Modeling of A Horizontal Active Magnetic Bearing System With Uncertainties in Deterministic Form

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Abstract

In this paper the derivation of mathematical model of a horizontal active magnetic bearing (AMB) system in deterministic form is presented. The system is open-loop unstable and highly coupled due to nonlinearities inherited in the system such as gyroscopic effect and mass imbalance. Based on the equation of motions of the rotor and dynamic equation of electromagnetic coils, the dynamic model of the system with eight inputs is derived and represented in state-space format in which the system matrix is 16x16 in size. By using the upper and lower bounds of the parameter and the state variables of the system, the model is transformed into deterministic form where it can be shown that the system contains mismatched uncertainties in the state and disturbance matrices. This final system model with its numerical values can be used for the design of a class of a dynamic controller for system stabilization.

1. Introduction

An active magnetic bearing (AMB) system is a collection of electromagnets used to suspend an object and stabilization of the system is performed by feedback control. The system is composed of a floating mechanical rotor and electromagnetic coils that provide the controlled dynamic force. Due to this non-contact operation, AMB system has many promising advantages for high-speed and clean-environment applications. Moreover, adjustable stiffness and damping characteristics also make the system suitable for elimination of system vibration. Although the system is complex and considered an advance topic in term of its structural and control design, the advantages it offers outweigh the design complexity. A few of the AMB applications that receive huge attentions from many research groups around the globe are the flywheel energy and storage device, compressor, turbo molecular pump, Left Ventrical Assist Device (LVAD) and artificial heart [1][2][3] and [4].

AMB system is considered an advance mechatronic system in which a successful design depends heavily on the mathematical models that represents the system behaviour at design stage [5]. Many of early works in AMB modeling involve the derivation of linear or linearized models which operate at certain operating condition. This procedure is performed in order to accommodate a linear dynamic controller for stabilization of the AMB system. The disadvantage of this approach is the model is valid at a very small operating condition and the system performance will degrade as the model of the physical system is perturbed from this operating point [6]. However, in order to maximize the performance of the system, the derived model needs to cover wider operational ranges that further will force the system into its nonlinear regime. In order to achieve this, a more sophisticated mathematical model that can describe the behaviour of the system within this boundary is required.

In this paper, a nonlinear mathematical model of a horizontal shaft AMB system will be derived in which the gyroscopic effect and mass imbalance are considered. The derived model will be presented in a state variable form that is suitable for the design of a class of robust controller. **Fig. 1** below shows the five degree-of-freedom (DOF) horizontal AMB system that requires four pairs of electromagnetic coils to perform the radial control. The thrust control of the system is performed independently by another pair of electromagnetic coil and is not shown in the figure. The fifth DOF, which is the rotation around the x-axis, is supplied by external rotating machine in which the rotational speed is considered as time-varying parameter. Eight voltage sources supplied to the coils will be the inputs to the system and the rotor-coil gap deviations and the electromagnet fluxes are selected as the state variables.

This paper will be organized as follows: In section 2, a review on the derivation of the equations of motion and the dynamic equation of electromagnetic coils will be performed based on [7] and [8]. In section 3, the integration of both sets of equations is carried out to form a state-space AMB model. Then, in section 4, based on the

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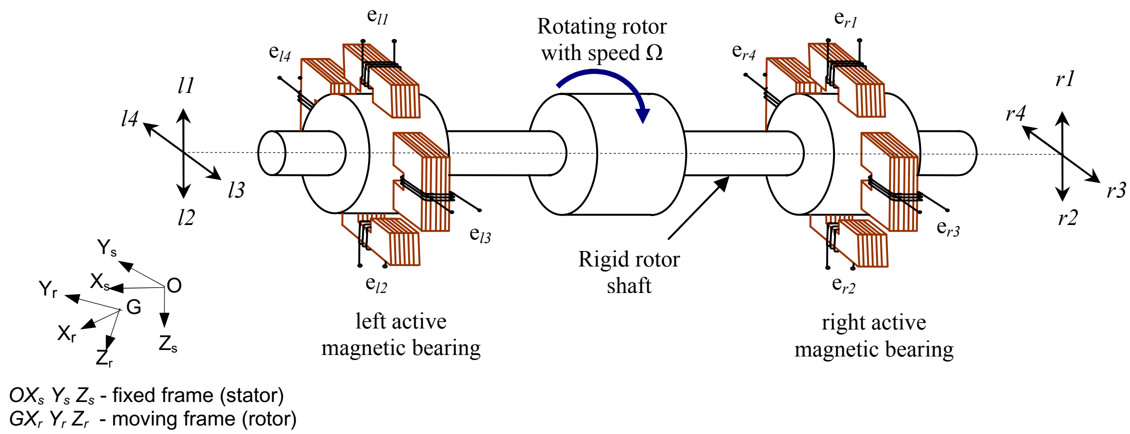

