T_j (Z_{s2} - Z_{a2}) \\
-k_j T_j (Z_{s3} - Z_{a3}) + k_j T_j (Z_{s4} - Z_{u4}) + T_j u_1 - T_j u_2 + T_r u_3 - T_r u_4
\]  
(2.23)

\[
I_p \ddot{\theta}_j = -b_j a(\dot{Z}_{s1} - \dot{Z}_{u1}) - b_j a(\dot{Z}_{s2} - \dot{Z}_{u2}) + b_j b(\dot{Z}_{s3} - \dot{Z}_{u3}) \\
+b_j b(\dot{Z}_{s4} - \dot{Z}_{u4}) - k_j a(Z_{s1} - Z_{u1}) - k_j a(Z_{s2} - Z_{u2}) \\
+ k_j b(Z_{s3} - Z_{u3}) + k_j b(Z_{s4} - Z_{u4}) + au_1 + au_2 - bu_3 - bu_4
\]  
(2.24)

\[
m_t \ddot{Z}_s = -b_j (\dot{Z}_{s1} - \dot{Z}_{u1}) - b_j (\dot{Z}_{s2} - \dot{Z}_{u2}) - b_j (\dot{Z}_{s3} - \dot{Z}_{u3}) \\
-b_j (\dot{Z}_{s4} - \dot{Z}_{u4}) - k_j (Z_{s1} - Z_{u1}) - k_j (Z_{s2} - Z_{u2}) \\
-k_j (Z_{s3} - Z_{u3}) - k_j (Z_{s4} - Z_{u4}) + u_1 + u_2 + u_3 + u_4
\]  
(2.25)
\[
m_{uf} \ddot{Z}_{u1} = b_f (\dot{Z}_{s1} - \dot{Z}_{u1}) + k_f (Z_{s1} - Z_{u1}) - k_{uf} Z_{u1} - u_1 + k_{uf} Z_{r1} \tag{2.26}
\]
\[
m_{uf} \ddot{Z}_{u2} = b_f (\dot{Z}_{s2} - \dot{Z}_{u2}) + k_f (Z_{s2} - Z_{u2}) - k_{uf} Z_{u2} - u_2 + k_{uf} Z_{r2} \tag{2.27}
\]
\[
m_{ur} \ddot{Z}_{u3} = b_r (\dot{Z}_{s3} - \dot{Z}_{u3}) + k_r (Z_{s3} - Z_{u3}) - k_{ur} Z_{u3} - u_3 + k_{ur} Z_{r3} \tag{2.28}
\]
\[
m_{ur} \ddot{Z}_{u4} = b_r (\dot{Z}_{s4} - \dot{Z}_{u4}) + k_r (Z_{s4} - Z_{u4}) - k_{ur} Z_{u4} - u_4 + k_{ur} Z_{r4} \tag{2.29}
\]

where

\[
Z_{s1} = T_f \phi_f + a \theta_s + Z_s \\
\dot{Z}_{s1} = T_f \dot{\phi}_f + a \dot{\theta}_s + \dot{Z}_s \tag{2.30}
\]

\[
Z_{s2} = -T_f \phi_f + a \theta_s + Z_s \\
\dot{Z}_{s2} = -T_f \dot{\phi}_f + a \dot{\theta}_s + \dot{Z}_s \tag{2.31}
\]

\[
Z_{s3} = T_r \phi_r - b \theta_s + Z_s \\
\dot{Z}_{s3} = T_r \dot{\phi}_r - b \dot{\theta}_s + \dot{Z}_s \tag{2.32}
\]

\[
Z_{s4} = -T_r \phi_r - b \theta_s + Z_s \\
\dot{Z}_{s4} = -T_r \dot{\phi}_r - b \dot{\theta}_s + \dot{Z}_s \tag{2.33}
\]

where

- \( m_k \) = mass of the car body or sprung mass (kg)
- \( m_{uf} \) and \( m_{ur} \) = front and rear mass of the wheel or unsprung mass (kg)
- \( I_p \) and \( I_r \) = pitch and roll of moment of inertia (kg m\(^2\))
- \( Z_s \) = car body displacement (m)
- \( Z_{s1}, Z_{s2}, Z_{s3}, \) and \( Z_{s4} \) = car body displacement for each corner (m)
- \( Z_{u1}, Z_{u2}, Z_{u3}, \) and \( Z_{u4} \) = wheel displacement (m)
- \( T_f \) and \( T_r \) = front and rear treat (m)
- \( a \) = distance from centre of sprung mass to front wheel (m)
- \( b \) = distance from centre of sprung mass to rear wheel (m)
- \( b_f \) and \( b_r \) = front and rear damping (Nm/s)
- \( k_f \) and \( k_r \) = stiffness of car body spring for front and rear (N/m)
- \( k_{uf} \) and \( k_{ur} \) = tire stiffness (N/m)
and u₂ = front right and left force actuators

