X-bar Control Chart Patterns Identification Using Nelson's Run Rules

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Abstract

Different types of run rules are proposed and applied to the control charts to identify unnatural variations as early as possible. Two opposite views are found in literature regarding pattern identification by run rules; one view in favour and the other view criticize the application of run rules for pattern identification. In this paper Nelson's run rules were investigated in detail for sensitivity and identification of control chart patterns. The goal is to gain deeper insights on the opposite views about the run rules. The rules were applied individually and in combinations to fully developed patterns. The results confirm that most of the run rules individually do not identify specific pattern. However, the run rules can be used to identify specific patterns when applied in appropriate combination. The results suggest that combinations of Nelson's run rules can lead to identification of shift, trend, cycle, and systematic control chart patterns.

Keywords
Control chart, Run rules, Pattern identification, Process variations, Short run production

1. Introduction

Process variation can be classified as either natural variation or unnatural variation. The natural variation is attributed to common causes which are inherent in the process, while the unnatural variations are due to assignable causes. The assignable causes can be removed from the process if detected properly and timely. Early detection of unnatural variation is necessary for preventive action and to avoid catastrophic deterioration (Montgomery 2013). Shewhart X-bar control chart is one of the widely used statistical process control (SPC) tool. It gives pictorial representation of natural and unnatural variations of a process. Shewhart control chart uses the process mean of a stable process as the centerline and $\pm 3\sigma$ from the centerline as an upper control limit (UCL) and a lower control limit (LCL) respectively. The averages of sampled process data are periodically plotted on the chart. In general, a process is considered as statistically stable if the observation point lies between the UCL and LCL. Only natural variation is presence if the plotted points behave randomly up and down within the control limits. Otherwise, unnatural variation is disturbing the process stability due the presence of special assignable causes. The Shewhart control chart can effectively differentiates between natural and unnatural variations (Khoo 2003).

Shewhart control chart by itself does not provide information on control chart patterns and their corresponding root causes. To overcome this limitation, supplementary run rules have been proposed by researchers for analyzing data points within the control limits. Some of well-known run rules are Western Electric rules, Nelson rules, ISO 2859 tests and some special run rules proposed by big manufacturing companies like Boeing and General Electric (Noskievičová, 2013). The most popular run rules are the Western Electric Run Rules (Montgomery, 2009). Nelson's run rules are the enhancement of the Western Electric Rules (Nelson, 1984). In applying run rules, the Shewhart control chart is divided into six different zones as shown in Figure 1. Each zone is one sigma wide. The three sigma on both side are upper control limit (UCL) and lower control limit (LCL). The two sigma limits are referred to as upper warning limit (UWL) and lower warning limit (LWL). The one sigma on each side is called upper one sigma limit (UOSL) and lower one sigma limit (LOSL). As the normal distribution is acceptable assumption in control chart, in stable process Zone C covers 68.27% of data. Meanwhile, Zone B covers 27.18% and Zone A covers 4.28% of the data. The Western electric run rules are proposed for detection of patterns such as shift, stratification, systematic variation, and mixtures (Noskievičová 2013). Table 1 lists the Nelson's runs rules.

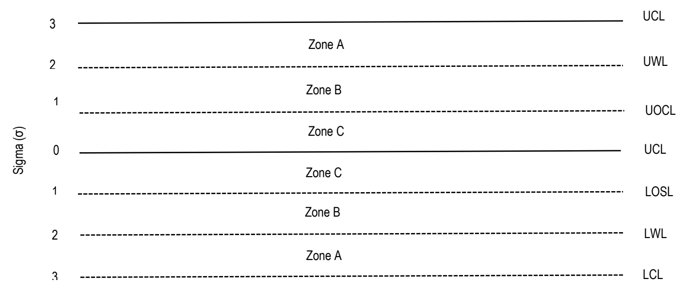

Figure 1. Control chart zones for different runs rules

Table 1. Nelson's runs rules (Nelson, 1984, 1985)