Fig. 1 Cylindrical Horizontal Active Magnetic Bearing System

upper and lower bounds of the system states and the rotational speed, the model is converted to its deterministic form where it can be shown that the system uncertainty and disturbance matrices suffer mismatched condition. However, the model produced is suitable for the design of a class on dynamic controller to stabilize the system. Finally, the conclusion in section 5 will summarize the work of this paper.

2. Dynamical equations of AMB system

For the derivation of the mathematical model, cylindrical horizontal AMB system as shown in **Fig. 1** above will be used. Equations of motions of the rotor and the nonlinear electromagnetic coils equations are the two sets of equations that describe the dynamical behaviour of the system.

2.1 Equations of motion

The equations of motion describe the dynamic movement of the rotor of the system. Assuming that the rotor is rigid floating body, the principle of flight dynamics is used to derive the equation. Based on the work in [7] and [8], the rotor's equations of motion for five DOF are

$$\begin{aligned}
\ddot{y}_o &= \frac{1}{m}[\alpha y_o + (f_{l3} - f_{l4} + f_{r4} - f_{r3}) + f_{dy}] \\
\ddot{z}_o &= \frac{1}{m}[\alpha z_o + (f_{l2} - f_{l1} + f_{r2} - f_{r1}) + f_{dz} + mg] \\
\ddot{\theta} &= \frac{-pJ_x}{J_y}\dot{\psi} + \frac{l}{J_y}((f_{l1} - f_{l2} + f_{r2} - f_{r1}) + f_{d\theta}) \\
\ddot{\psi} &= \frac{pJ_x}{J_y}\dot{\theta} + \frac{l}{J_y}((f_{l3} - f_{l4} + f_{r4} - f_{r3}) + f_{d\psi})
\end{aligned} \tag{1}$$

where: m is the mass of the rotor, l is the longitudinal of length between the rotor mass centre to the electromagnetic coil, J_x is the moment of inertia around X_r, J_y is the moment around Y_r, α is the radial eccentricity coefficient, ψ and θ are angular displacement of rotor axis about Y_s and Z_s axes, y_o and z_o are the coordinates of rotor mass centre on Y_s and Z_s axes, f_{l1}, f_{l2}, f_{l3}, f_{l4}, f_{r1}, f_{r2}, f_{r3}, and f_{r4} are the nonlinear magnetic force produced by the bearings (stator) and exerted on the rotor, and f_{dy}, f_{dz}, $f_{d\psi}$ and $f_{d\theta}$ are terms for the imbalances that present in the system. It can be noticed from equation (1) that the imbalances act like external disturbances to the system which will cause the vibration to the rotor. The imbalance forces can be modeled as follows [9]

$$\begin{aligned}
f_{dy} &= m_o \varepsilon p^2 \cos(pt + \kappa) \\
f_{dz} &= m_o \varepsilon p^2 \sin(pt + \kappa) \\
f_{d\theta} &= \frac{(J_y - J_x)}{l}\tau\, p^2 \cos(pt + \lambda) \\
f_{d\psi} &= \frac{(J_y - J_x)}{l}\tau\, p^2 \sin(pt + \lambda)
\end{aligned} \tag{2}$$

where: m_o is the mass of unbalance, ε and τ are static and dynamic imbalances, κ and λ are initial phase values.