u₃ and u₄ = rear right and left force actuators

State space form is shown in equation (2.4). Thus, equations (2.23) ~ (2.29) can be written as below

\[ \dot{X}_1 = \dot{\phi} \approx X_8 \]
\[ \dot{X}_2 = \dot{\theta} \approx X_9 \]
\[ \dot{X}_3 = \dot{Z} \approx X_{10} \]
\[ \dot{X}_4 = \dot{Z}_{u1} \approx X_{11} \]
\[ \dot{X}_5 = \dot{Z}_{u2} \approx X_{12} \]
\[ \dot{X}_6 = \dot{Z}_{u3} \approx X_{13} \]
\[ \dot{X}_7 = \dot{Z}_{u4} \approx X_{14} \]
\[ \dot{X}_8 = \dot{\phi} \approx [-b_f T_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + b_f T_f ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) \]
\[ \quad -b_f T_f ((T_f X_8 - bX_9 + X_{10}) - X_{13}) + b_f T_f ((-T_f X_8 - bX_9 + X_{10}) - X_{14}) - k_f T_f ((T_f X_1 + aX_2 + X_3) - X_4) \]
\[ \quad + k_f T_f ((-T_f X_1 + aX_2 + X_3) - X_5) - k_f T_f ((-T_f X_1 - bX_2 + X_3) - X_6) + k_f T_f ((-T_f X_1 - bX_2 + X_3) - X_7) \]
\[ \quad + T_f u_1 - T_f u_2 + T_f u_3 - T_f u_4] / I_f \]
\[ \dot{X}_9 = \theta \approx [-b_f a((T_f X_8 + aX_9 + X_{10}) - X_{11}) - b_f a((-T_f X_8 + aX_9 + X_{10}) - X_{12}) \]
\[ \quad + b_f b((T_f X_8 - bX_9 + X_{10}) - X_{13}) + b_f b((-T_f X_8 - bX_9 + X_{10}) - X_{14}) - k_f a((T_f X_1 + aX_2 + X_3) - X_4) \]
\[ \quad - k_f a((-T_f X_1 + aX_2 + X_3) - X_5) + k_f b((-T_f X_1 - bX_2 + X_3) - X_6) + k_f b((-T_f X_1 - bX_2 + X_3) - X_7) \]
\[ \quad + au_1 + au_2 - bu_3 - bu_4] / I_p \]
\[ \dot{X}_{10} = \dot{Z} \approx [-b_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) - b_f ((-T_f X_8 + aX_9 + X_{10}) - X_{12}) \]
\[ \quad - b_f ((T_f X_8 - bX_9 + X_{10}) - X_{13}) - b_f ((-T_f X_8 - bX_9 + X_{10}) - X_{14}) - k_f ((T_f X_1 + aX_2 + X_3) - X_4) \]
\[ \quad - k_f ((-T_f X_1 + aX_2 + X_3) - X_5) - k_f ((-T_f X_1 - bX_2 + X_3) - X_6) - k_f ((-T_f X_1 - bX_2 + X_3) - X_7) \]
\[ \quad + u_1 + u_3 + u_4] / m_1 \]
\[ \dot{X}_{11} = \dot{Z}_{u1} \approx [b_f ((T_f X_8 + aX_9 + X_{10}) - X_{11}) + k_f ((T_f X_1 + aX_2 + X_3) - X_4) \]
\[ \quad - k_{gf} X_4 - u_1 + k_{gf} \dot{Z}_{u1}] / m_{gf} \]
\[
\begin{align*}
\dot{X}_{12} &= \ddot{Z}_{u_2} \approx \left[ b_f \left( -T_f X_8 + a X_9 + X_{10} \right) - X_{12} \right] + \left[ k_f \left( -T_f X_1 + a X_2 + X_3 \right) - X_5 \right] \\
&\quad - k_f X_5 - u_2 + k_{\theta_2} \dot{Z}_{u_2} / m_f \\
\dot{X}_{13} &= \ddot{Z}_{u_2} \approx \left[ b_r \left( (T_f X_8 - b X_9 + X_{10}) - X_{13} \right) + k_r \left( (T_f X_1 - b X_2 + X_3) - X_6 \right) \\
&\quad - k_s X_6 - u_3 + k_{\theta_1} \dot{Z}_{u_2} \right] / m_s \\
\dot{X}_{14} &= \ddot{Z}_{u_2} \approx \left[ b_g \left( -T_f X_8 - b X_9 + X_{10} \right) - X_{14} \right] + \left[ k_g \left( -T_f X_1 - b X_2 + X_3 \right) - X_7 \right] \\
&\quad - k_s X_7 - u_4 + k_{\theta_1} \dot{Z}_{u_2} \right] / m_s \\
\end{align*}
\]