Rule No.	Description	Label
Rule 1	One point beyond Zone A	R1
Rule 2	Nine points in a row in Zone C or beyond	R2
Rule 3	Six points in a row steadily increasing or decreasing	R3
Rule 4	Fourteen points in a row alternating up and down	R4
Rule 5	Two out of three points in a row in Zone A or beyond	R5
Rule 6	Four out of five points in a row in Zone B or beyond	R6
Rule 7	Fifteen points in a row in Zone C (above and below center line)	R7
Rule 8	Eight points in a row on both sides of center line with none in Zone C	R8

Some researcher used Nelson's run rules individually to improve the performance of the Shewhart Control chart. Camp and Woodall (1987) investigated performance of the Shewhart control chart considering different run rules. Hurwitz and Mathur (1992) proposed a simple two-of-two rule for detecting small shift. Klein (2000) suggested two of two and two of three rules by Markov chain method. Khoo et al. (2006) improved the Klein two rules for small and large shift detections. Few researchers have investigated the use of Nelson's rules to identify the unnatural variation patterns on control chart. Wang et al. (1998) described Nelson's rules as eight types of patterns on the control chart. Noskievicova (2013) reported different types of run rules and recommends some runs rules for different types of unnatural patterns. Cheng (1997), Guh (2005) and Hachicha et al. (2012) argued that run rules do not explicitly shows unnatural pattern recognition and identification. Some researchers noted that run rules indicate the presence of abnormal patterns, but do not clearly indicate which pattern occurs. There are no specified run rules for specified type of patterns on control chart. Hassan (2011) implemented run rules for stability test before proceeding with artificial neural network patterns classifier.

The objective of this study is to investigate whether run rules individually or in combinations are suitable for pattern identification. This study was motivated by the contrasting views in literature on the ability of runs rules to discriminate different types of control chart patterns on Shewhart X-bar chart. It aims to provide better understanding on ability of run rules to identify different control chart pattern. The eight types of commonly investigated patterns adopted from Zaman and Hassan (2021) as shown in Figure 2 were studied. The rest of this paper is organized as follow. Section 2 presents the methodology of investigation, Section 3 discusses the data generation, Section 4 reports the results, Section 5 discusses the findings, and finally Section 6 concludes the paper.

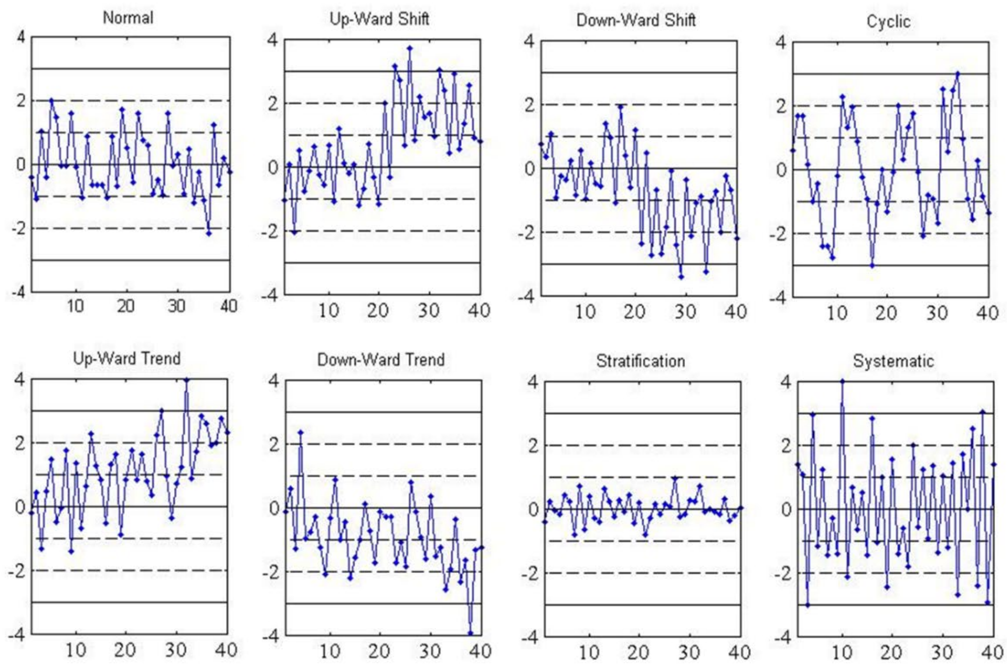

Figure 2. Eight typical process variation patterns on *X*-bar control chart (Zaman and Hassan, 2021)