2.2 Electromagnetic equations

There are two ways to model the dynamic of electromagnetic coils which are by using force-to-flux or force-to-current relation. For this model, as claimed in [10], the force-to-flux relation for the dynamic coil is used due to the fact that both the force and flux depend inversely to the time varying airgap length which will give a better system performance under feedback control. The electromagnetic force f_j produced by jth electromagnetic coils is expressed in term of the airgap flux ϕ_j and the gap length g_j as shown below

$$f_j = k\phi_j^2\left(1+\frac{2g_j}{\pi h}\right)\ ,\quad j = l1, \cdots, l4, r1, \cdots, r4 \qquad (3)$$

where: k is a constant and h is the width of the electromagnetic pole. The electric circuit equation that relates the airgap flux ϕ_j, the airgap length g_j and the input voltage e_j of the jth electromagnet is

$$e_j = N\frac{d\phi_j}{dt} + \frac{2R}{\mu_o AN} g_j \phi_j\ ,\ j = l1, \cdots, l4, r1, \cdots, r4 \qquad (4)$$

where: N is the number of turn in each coil, R is the coil resistance, A is the area under one electromagnetic pole and $\mu_o = 4\pi \times 10^{-7}$ H/m is the permeability of free space. Notice that equation (4) is valid by assuming that the speed e.m.f and the leakage inductance produced by the coil are negligibly small.

2.3 Changes of state variables – transformation matrix

From the control point of view, it is preferable to have the gap deviations as the state variables of the system instead of the coordinates of the mass center, yaw and pitch angles of the rotor. This is due to the fact that the gap deviations are easier to be measured than the rotor mass center either by using sensors or by designing observers [8]. The relation between the jth airgap of the electromagnetic coils and the rotor can be expressed as follows:

$$g_j = D_o + g_j'\ ,\quad j = l1, \cdots, l4, r1, \cdots, r4 \qquad (5)$$

where: D_o is the steady state gap length at equilibrium and g_j' is the gap length deviation from steady state value D_o.

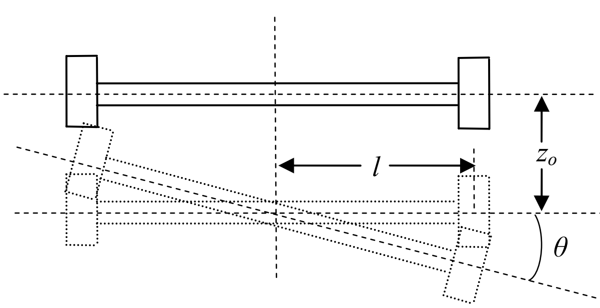

Fig. 2 Movement of rotor in z-axis

Based on **Fig. 2** above which shows the exaggerated view of the movement of the rotor in z-axis, assuming the angle θ is small such that sin $\theta \approx \theta$, the relation between the airgaps in the AMB with rotor mass centre coordinate can be related as follows:

$$\boldsymbol{g} = \begin{bmatrix} g'_{l1} \\ g'_{r1} \\ g'_{l3} \\ g'_{r3} \end{bmatrix} = - \begin{bmatrix} g'_{l2} \\ g'_{r2} \\ g'_{l4} \\ g'_{r4} \end{bmatrix} = \begin{bmatrix} (z_o - l\theta) \\ (z_o + l\theta) \\ (-y_o - l\psi) \\ (-y_o + l\psi) \end{bmatrix} \qquad (6)$$

With this equation a transformation matrix, T, can be established to perform the change the system variables.

$$T = \begin{bmatrix} 0 & 1 & -l & 0 \\ 0 & 1 & l & 0 \\ -1 & 0 & 0 & -l \\ -1 & 0 & 0 & l \end{bmatrix} \qquad (7)$$