After that, rewrite equation above into the matrix yield

\[
\dot{X}(t) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} [X(t)] + B u(t) + f(t) \tag{2.30}
\]

where

\[
\begin{align*}
\dot{X}(t) &= \begin{bmatrix} \dot{X}_1 & \dot{X}_2 & \dot{X}_3 & \dot{X}_4 & \dot{X}_5 & \dot{X}_6 & \dot{X}_7 & \dot{X}_8 & \dot{X}_9 & \dot{X}_{10} & \dot{X}_{11} & \dot{X}_{12} & \dot{X}_{13} & \dot{X}_{14} \end{bmatrix}^T \\
X(t) &= \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 & X_{10} & X_{11} & X_{12} & X_{13} & X_{14} \end{bmatrix}^T \\
\end{align*}
\]

The complete elements matrix A and f(t) and about matrix B and u(t), are shown in Appendix A and B respectively.
CHAPTER 3

CONTROLLER DESIGN USING LINEAR QUADRATIC REGULATOR

3.1 Introduction

The main objective of this chapter is to design controller for the active suspension system. The controller generates forces to control output parameters such as body displacement, wheel displacement, suspension travel body acceleration and wheel deflection. An optimal controller is one of the controller that can provides good performance with respect to some given measure performance.

3.2 Control Strategy

In optimal control, the attempts to find controller that can provide the best possible performance. Control strategy is a very importance part for the active suspension system. With the correct control strategy, it will give better compromise between ride comfort and vehicle handling. Nowadays there a lot of researches have been done to improve the performance of active suspension by introducing various control strategies.

In most of the research has been done, a linearized quarter car model is used. One of the reason is it can be derived easily and it can capture basic features of a real vehicle problem. Most of the studies are concerning on controlling the forces created by the suspension damper and springs. In Section 3.2.1 ~ Section 3.2.5 reviews relevant literature and previous work regarding control strategy.
3.2.1 Linear Quadratic Regulator

The LQR approach of vehicle suspension control is widely used in background of many studies in vehicle suspension control. It has been use in a simple quarter car model, half-car, and also in full car model. The strength of LQR approach is that in using it the factors of the performance index can be weighted according to the designer’s desires or other constraints. With this type of approach, an optimal result can be obtained when all the factors performance index are taken into account [4].

Application of the LQR method to the active suspension system has been proposed [5]. In this study, the LQR method is used to improve the road handling and the ride comfort for a quarter car model.

3.2.2 Fuzzy Logic

Fuzzy logic methods can be utilized in many ways in controlled suspension systems. With appropriate membership functions and rule bases it considered to be insensitive to model. Fuzzy logic is a method of controlling a system where all input conditions are not well defined. The implementation of fuzzy logic allows the use of rule based control whereby the controller is defined by abstract that give them a fuzzy quality. The linguistic control strategy of the fuzzy algorithm serves as a fuzzy process model. Because of the linguistic statements from the rule base of the fuzzy logic controller, the control strategy resembles human thinking process. Application of fuzzy logic method is presented [6].

An, appropriate input and output variable were decided. For example in active suspension system, using numerical simulation techniques, the optimum choice for state variables was the velocity and the acceleration of the sprung mass. The output of the controller was force and represented with \( u \) (force of the actuator). The universe of discourse for both the input and the output variables was divided into three sections using the following linguistic variables, P (positive), Z (zero), and N
(negative). The universe of discourse for the input variable was found by subjecting the passive suspension to several different input conditions and viewing the maximum and minimum values for each particular input variable. Triangular membership functions were initially chosen but in triangular membership function there is a problem due to the inherent sharpness in the triangular membership function’s shape. The controller will act very fast to the slightest change in velocity or acceleration. The trepizoidal membership function can produced smoother control action due to the flatness at the top of the trapezoid shape.

### 3.2.3 Adaptive Control

Adaptive control is a type of controller that can modify its behavior in response to changes in the dynamics of the process and the character of the disturbances. This type of controller has an adjustable parameter and a mechanism for adjusting purpose. Therefore the controller became nonlinear due to adjustable parameters. An adaptive control system can be describes of having two loops as shown is Figure 3.1. The first loop is a normal feedback with the process and the controller and second loop is the parameter adjustment loop.