3. Methodology

This study focused on identifications of different control chart patterns by implementing Nelson's runs rules. These patterns are namely trends-up, trend-down, shift-up, shift-down, stratification, systematic and cyclic along with normal process patterns. These patterns were synthetically generated and coded in MATLAB. Ideally, actual process data should be used. Since a large amount of data samples were required for each pattern types, it was not possible to economically obtain them from real life processes. The data generation proposed by De la Torre Gutierrez and Pham (2016) were adopted for random number generations. The equations and parameters range used for patterns generation are given in Table 2. A window size of 20 observations was adopted form Hassan, et al. (2003). A total of 63600 fully developed patterns were tested using the eight Nelson's run rules. Each Nelson's run rule required different minimum number of data points to implement the tests. The sensitivity of Nelson's run rules regarding pattern identification was studied individually and in various combinations for each type of pattern. Table 3 provides the generic pseudo-code for the pattern identification procedure. The Nelson's run rules tests were sequentially applied to all data points in each pattern category.

Table 2. Standard equations and parameters range for patterns

Pattern type	Parameter	Parameter range (σ)	Equation for pattern generators
Trend Up or Down	Slope (γ_1)	0.005 to 0.025	$y_t = \mu + N_t \pm \gamma_1 t$
Shift-Up or Down	Shift (γ_2)	0.005 to 2.5	$y_t = \mu + N_t \pm \gamma_2$
Cyclic (frequency(γ_4)=10) (Amplitude=variable)	Amplitude (γ_3)	0 to 1.8	$y_t = \mu + N_t \pm \gamma_3 \sin(\frac{2\pi t}{\gamma_4})$
Stratification	Stratification (γ_5)	0.1 to 0.6	$y_t = \mu + \gamma_5 N_t)$
Systematic	Departure (γ_6)	0.005 to 2.5	$y_t = \mu + N_t \pm \gamma_6(-1)^t$
Stable Process (Normal)		N (0,1)	$y_t = \mu + N_t$

Table 3. Pseudo-codes for pattern generation and identification using Nelson's run rules

Step no	Pseudo-codes
	Start
1	Declare *n* number of patterns for each type to generate
2	Declare parameters for pattern generation (from Table 2)
3	Generate random data
4	Repeat *n* times pattern data generation
4a	*n* time repetition for each type of pattern
4b	Standardize each pattern *n* time
4c	Store in array form and display graphically
5	Apply Nelson's run rules to each pattern generated
5a	Specify unique mark of identification for each rule
5b	Repeat *n* time application of Nelson's rules
5c	Count recognition accuracy for different combination
5d	Store results
6	Analyze results
	End

4. Results

The parameter ranges of different types of patterns are shown in Table 2. The labelling symbols and minimum numbers of point for testing of each Nelson's run rule are summarized in Table 4. Examples of Nelson's run rules applied to various types of pattern are shown in Figure 3. A unique mark of identification for each Nelson's run rule was automatically labelled on control the chart whenever a run rule was satisfied. The sensitivity of different combination of Nelsons run rules in identification of patterns as the respective magnitudes were varied are showed in Figure 4 to Figure 8.

The shift magnitude was equally divided between 0 and 2.5 sigma range. At each shift magnitude 600 patterns were tested. A total of 15600 patterns were tested to identify shift patterns between 0 to 2.5 sigma. Nelson's run rules combination was applied to each patterns. Combinations of two, three, four and five run rules were implemented at a time for the shift patterns. Appropriate alternative combinations were selected based on extensive preliminary simulation study. Those combinations of Nelson's run rules were chosen if they indicated sensitivity to the patterns and the rest were ignored.