3. AMB model in state-space form

Equations (1), (2), (3), (4) and (7) can now be easily integrated to form the model of horizontal AMB system in state-space form. Let the 16 state variables and 8 inputs of the system to be defined as follows:

$$\begin{array}{llll}
x_1 = g'_{l1}\ , & x_2 = g'_{r1}\ , & x_3 = g'_{l3}\ , & x_4 = g'_{r3}\ , \\
x_5 = \dot{x}_1\ , & x_6 = \dot{x}_2\ , & x_7 = \dot{x}_3\ , & x_8 = \dot{x}_4\ , \\
x_9 = \phi_{l1}\ , & x_{10} = \phi_{l2}\ , & x_{11} = \phi_{r1}\ , & x_{12} = \phi_{r2}\ , \\
x_{13} = \phi_{l3}\ , & x_{14} = \phi_{l4}\ , & x_{15} = \phi_{r3}\ , & x_{16} = \phi_{r4}\ , \\
u_1 = e_{l1}\ , & u_2 = e_{l2}\ , & u_3 = e_{r1}\ , & u_4 = e_{r2}\ , \\
u_5 = e_{l3}\ , & u_6 = e_{l4}\ , & u_7 = e_{r3}\ , & u_8 = e_{r4}\ .
\end{array}$$

Then, equation (8) below is the representation of the AMB system in state-space form.

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{x}, p, t)\mathbf{x}(t) + \mathbf{BU}(t) + \mathbf{D}(p, t) \qquad (8)$$

It can be observed from this nonlinear model that the system matrix **A** (**x**, p, t) is dependent to the state variables and the time-varying speed, p, while the disturbance matrix **D**(p,t) is only dependent on p. In order to facilitate the process to develop of dynamic controller, the equation above can be partitioned as shown below

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \\ \hline \dot{\mathbf{x}}_3 \\ \dot{\mathbf{x}}_4 \end{bmatrix} = \left[\begin{array}{cc|cc} 0 & \mathbf{I} & 0 & 0 \\ \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3 & \mathbf{A}_4 \\ \hline 0 & 0 & \mathbf{A}_5 & 0 \\ 0 & 0 & 0 & \mathbf{A}_6 \end{array}\right] \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \hline \mathbf{x}_3 \\ \mathbf{x}_4 \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \hline \mathbf{B}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 \end{bmatrix} \mathbf{U} + \begin{bmatrix} \mathbf{0} \\ \mathbf{D}_1 \\ \hline \mathbf{0} \\ \mathbf{0} \end{bmatrix} \qquad (9)$$

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where $\dot{\mathbf{x}}_1 = [g'_{l1}, g'_{r1}, g'_{l3}, g'_{r3}]$, $\dot{\mathbf{x}}_2 = [\dot{g}'_{l1}, \dot{g}'_{r1}, \dot{g}'_{l3}, \dot{g}'_{r3}]$, $\dot{\mathbf{x}}_3 = [\phi_{l1}, \phi_{l2}, \phi_{r1}, \phi_{r2}]$ and $\dot{\mathbf{x}}_3 = [\phi_{l1}, \phi_{l2}, \phi_{r1}, \phi_{r2}]$. The elements of the matrices are shown in the appendix.

4. AMB model as uncertain system

In order to synthesize a type of robust controller for this class of system, the AMB model derived in previous section will be treated as uncertain system in which deterministic approach to classify the system will be used. By using this approach, the AMB model can be decomposed into its nominal and uncertain parts as shown below

$$\begin{aligned}\mathbf{x}(t) &= [\mathbf{A} + \mathbf{\Delta A}(\mathbf{x}, p, t)]\mathbf{x}(t) + \mathbf{BU}(t) + \mathbf{D}(p,t) \\ \mathbf{y}(t) &= \mathbf{Cx}(t)\end{aligned} \tag{10}$$

where $\mathbf{\Delta A}(\mathbf{x},p,t)$ represents the uncertainty of the system matrix and $\mathbf{D}(p,t)$ is the disturbance matrix associated with speed dependent of imbalance. **A** and **B** are the nominal constant matrices. The decomposition into this deterministic form is possible due to the fact that the all the maximum and minimum values of state variables and rotor speed are known. The elements of the $\mathbf{\Delta A}(\mathbf{x},p,t)$ and $\mathbf{D}(p,t)$ system and disturbance matrices, respectively, can be calculated based on these bounds. The minimum and maximum bounds of all the state variables and the rotor speed are as follows:

$$-0.55\ mm \le x_i \le 0.55\ mm \text{ , for } i = 1,2,3,4, \tag{11a}$$

$$0\ m \le x_i \le 1.87\ m/\sec \text{ , for } i = 5,6,7,8, \tag{11b}$$

$$0\ Wb \le x_i \le 10.0 \times 10^{-4}\ Wb \text{ , for } i = 9, \dots, 16, \tag{11c}$$