![Block Diagram of an Adaptive System](image)

**Figure 3.1: Block Diagram of an Adaptive System**

In adaptive control it can be divided into two types of adaptive control. First is Model Reference Adaptive control and second is Self-Tuning control. For
literature research only discuss on MRAC (Model Reference Adaptive Control). Figure 3.2 shows that the MRAC blocks diagram. The system has an ordinary feedback loop composed of the process and the controller and second feedback loop that changes the controller parameters [7]. [8] present the application of adaptive control. In this study the stability of the proposed controller is guaranteed.

![Figure 3.2: MRAC Block Diagram](image)

In MRAC the desired behavior of the system is specified by a model, and the parameters of the controller are adjusted based on the error. The error is a difference between the output of close-loop system and the model. In MRAC it has two methods, first is by using MIT rule also known as gradient method and second is Lyapunov method. The law of Lyapunov method is almost the same with the MIT rule but it has been claimed that the Lyapunov method promises stability compare with the MIT rule.

### 3.2.3.1 MIT Rule

The MIT rule is the original approach of MRAC. In MIT rule consideration of closed loop system, the controller has one adjustable parameter $\theta$. One possibility is to adjust parameters by minimizing the cost function, $J$.

$$J(\theta) = \frac{1}{2} e^2$$  \hspace{1cm} (3.1)
To make $J$ small, it is reasonable to change the parameter in the direction of the negative direction of gradient of $J$, that is,

$$\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta \theta} = -\gamma e \frac{\delta e}{\delta \theta}$$  \hspace{1cm} (3.2)

### 3.2.3.2 Lyapunov Theory

There is no guarantee that an adaptive controller based on the MIT rule will give a stable closed-loop system. Fundamental of the contribution to the stability theory for nonlinear system were made by the Russian mathematician named Lyapunov. To design the controller by using Laypunov method, first is derive a differential equation for error that contains adjustable parameter. Next, find a suitable Lyapunov function $V(e, \theta)$ usually in quadratic form. Finally derive an adaptation mechanism based on $V(e, 0)$ so that the error will be zero. Lyapunov investigate the nonlinear differential equation.

$$\frac{dx}{dt} = f(x) \hspace{1cm} f(0) = 0$$  \hspace{1cm} (3.3)

Since $f(0) = 0$, the equation has the solution $x(t) = 0$. To guarantee that a solution exists and unique, it is necessary to make some assumption about $f(x)$.

### 3.2.4 $H_\infty$ Control

$H_\infty$ controller is design in term of feasibility of certain delay dependent matrix inequalities. It confirms that $H_\infty$ control of active suspension system using the optimization of either a weighted single objective functional with hard constrains or multi objective functional is an effective way to deal with the conflicting vehicle suspension performance problem [9].
$H_\infty$ of transfer function

$$G(s) \in C^{p \times q}$$

is defined by

$$\|G(s)\|_\infty = \max \omega \in \mathbb{R} \|G(j\omega)\|$$  \hspace{1cm} (3.4)

Where, $\sigma_1$ denotes the largest singular value of the complex matrix $G(j\omega)$

Consider the system given by

$$\begin{align*}
    x &= Ax + B_1w + B_2u \\
    z &= C_1x + D_12w \\
    y &= C_2x + w
\end{align*}$$  \hspace{1cm} (3.5)

$w(t)$ is a disturbance acting on the system, while $u(t)$ is the control action and $z(t)$ is a performance index. There is also a problem with $H_\infty$ control type which is, about finding the stabilizing control law $u(t) = F(y(t))$.

The equations, stabilizes the system and minimizes the effect of the disturbance $w(t)$ on the performance index $z$. This goal can be achieved by minimizing the $H_\infty$ norm of the transfer function [9].

### 3.2.5 Sliding Mode Control

Proportional Integral Sliding Mode Control of a Quarter Car Active suspension system is presented in [2]. Propose of this study is to present new approach in controlling an active suspension system. The approach utilized the proportional integral sliding mode control scheme which asymptotic stability of the system during sliding mode is assured. Model of the suspension system used in the study is a quarter car linear suspension systems with 2 DOF. Result from the study shows that by applying the control approach, both of the control objectives, the ride comfort and the car handling performance have been improved.
3.3 Linear Quadratic Regulation (LQR) Controller Design

For the comparison purpose, the LQR approach will be utilized. LQR is one of the most popular control approaches normally been used by many researches in controlling the active suspension system. [10] has presented the LQR control approach in controlling a linear active suspension system. The study concluded that LQR control approach will result in better performance in terms of ride comfort.

Consider a state variable feedback regulator for the system as

$$u(t) = -Kx(t)$$ (3.6)

K is the state feedback gain matrix.