For trend pattern, a total of 12600 patterns were tested. The slope value was equally divided between 0 and 0.025 uniformly, and at each value, 600 patterns were tested for different combination of the Nelson's run rules. The results are summarized in the Figure 5. For cyclic pattern, a total of 12600 patterns, for systematic pattern a total of 11400 patterns and for stratification pattern a total of 10800 patterns were tested, respectively. Their corresponding recognition results are shown in Figures 6 to 8, respectively. For normal patterns, a total of 600 sample patterns were tested using different combinations of run rules. Patterns which were not recognized by any combination of the Nelson's run rules were tested by using individual run rule.

Table 4. Nelson's run rules, minimum data point and symbols used for labelling on patterns

Rule Number	Minimum data Points	Symbol for labelling
R1	1	Big Square
R2	9	Medium Square
R3	6	Triangle
R4	14	Small Square
R5	3	Small Circle
R6	5	Big Circle
R7	15	Asterisk
R8	8	Star Shape

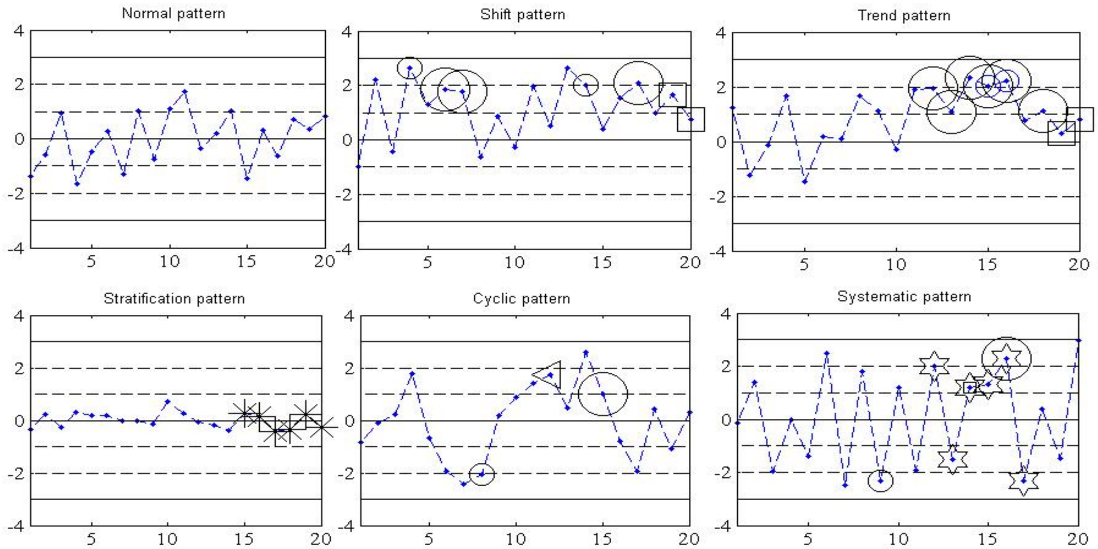

Figure 3. Examples of Nelson's run rules triggered and labelled on various control chart patterns