and

$$0\ rad/\sec \le p \le 3142\ rad/\sec . \tag{11d}$$

Then, by using these values and the other system parameter values as shown in **Table 1** below, each element of the system and disturbance matrices can be calculated and specified in the following form:

$$\underline{a}_{ij} \le a_{ij}(x,p,t) \le \overline{a}_{ij} \tag{12a}$$

$$\underline{d}_j \le d_j(p,t) \le \overline{d}_j \tag{12b}$$

for $i = 1, \dots, 18$, and $j = 1, \dots, 18$, where $a_{ij}(x,p,t)$ and $d_j(p,t)$ are the element of $\mathbf{A}(\mathbf{x},p,t)$ and $\mathbf{D}(p,t)$ matrices respectively. The upper and lower bars indicate the maximum and minimum values of the elements. Since these bounds are known, the system matrix can be written in the following form:

Table 1. Parameter for Horizontal AMB system [8][9]

Symbol	Parameter	Value [Unit]
m	Mass of Rotor	1.39×10^{1} [kg]
A	Area of coil	1.532×10^{-3} [m^2]
J_x	Moment of Inertia about X	1.34×10^{-2} [kg.m^2]
J_y	Moment of Inertia about X	2.32×10^{-1} [kg.m^2]
G_o	Steady airgap	5.50×10^{-4} [m]
R	Coil Resistance	1.07×10 [Ω]
L	Coil Inductance	2.85×10^{-1} [H]
l	Distance between Mass centre to coil	1.30×10^{-1} [m]
α	Rotor radial eccentricity coefficient	0.1g [N/m]
ε	Static imbalance	1.0×10^{-4} [m]
τ	Dynamic imbalance	4.0×10^{-4} [rad]

$$\mathbf{A}(\mathbf{x}, p, t) = \mathbf{A} + \mathbf{\Delta A}(\mathbf{x}, p, t) \tag{13}$$

For this class of AMB system model, it can be noticed that for the disturbance matrix, $\mathbf{D}(p,t)$, only the maximum value of the elements are needed since these values represent the highest disturbance values caused by the imbalance which should be eliminated from the system. By using the values of the bounds given by (11), and the deterministic form of system matrix given by (13), the nominal and uncertain values of system matrix, a well as the values of disturbance matrix are calculated and given in the appendix. The norm for these matrices can also be calculated and the values are as follows:

$$\|\mathbf{\Delta A}\| = 5.6003 \times 10^{4} \text{ , } \|\mathbf{D}\| = 1.5595 \times 10^{3} . \tag{14}$$

From the structure of the matrices, it can be shown that both the uncertainty and disturbance matrices suffer mismatched condition which means that the elements of the uncertainty and disturbance matrices do not lie in the range space of the input matrix **B**. Also, the mismatched condition can be checked by using the rank test as shown below in which the results agree with the aforementioned mismatched condition.

$$\begin{aligned}&\text{rank}[\mathbf{B}] \neq \text{rank}\,[\mathbf{B}, \mathbf{\Delta A}(\mathbf{x},p,t)], \\ &\text{rank}[\mathbf{B}] \neq \text{rank}\,[\mathbf{B}, \mathbf{D}(p,t)].\end{aligned} \tag{15}$$

By having this mismatched condition, the input voltages of the system do not have direct access to the mismatched elements. Thus, this has made the design of the robust controller that can eliminate the disturbance and to achieve robustness towards system uncertainty a challenging process [3][11]. Only after overcoming this mismatched condition the system is guaranteed to achieve required stability and robust performance.

5. Conclusion

This paper concerns with the derivation of a mathematical model of a horizontal AMB system which is highly coupled and nonlinear. The derivation starts with the equation of motions of the rotor and the dynamic equation of electromagnetic coils. The integration of these nonlinear equations forms the state-space model of the system with 16 states variables and eight inputs. The model then is treated as uncertain system by using deterministic approach in which it has been shown that the system inherits mismatched condition in its system and disturbance matrices. The final system model is suitable for the design of a class of dynamic controller, however, to eliminate the mismatched disturbance and to achieve robust system performance towards the mismatched system uncertainties will be great challenge for the design of controller.