The optimization procedure consists of determining the control input U, which minimizes the performance index. The performance index J represents the performance characteristic requirement as well as the controller input limitation. The optimal controller of given system is defined as controller design which minimizes the following performance index.

$$J = \frac{1}{2} \int_0^T \left( x^T Q x + u^T R u \right) dt$$ (3.7)

The matrix gain K is represented by:

$$K = R^{-1} B' P$$ (3.8)

The matrix P must satisfy the reduced-matrix Riccati equation

$$A' P + PA - PBR^{-1} B' P + Q = 0$$ (3.9)
Then the feedback regulator $U$

$$u(t) = - (R^{-1}B'P)x(t)$$
$$= -Kx(t)$$  \hspace{1cm} (3.10)

### 3.4 Conclusion

In this chapter discussed several control strategies used in active suspension system. Implementation of Linear Quadratic Regulator control strategy in linear system of active suspension for a full car model. The designed matrix for feedback gain is also presented in this chapter. The crucial step is to vary the value of matrix $Q$ and matrix $R$. It is because there is effect at the transients output if matrix $Q$ too large and there also effect at the usage of control action if matrix $R$ is too large.
CHAPTER 4

SIMULATION RESULT

4.1 Introduction

This chapter discusses about linear active suspension system’s performance as described in chapter two. Simulation based on the mathematical model for quarter car and full car model by using MATLAB/SIMULINK software will be performed. Performances of the suspension system in term of ride quality and car handling will be observed, where road disturbance is assumed as the input for the system. Parameters that will be observed are the suspension travel, wheel deflection and the car body acceleration for quarter car and full car model. Road disturbance or road profile is taken from [11]. The aim is to achieve small amplitude value for suspension travel, wheel deflection and the car body acceleration. The steady state for each part also should be fast.

4.2 Simulation Parameter

In chapter two, the derivation of mathematical model for quarter and full car model has been presented for passive and active suspensions systems. The parameters also have been given for quarter and full car suspension. Mathematical model for quarter car in chapter 2 is based on [2], therefore the parameters are also based on [2].
4.3 Road profile 1 for Quarter Car Simulation

The road profile is assumed to be a single bump taken from [2]. The disturbance input representing in [2] is shown below where $a$ denotes the bump amplitude.

$$r(t) = \begin{cases} a(1 - \cos 8\pi t)/2, & 0.5 \leq t \leq 0.75 \\ 0 & \text{otherwise} \end{cases} \quad (4.1)$$

![Figure 4.1: Road Profile 1](image)

4.4 Road Profile 2 for Quarter Car Simulation

The road profile is assumed have 2 bumps as below where $a$ denotes the bump amplitude.

$$r(t) = \begin{cases} a(1 - \cos 8\pi t)/2, & 0.5 \leq t \leq 0.75 \text{ and } 3.00 \leq t \leq 3.25 \\ 0 & \text{otherwise} \end{cases} \quad (4.2)$$
4.5 Comparison between Passive and Active Suspension for Quarter Car Model

Computer simulation work is based on the equation (2.4) has been performed. Comparison between passive and active suspension for quarter car model is observed. For the LQR controller, the parameters Q and R is set to be as below.

\[
Q = \begin{bmatrix}
1000 & 0 & 0 & 0 \\
0 & 1000 & 0 & 0 \\
0 & 0 & 1000 & 0 \\
0 & 0 & 0 & 1000
\end{bmatrix}
\]

and \[ R = 0.0001 \]

Therefore, the value of gain K

\[
K = \begin{bmatrix}
-2750 & -9720 & 206400 & 8240
\end{bmatrix}
\]
Figure 4.3: Force Generated by Using LQR Controller with Road Profile 1

Figure 4.4: Car Body Displacement with Road Profile 1
Figure 4.5: Car Body Acceleration with Road Profile 1

Figure 4.6: Wheel Displacement with Road Profile 1
Figure 4.7: Wheel Deflection with Road Profile 1

Figure 4.8: Suspension Travel with Road Profile 1
Figure 4.9: Force Generate by Using LQR controller with Road Profile 2

Figure 4.10: Body Displacement with Road Profile 2
Figure 4.11: Body Acceleration with Road Profile 2

Figure 4.12: Wheel Displacement with Road Profile 2
By comparing the performance of the passive and active suspension system using LQR control technique it is clearly shows that active suspension can give lower amplitude and faster settling time. Suspension Travel for both types of Road Profile can reduce the amplitude and settling time compare to passive suspension system. Body Displacement also improve even the amplitude is slightly higher compare with passive suspension system but the settling time is very fast. Body Displacement is used to represent ride quality. Force generate by the actuator for both Road Profile is
shown in Figure 4.3 Figure 4.9. Wheel Displacement also gives slightly higher amplitude but assure fast settling time. The main purpose to observe Wheel Displacement it represents car handling performance.

