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FLUID DYNAMICS RESEARCH

Discussion

Comments on: "Steady two-dimensional oblique stagnation-point flow towards a stretching surface" M. Reza and A.S. Gupta, Fluid Dynamics Research 37 (2005) 334–340

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In a recent paper by Reza and Gupta(2005) it has been shown that the boundary layer equations describing the two-dimensional oblique stagnation-point flow towards a stretching surface can be reduced to the following two equations:

$$F'^2 - FF'' - F''' = C_1, \tag{1}$$

$$F'W' - FW'' - W''' = C_2, \tag{2}$$

where primes denote differentiation with respect to η, and C_1 and C_2 are constants of integration which can be determined upon using the following boundary conditions:

$$F = 0, \quad F' = 1, \quad W = 0, \quad W' = 0 \quad \text{at} \quad \eta = 0, \tag{3}$$

and

$$F'(\eta) \sim \frac{a}{c} \text{ and } W'(\eta) \sim 2\frac{b}{c}\eta \quad \text{as } \eta \to \infty. \tag{4}$$

It should be noticed that the parameter a/c is defined by $u_e(x)/u_w(x) = a/c$, where $u_e(x) = ax$ is the velocity of the flow outside the boundary layer (inviscid flow) and $u_w(x) = cx$ is the velocity of the

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stretching sheet, respectively, with a and c being positive constant. Also, $b(\geqslant 0)$ is a constant characterizing the obliqueness of oncoming flow. Using the boundary conditions (4), Reza and Gupta(2005) found that $C_1 = a^2/c^2$ and $C_2 = 0$. Thus, Eqs. (1) and (2) reduce to

$$F'^2 - FF'' - F''' = \frac{a^2}{c^2}, \tag{5}$$

$$F'W' - FW'' - W''' = 0. \tag{6}$$

We wish to point out in this note that Eq. (6) is wrong. Thus, from the boundary conditions (4) we get

$$F(\eta) \sim \frac{a}{c}\eta + A, \quad F'(\eta) \sim \frac{a}{c}, \quad W'(\eta) \sim 2\frac{b}{c}\eta, \quad W''(\eta) \sim 2\frac{b}{c} \quad \text{as } \eta \to \infty, \tag{7}$$

where A is a constant which can be determined by solving numerically Eq. (5) subject to the boundary conditions (3) and (4) for the function $F(\eta)$, see Labropulu et al. (1996). However, using (7) in Eq. (2), we get

$$C_2 = -2\frac{b}{c}A, \tag{8}$$

so that Eq. (2) becomes

$$F'W' - FW'' - W''' = -2\frac{b}{c}A, \tag{9}$$

which is different from Eq. (6) reported by Reza and Gupta(2005). The incorrect Eq. (6) appears, unfortunately, also in the papers by Tamada (1979), Takemitsu and Matunobu(1979), Wang (1985), Dorrepaal (2000), and Weidman and Putkaradze(2003). However, Weidman and Putkaradze(2005) have partially corrected the error in their paper.

Substituting (5) and (9) into Eqs. (8) and (9), and after integration we found that the correct form of the dimensionless pressure for the viscous oblique stagnation-point flow is given by

$$-P(\xi,\eta) = \frac{1}{2}\left(\frac{a^2}{c^2}\xi^2 + F^2\right) + F' - 2\frac{b}{c}A\xi + \text{constant}. \tag{10}$$

Since this flow violates the no-slip boundary condition at the wall, the dimensionless outer inviscid (but rotational) velocity, is given by, see Reza and Gupta(2005),

$$U_e(\xi,\eta) = \frac{a}{c}\xi + 2\frac{b}{c}\eta, \qquad V_e(\xi,\eta) = -\frac{a}{c}\eta. \tag{11}$$

Using the Bernoulli equation for the inviscid but rotational flow, we can show that the dimensionless pressure of the outer field in the present problem, is given by

$$-P_e(\xi,\eta) = \frac{1}{2}\frac{a^2}{c^2}(\xi^2 + \eta^2) + 2\frac{ab}{c^2}\xi\eta + 2\frac{b^2}{c^2}\eta^2 + \text{constant}. \tag{12}$$

If $b=0$, the linear shear flow (shear stress b) is absent, the external flow reduces to the potential irrotational flow for the normal planar stagnation-point flow or Hiemenz (1911) problem for the stagnation-point flow