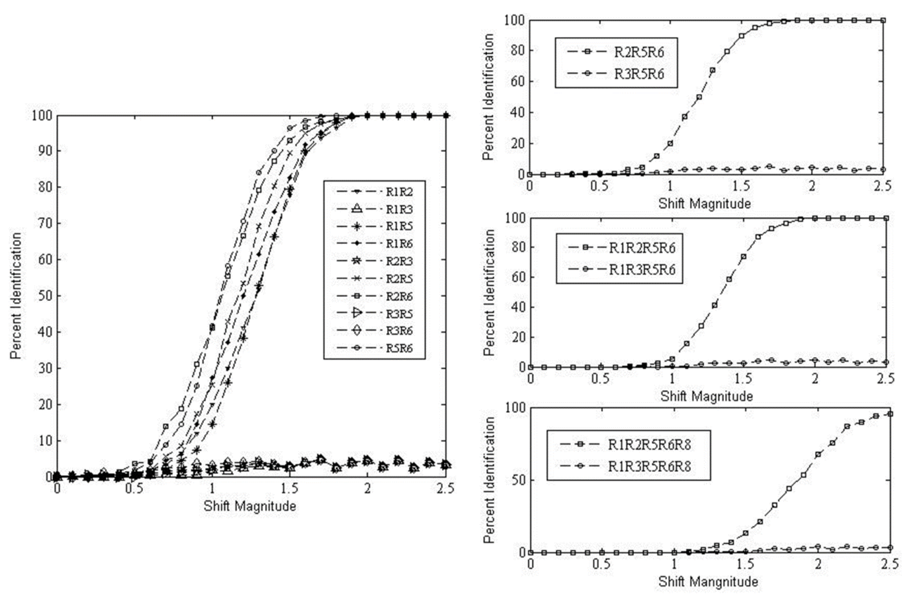

Figure 4. Sensitivity of different combination of Nelsons run rules in identification of shift patterns

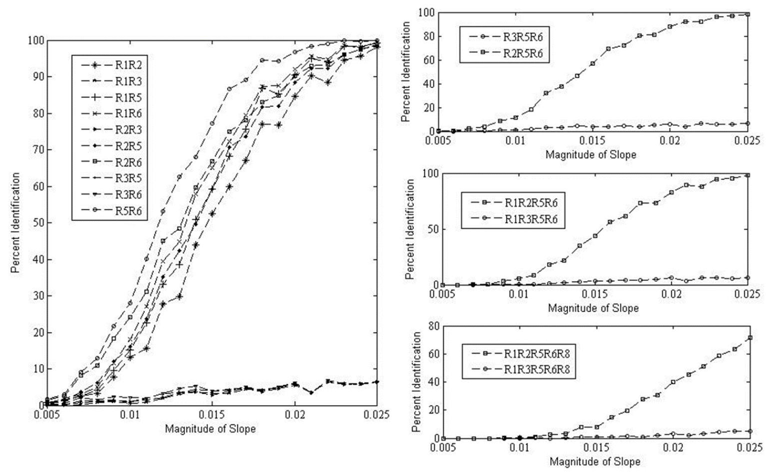

Figure 5. Sensitivity of different combination of Nelsons run rules in identification of trend patterns

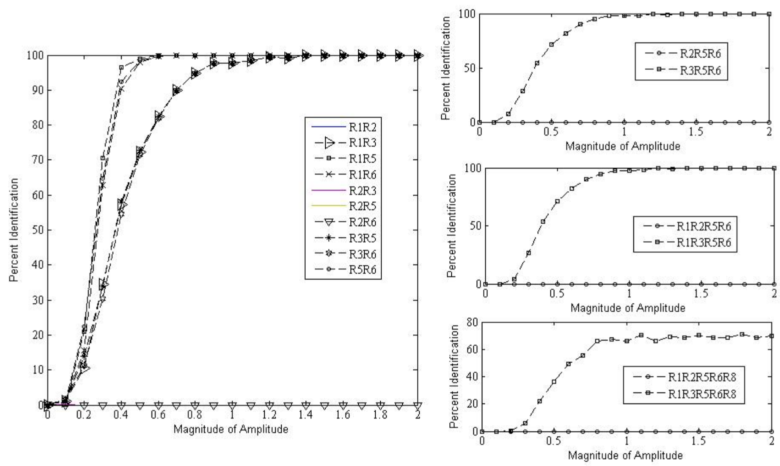

Figure 6. Sensitivity of different combination of Nelsons run rules in identification of cyclic patterns