4.6 Road Profile 1 for a Full Car Simulation

The road profile is assumed to be a single bump. The disturbance input is taken from [12]. Where \( a \) is set at 0.05.

\[
d(t) = \begin{cases} 
  a(1 - \cos 8\pi t) & \text{if } 0.5s \leq t \leq 0.75s \\
  0 & \text{otherwise} 
\end{cases} 
\]  

(4.3)

\[
d(t) = \begin{cases} 
  a(1 - \cos 8\pi t) & \text{if } 3.0s \leq t \leq 3.25s \\
  0 & \text{otherwise} 
\end{cases} 
\]  

(4.4)

Equation (4.3) is an input disturbance for front right and left wheel and equation (4.4) is an input disturbance for rear right and left wheel. There is an assumption has to be made which is assume that front wheel for right and left reach the bump at the same time. This condition also refers to the rear wheel.
Figure 4.15: Road Profile 1 for Front Right and Left Wheel

Figure 4.16: Road Profile 1 for Rear Right and Left Wheel
4.7 Road Profile 2 for a Full Car Simulation

The road profile is assumed to have 2 bumps as below where \( a \) is set at 0.05.

\[
d(t) = \begin{cases} 
    a(1 - \cos 8\pi t) & 0.5s \leq t \leq 0.75s \\
    a(1 - \cos 8\pi t) / 2 & 6.5s \leq t \leq 6.75s \\
    0 & \text{otherwise} 
\end{cases} \quad (4.5)
\]

\[
d(t) = \begin{cases} 
    a(1 - \cos 8\pi t) & 3.0s \leq t \leq 3.25s \\
    a(1 - \cos 8\pi t) / 2 & 9.0s \leq t \leq 9.25s \\
    0 & \text{otherwise} 
\end{cases} \quad (4.6)
\]

Figure 4.17: Road Profile 2 for Front Right and Left Wheel
Equation (4.5) is an input disturbance for front right and left wheel and equation (4.6) is an input disturbance for rear right and left wheel. Assumption for this Road Profile is as same as stated in section 4.6.

4.8 Comparison between Passive and Active Suspension for a Full Car Model

Computer simulation work is based on the state space equation obtain from equation (2.4). Comparison between passive and active suspension for quarter car model is presented. For LQR controller, the parameters Q and R is set to obtain suitable feedback gain K. All the parameters refer to Appendix C. Feedback gain K obtained from the computer simulation implemented for both road profiles.
Figure 4.19: Force Generate for Front Right and Left Actuator by Using LQR controller with Road Profile 1

Figure 4.20: Force Generate for Rear Right and Left Actuator by Using LQR controller with Road Profile 1
Figure 4.21: Body Displacement with Road Profile 1

Figure 4.22: Body Acceleration with Road Profile 1
Figure 4.23: Wheel Displacement for Front Right and Left with Road Profile 1

Figure 4.24: Wheel Displacement for Rear Right and Left with Road Profile 1
Figure 4.25: Wheel Deflection for Front Right and Left with Road Profile 1

Figure 4.26: Wheel Deflection for Rear Right and Left with Road Profile 1
Figure 4.27: Suspension Travel Front Right and Left with Road Profile 1

Figure 4.28: Suspension Travel Rear Right and Left with Road Profile 1
Figure 4.29: Force Generate for Front Right and Left Actuator by Using LQR controller with Road Profile 2

Figure 4.30: Force Generate for Rear Right and Left Actuator by Using LQR controller with Road Profile 2
Figure 4.31: Body Displacement with Road Profile 2

Figure 4.32: Body Acceleration with Road Profile 2
Figure 4.33: Wheel Displacement for Front Right and Left with Road Profile 2

Figure 4.34: Wheel Displacement for Rear Right and Left with Road Profile 2
Figure 4.35: Wheel Deflection for Front Right and Left with Road Profile 2

Figure 4.36: Wheel Deflection for Rear Right and Left with Road Profile 2
By comparing the performance of the passive and active suspension system using LQR control technique for full car model, it is clearly shows that there is a problem with robustness. The output performances with two bumps happen to have slightly higher amplitude compare with single bump active suspensions performances for certain parameters such as Body Acceleration and Wheel Deflection for each wheel. Body Displacement improved even the amplitude is slightly higher compare with passive suspension system but the settling time is very
fast. Body Displacement is used to represent ride quality. Each pairs of wheel set have same output performance due to mathematical modeling that has been use shows that there are relationship exist among these wheel. The conclusion for the relationship between these wheels is when front wheel right and left receive same types of disturbance therefore output performance will be exactly same. This also refers to the rear wheel right and left. Force generate by the actuator for both Road Profile is shown in Figure 4.19, Figure 4.20, Figure 4.29 and Figure 4.30. It shows that generated force for Road Profile 1 around 3000N/m. Next simulation increase parameter Q to observe the effects on the systems.