Table 1
Values of the constant A and $F''(0)$ for different values of the parameter a/c

a/c	A	$F''(0)$		
		Mahapatra and Gupta (2002)	Nazar et al. (2004)	Present results
0.01	0.9747		−0.9980	−0.9980
0.02	0.9507		−0.9958	−0.9958
0.05	0.8853		−0.9876	−0.9876
0.10	0.7917	−0.9694	−0.9694	−0.9694
0.20	0.6407	−0.9181	−0.9181	−0.9181
0.50	0.3286	−0.6673	−0.6673	−0.6673
0.80	0.1145			−0.2994
1.00	0.0000		0.0000	0.0000
2.00	−0.4104	2.0175	2.0175	2.0176
3.00	−0.6931	4.7293	4.7296	4.7297
4.00	−0.9166			8.0014
5.00	−1.1053		11.7537	11.7538
10.00	−1.7997		36.2687	36.2687
20.00	−2.7196		106.5744	106.5744

towards a stretching surface. In this case $P_e(\xi, \eta)$ given by (12) reduces to

$$P_e(\xi, \eta) = \frac{a^2}{c^2}(\xi^2 + \eta^2) + \text{constant}. \tag{13}$$

We have now solved numerically Eqs. (5) and (9) subject to the boundary conditions (3) and (7) using the Keller-box method described in the book by Cebeci and Bradshaw (1984). We notice that the problem

$$\begin{gathered} F'^2 - FF'' - F''' = \frac{a^2}{c^2}, \\ F(0) = 0, \quad F'(0) = 1, \quad F'(\infty) = \frac{a}{c}, \end{gathered} \tag{14}$$

describes the flow towards the orthogonal stagnation point on a horizontal stretching sheet. This problem has been studied by Mahapatra and Gupta (2002) and Nazar et al. (2004). In order to verify the accuracy of the present results, we have compared the values of the reduced skin friction $F''(0)$ for some values of the parameter a/c in Table 1. Values of the constant A are also included in Table 1. It is seen that the present values of $F''(0)$ are in excellent agreement with those obtained by Mahapatra and Gupta (2002) and Nazar et al. (2004). Therefore, we are confident that the results obtained in this paper are very accurate.

Fig. 1 shows the variation of $U(\xi, \eta) = \xi F'(\eta) + W'(\eta)$ with η at $\xi = 0.5$ and $b/c = 1$ for several values of a/c. It is seen that this figure is the same with Fig. 3 in Reza and Gupta(2005)'s paper. However, graphs for $a/c = 0.2$ and 0.8 do not intersect as in the paper by Reza and Gupta(2005). On the other hand, it can be seen from this figure that the flow has a boundary layer behavior when $a/c > 1$ and it has an inverted boundary layer structure when $a/c < 1$. In our opinion, the claim by Reza and Gupta(2005)

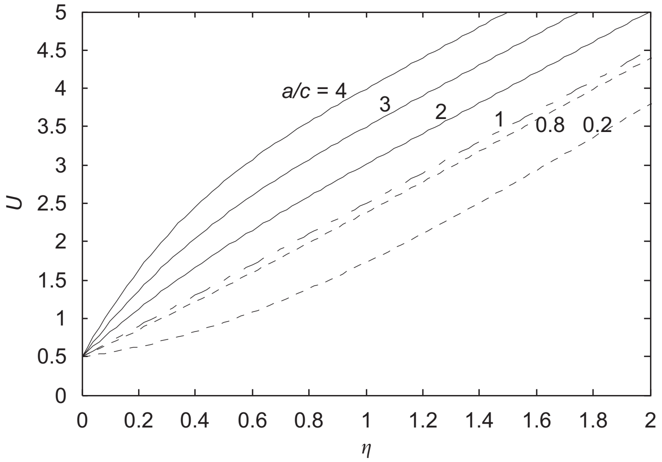

Fig. 1. Variation of $U(\xi, \eta)$ with η at fixed value of $\xi = 0.5$ for several values of a/c when $b/c = 1$.

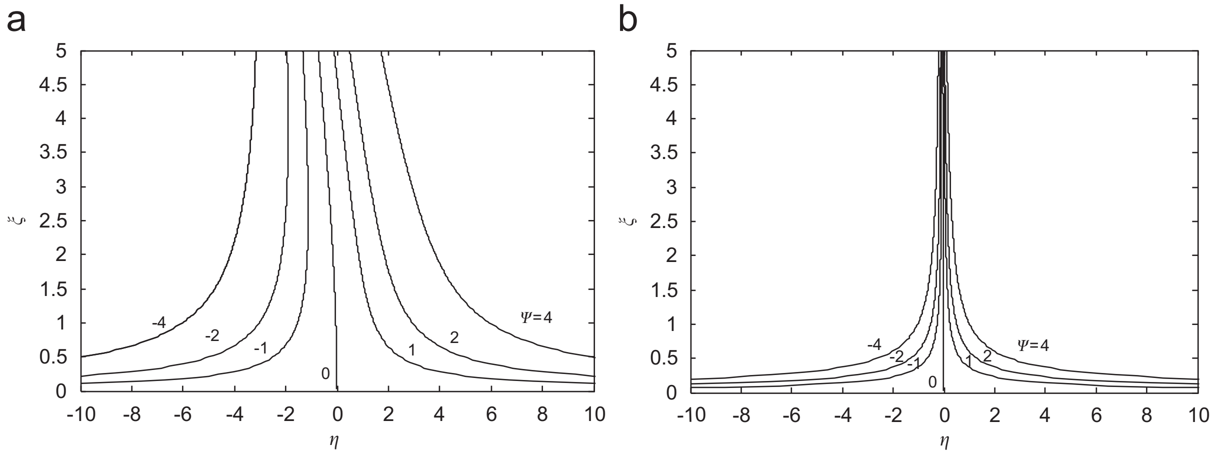

Fig. 2. Streamlines pattern for $b/c = 0.05$. (a) $a/c = 0.2$; (b) $a/c = 5$.

that the boundary layer structure is destroyed in the presence of considerable shear in the free stream is actually not true.