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Appendix

$$\mathbf{A}_1 = \left[\begin{array}{cc|cc} a_1^{11} & a_1^{12} & 0 & 0 \\ a_1^{21} & a_1^{22} & 0 & 0 \\ \hline 0 & 0 & a_1^{33} & a_1^{34} \\ 0 & 0 & a_1^{43} & a_1^{44} \end{array}\right], \mathbf{A}_2 = \left[\begin{array}{cc|cc} 0 & 0 & a_2^{13} & a_2^{14} \\ 0 & 0 & a_2^{23} & a_2^{24} \\ \hline a_2^{31} & a_2^{32} & 0 & 0 \\ a_2^{41} & a_2^{42} & 0 & 0 \end{array}\right],$$

$$\mathbf{A}_3 = \left[\begin{array}{cc|cc} a_3^{11} & a_3^{12} & a_3^{13} & a_3^{14} \\ a_3^{21} & a_3^{22} & a_3^{23} & a_3^{24} \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right], \mathbf{A}_4 = \left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline a_4^{31} & a_4^{32} & a_4^{33} & a_4^{34} \\ a_4^{41} & a_4^{42} & a_4^{43} & a_4^{44} \end{array}\right],$$

$$\mathbf{A}_5 = \left[\begin{array}{cc|cc} a_5^{11} & 0 & 0 & 0 \\ 0 & a_5^{22} & 0 & 0 \\ \hline 0 & 0 & a_5^{33} & 0 \\ 0 & 0 & 0 & a_5^{44} \end{array}\right], \mathbf{A}_6 = \left[\begin{array}{cc|cc} a_6^{11} & 0 & 0 & 0 \\ 0 & a_6^{22} & 0 & 0 \\ \hline 0 & 0 & a_6^{33} & 0 \\ 0 & 0 & 0 & a_6^{44} \end{array}\right],$$

$$\mathbf{B}_1 = \left[\begin{array}{cc|cc} b_1^{11} & 0 & 0 & 0 \\ 0 & b_1^{22} & 0 & 0 \\ \hline 0 & 0 & b_1^{33} & 0 \\ 0 & 0 & 0 & b_1^{44} \end{array}\right], \mathbf{B}_2 = \left[\begin{array}{cc|cc} b_2^{11} & 0 & 0 & 0 \\ 0 & b_2^{22} & 0 & 0 \\ \hline 0 & 0 & b_2^{33} & 0 \\ 0 & 0 & 0 & b_2^{44} \end{array}\right],$$

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$$\mathbf{D}_1 = \begin{bmatrix} (g + \frac{m_o}{m}\varepsilon p^2 \sin(pt) - \left(\frac{J_y - J_x}{J_y}\right) l\tau\, p^2 \cos(pt)) \\ (g + \frac{m_o}{m}\varepsilon p^2 \sin(pt) + \left(\frac{J_y - J_x}{J_y}\right) l\tau\, p^2 \cos(pt)) \\ -\left(\frac{m_o}{m}\varepsilon p^2 \cos(pt) + \left(\frac{J_y - J_x}{J_y}\right) l\tau\, p^2 \sin(pt)\right) \\ -\left(\frac{m_o}{m}\varepsilon p^2 \cos(pt) - \left(\frac{J_y - J_x}{J_y}\right) l\tau\, p^2 \sin(pt)\right) \end{bmatrix},$$

$$a_1^{11} = a_1^{12} = a_1^{21} = a_1^{22} = a_1^{33} = a_1^{34} = a_1^{43} = a_1^{44} = \frac{\alpha}{2m},$$

$$a_2^{13} = a_2^{24} = a_2^{32} = a_2^{41} = -\frac{pJ_y}{2J_x},$$

$$a_2^{14} = a_2^{23} = a_2^{31} = a_2^{42} = \frac{pJ_y}{2J_x},$$

$$a_3^{11} = -H_1 k\left(1 + \frac{2(D_o + x_1)}{\pi h}\right)x_9, \quad a_3^{12} = H_1 k\left(1 + \frac{2(D_o - x_1)}{\pi h}\right)x_{10},$$