4.9 Comparison between Passive and Active Suspension for Full Car Model with Different Q

In this section, parameter Q is set higher compare previous simulation as stated at APPENDIX B. When parameter Q (Appendix D) changed, it also affects the designed matrix K (Appendix D). Road Profile 1 for a full car model is used as input for the system.

![Figure 4.39: Force Generate for Front Right and Left Actuator by Using LQR controller with Road Profile 1](image)
Figure 4.40: Force Generate for Rear Right and Left Actuator by Using LQR controller with Road Profile 1

Figure 4.41: Body Displacement with Road Profile 1
Figure 4.42: Body Acceleration with Road Profile 1

Figure 4.43: Wheel Displacement for Front Right and Left with Road Profile 1
Figure 4.44: Wheel Displacement for Rear Right and Left with Road Profile 1

Figure 4.45: Wheel Deflection for Front Right and Left with Road Profile 1
Figure 4.46: Wheel Deflection for Rear Right and Left with Road Profile 1

Figure 4.47: Suspension Travel Front Right and Left with Road Profile 1
From the results, it clearly shows that there is a limitation in setting parameter $Q$. It cannot be too large because it affects force generator output. Force generator cannot be too high as shown in the Figure 4.41 ~ Figure 4.48. It prove that the amplitude of each parameter observe goes very high.

4.10 Conclusion

This chapter has described on output performance for active and passive suspensions systems. From computer simulations it shows that the output performance for full car model using LQR controller faces problem with robustness. It cannot improve amplitude for body displacement even though the settling time is good. LQR also does not have capability to adapt variations in road profiles.
CHAPTER 5

CONCLUSION AND FUTURE WORK

5.0 Conclusion

The objectives of this project have been achieved. Dynamic model for linear full car suspensions systems has been formulated and derived. Only one type of controller is used to test the systems performance which is LQR.

There is some problem occur using LQR to control actuator force. It happens to have problem with robustness. First it cannot reduce amplitude for most of parameters measured compare with passive suspension. However LQR can give faster settling time compare to passive suspensions system. Another problem occurs is LQR does not have capability to adapt variations road disturbance. It already proves when there is changes in road disturbance the performance become worst. This also shows that LQR active suspension for full car model cannot perform in rough road disturbances.

5.1 Recommendation and Future Work

Work study that has been done, on control active suspension does not consider the dynamics of the force actuators. In this project, the mathematical modeling for full car and quarter car does not include the dynamic model of the actuators. Force actuators generate the needed forces to achieve the desired objectives. Thus the study of these actuators is important so it can give real time performance.
Parameter that has been used also can affect the output performance either for active or passive suspension systems. Therefore, using different type of parameter it can give different output performance. Finally, use other type of controller that more robust than LQR.
REFERENCES


11. Sam Y.M. *Proportional Integral Sliding Mode Control of an Active Suspension System*. Malaysia University of Technology. PHD Dissertation. Malaysia University of Technology; 2004


Appendix A

State Space Equation for a Passive Suspension

\[ \dot{X}(t) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X(t) \\ f(t) \end{bmatrix} \]

Matrix A

\[ A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \]

\[ A_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ A_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]
\[
A_{21} = \begin{bmatrix}
2(-k_f T_f - k_r T_r) & 0 & 0 & -1 & -1 & -1 & -1 \\
I_r & 0 & 0 & I_r & I_r & I_r & I_r \\
0 & 2(-k_f a - k_r b b) & 2(-k_f a + k_r b) & -1 & -1 & -1 & -1 \\
I_p & I_p & I_p & I_p & I_p & I_p & I_p \\
0 & 2(-k_f a + k_r b) & 2(-k_f - k_r) & -1 & -1 & -1 & -1 \\
m_s & m_s & m_s & m_s & m_s & m_s & m_s \\
\frac{k_f T_f}{m_{uf}} & \frac{k_f a}{m_{uf}} & \frac{k_f}{m_{uf}} & -1 & -1 & 0 & 0 & 0 \\
\frac{-k_f T_f}{m_{uf}} & \frac{k_f a}{m_{uf}} & \frac{k_f}{m_{uf}} & 0 & -1 & 0 & 0 & 0 \\
\frac{k_r T_r}{m_{ur}} & \frac{-k_r b}{m_{ur}} & \frac{k_r}{m_{ur}} & 0 & 0 & -1 & -1 & 0 \\
\frac{-k_r T_r}{m_{ur}} & \frac{-k_r b}{m_{ur}} & \frac{k_r}{m_{ur}} & 0 & 0 & 0 & -1 & m_{ur} \\
\end{bmatrix}
\]