The streamline patterns for the oblique flows are shown in Figs. 2 and 3 for $a/c = 0.2$, 0.5, 2 and 5 corresponding to a small shear in the free stream ($b/c = 0.05$) or moderate value of shear ($b/c = 1$). It is found that the obliqueness of the streamlines are very obvious for the case of moderate shear compared to small shear. However, it is seen that the location of stagnation point is always at the origin, which can be explained due to the fact that the plate linearly stretches with a velocity $u_w(x) = cx$. However, this is quite opposite to the case of a fixed plate shown recently by Lok et al. (2006).

Observation: We would like to point out that it is not our intention to criticize any paper mentioned here but only to draw the attention that the results presented in some papers are not accurate. This is very important, especially, for those who wish to deal with the topic of oblique stagnation-point flow.

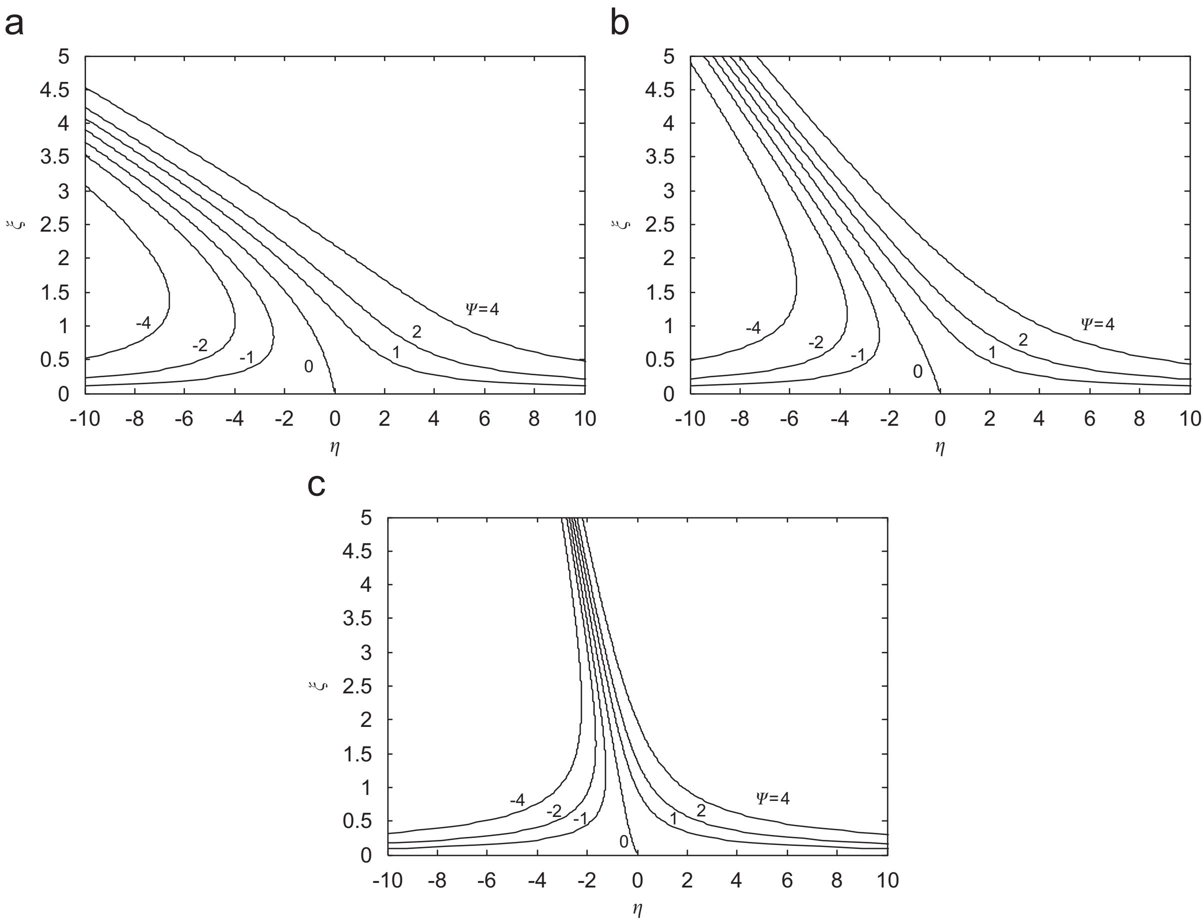

Fig. 3. Streamlines pattern for $b/c = 1$. (a) $a/c = 0.2$; (b) $a/c = 0.5$; (c) $a/c = 2$.

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