$$a_3^{13} = H_2 k\left(1 + \frac{2(D_o + x_2)}{\pi h}\right)x_{11}, \quad a_3^{14} = -H_2 k\left(1 + \frac{2(D_o - x_2)}{\pi h}\right)x_{12},$$

$$a_3^{21} = H_2 k\left(1 + \frac{2(D_o + x_1)}{\pi h}\right)x_9, \quad a_3^{22} = -H_2 k\left(1 + \frac{2(D_o - x_1)}{\pi h}\right)x_{10},$$

$$a_3^{23} = -H_1 k\left(1 + \frac{2(D_o + x_2)}{\pi h}\right)x_{11}, \quad a_3^{24} = H_1 k\left(1 + \frac{2(D_o - x_2)}{\pi h}\right)x_{12},$$

$$a_4^{31} = -H_1 k\left(1 + \frac{2(D_o + x_3)}{\pi h}\right)x_{13}, \quad a_4^{32} = H_1 k\left(1 + \frac{2(D_o - x_3)}{\pi h}\right)x_{14},$$

$$a_4^{33} = H_2 k\left(1 + \frac{2(D_o + x_4)}{\pi h}\right)x_{15}, \quad a_4^{34} = -H_2 k\left(1 + \frac{2(D_o - x_4)}{\pi h}\right)x_{16},$$

$$a_4^{41} = H_2 k\left(1 + \frac{2(D_o + x_3)}{\pi h}\right)x_{13}, \quad a_4^{42} = -H_2 k\left(1 + \frac{2(D_o - x_3)}{\pi h}\right)x_{14},$$

$$a_4^{43} = -H_1 k\left(1 + \frac{2(D_o + x_4)}{\pi h}\right)x_{15}, \quad a_4^{44} = H_1 k\left(1 + \frac{2(D_o - x_4)}{\pi h}\right)x_{16},$$

$$a_5^{11} = -q(D_o + x_4)x_9, \quad a_5^{22} = -q(D_o - x_1)x_{10},$$

$$a_5^{33} = -q(D_o + x_2)x_{11}, \quad a_5^{44} = -q(D_o - x_2)x_{11},$$

$$a_6^{11} = -q(D_o + x_3)x_{12}, \quad a_6^{22} = -q(D_o - x_3)x_{13},$$

$$a_6^{33} = -q(D_o + x_4)x_{15}, \quad a_6^{44} = -q(D_o - x_4)x_{16},$$

$$b_1^{11} = b_1^{22} = b_1^{33} = b_1^{44} = b_2^{11} = b_2^{22} = b_2^{33} = b_2^{44} = \frac{1}{N}$$

$$\Delta\mathbf{A}_2 = 10^4 \left[\begin{array}{cc|cc} 0 & 0 & 1.355 & 1.355 \\ 0 & 0 & 1.355 & 1.355 \\ \hline 1.355 & 1.355 & 0 & 0 \\ 1.355 & 1.355 & 0 & 0 \end{array}\right],$$

$$\Delta\mathbf{A}_3 = 10^4 \left[\begin{array}{cc|cc} 3.4441 & 3.441 & 0.0215 & 0.0215 \\ 0.0215 & 0.0215 & 3.4441 & 3.4441 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right],$$

$$\Delta\mathbf{A}_4 = 10^4 \left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 3.4441 & 3.4441 & 0.0215 & 0.0215 \\ 0.0215 & 0.0215 & 3.4441 & 3.4441 \end{array}\right],$$

$$\Delta\mathbf{A}_5 = \Delta\mathbf{A}_6 = \left[\begin{array}{cc|cc} 38.216 & 0 & 0 & 0 \\ 0 & 38.216 & 0 & 0 \\ \hline 0 & 0 & 38.216 & 0 \\ 0 & 0 & 0 & 38.216 \end{array}\right],$$

$$\mathbf{D}_1 = \begin{bmatrix} -43.204 \\ 844.676 \\ -1097.956 \\ -714.944 \end{bmatrix}.$$