\[
A_{22} = \begin{bmatrix}
2(-b_f T_f - b_r T_r) & 0 & 0 & -1 & -1 & -1 & -1 \\
I_r & 0 & 0 & I_r & I_r & I_r & I_r \\
0 & 2(-b_f a - b_r b b) & 2(-b_f a + b_r b) & -1 & -1 & -1 & -1 \\
I_p & I_p & I_p & I_p & I_p & I_p & I_p \\
0 & 2(-b_f a + b_r b) & 2(-b_f - b_r) & -1 & -1 & -1 & -1 \\
m_s & m_s & m_s & m_s & m_s & m_s & m_s \\
\frac{b_f T_f}{m_{uf}} & \frac{b_f a}{m_{uf}} & \frac{b_f}{m_{uf}} & 1 & 0 & 0 & 0 \\
\frac{-b_f T_f}{m_{uf}} & \frac{b_f a}{m_{uf}} & \frac{b_f}{m_{uf}} & 0 & 1 & 0 & 0 \\
\frac{b_r T_r}{m_{ur}} & \frac{-b_r b}{m_{ur}} & \frac{b_r}{m_{ur}} & 0 & 0 & -1 & m_{ur} \\
\frac{-b_r T_r}{m_{ur}} & \frac{-b_r b}{m_{ur}} & \frac{b_r}{m_{ur}} & 0 & 0 & 1 & -m_{ur} \\
\end{bmatrix}
\]
Matrix $f(t)$

$$f(t) = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
\dot{Z}_{r_1} \\
\dot{Z}_{r_2} \\
\dot{Z}_{r_3} \\
\dot{Z}_{r_4} \\
\end{bmatrix}
$$

\[
\begin{bmatrix}
\mathbf{0} & 0 & 0 & 0 \\
0 & \frac{k_{ff}}{m_{ff}} & 0 & 0 \\
0 & 0 & \frac{k_{fr}}{m_{fr}} & 0 \\
0 & 0 & 0 & \frac{k_{rr}}{m_{rr}} \\
\end{bmatrix}
\]
Appendix B

State Space Equation for an Active Suspension

\[ \dot{X}(t) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} [X(t)] + Bu(t) + f(t) \]

Matrix \( Bu(t) \)

\[
Bu(t) = \begin{bmatrix}
T_f & -T_f & T_f & -T_f \\
I_f & I_f & I_f & I_f \\
\frac{a}{I_p} & \frac{a}{I_p} & -\frac{b}{I_p} & -\frac{b}{I_p} \\
\frac{1}{m_i} & \frac{1}{m_i} & \frac{1}{m_i} & \frac{1}{m_i} \\
\frac{-1}{m_{uf}} & 0 & 0 & 0 \\
0 & \frac{-1}{m_{uf}} & 0 & 0 \\
0 & 0 & \frac{-1}{m_{wr}} & 0 \\
0 & 0 & 0 & \frac{-1}{m_{wr}} \\
\end{bmatrix}
\begin{bmatrix}
u_1 \\ u_2 \\ u_3 \\ u_4 \\ \end{bmatrix}
Appendix C

Matrix Q

\[
Q = \begin{bmatrix}
10^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 10^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 10^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 10^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 10^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 10^5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 10^5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10^5 \\
\end{bmatrix}
\]

Matrix B

\[
B = \begin{bmatrix}
0.1 & 0 & 0 & 0 \\
0 & 0.1 & 0 & 0 \\
0 & 0 & 0.1 & 0 \\
0 & 0 & 0 & 0.1 \\
\end{bmatrix}
\]

Matrix K

\[
K = \begin{bmatrix}
\end{bmatrix}
\]
### Appendix D

#### Matrix Q

\[ Q = \begin{bmatrix}
10e6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 10e6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 10e6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 10e6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 10e6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 10e6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 10e6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 10e6 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

#### Matrix B

\[ B = \begin{bmatrix}
0.1 & 0 & 0 & 0 \\
0 & 0.1 & 0 & 0 \\
0 & 0 & 0.1 & 0 \\
0 & 0 & 0 & 0.1 \\
\end{bmatrix} \]

#### Matrix K

\[ K = \begin{bmatrix}
\end{bmatrix} \]