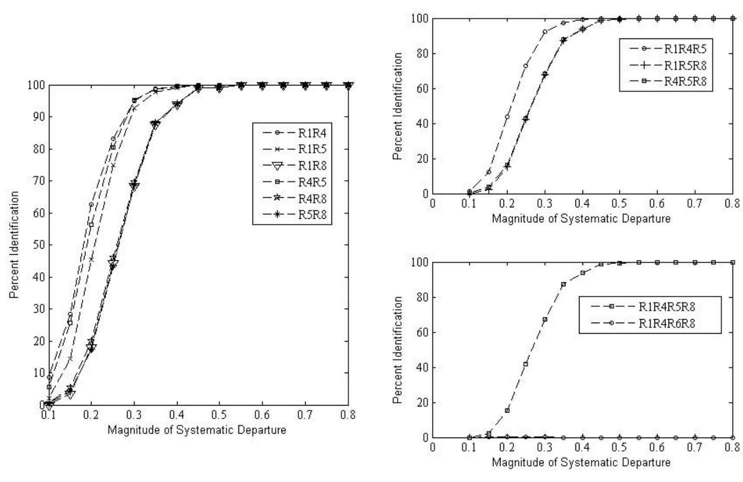

Figure 7. Sensitivity of different combination of Nelson's run rules in identifying systematic pattern

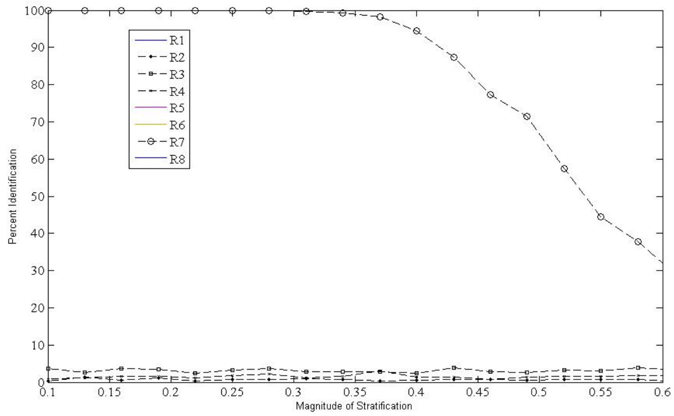

Figure 8. Sensitivity of different combination of Nelson's run rules in identifying stratification patterns

5. Discussion

Figure 4 shows that the shift patterns are more sensitive to the combination of run rules R1, R2, R5, R6 and R8. Any two combinations of run rules R2, R5, or R6 gave more than 90% identification accuracy when the shift magnitude was 1.5σ or more. The combination of three rules (R2, R5 and R6) gave more than 85% accuracy when identifying patterns with shift magnitude of 1.5σ and more. For combination of four run rules (R1, R2, R5 and R6), the accuracy was reduced to less than 70%. Meanwhile, for five rules combination, the accuracy further reduced to less than 50%. The result suggests that run rules and their combinations are not effective for detection of shifts with magnitude lower than 1.5σ.

The result for trend patterns identification is shown in Figure 5. The combined rules gave almost the same results as for the shift patterns. The combined run rules R1, R2, R5, and R6 was also sensitive to the trend patterns. Figure 4 shows that the three rules R2, R5 and R6 combination was effective in identifying shift patterns of 1.5σ to 2.5σ magnitude. Meanwhile, Figure 5 shows the same combination was effective for the trend patterns with slope between 0.018 to .025.

Figure 6 shows that cyclic patterns are more sensitive to the run rules combination of R1, R3, R5, R6 and R8 compared to the combination of R1, R2, R5, R6 and R8. When we applied combination of R3, R5 and R6, it gave about 90% accuracy for cyclic amplitude 0.7σ or higher. The result suggests that existence of rule R3 in any rules combination degraded identification effectiveness for the trend pattern. However, the existence of rule R3 in the rule's combination is useful in the identification of cyclic patterns. The two rules combination, namely R1 and R5, R1 and R6, and R5 and R6 were found to be effective for identifying cyclic patterns.

For systematic pattern, it was found that combination of run rules R1, R4, R5 and R8 was relatively more sensitive compared to combination of R1, R4, R6 and R8 as shown in Figure 7. The results suggest that combination of R1, R4, and R5 is effective for identification of the systematic patterns with accuracy of 90% when systematic departure is more than 0.4. The results suggest that the inclusiveness of R5 improves the effectiveness in identifying systematic patterns.

Figure 8 suggests that the identification of stratification pattern can be done using a single rule R7. It scored the identification accuracy of more than 90% when the stratification magnitude was less than 0.4. The other rules were not sensitive to the stratification pattern. For the normal pattern, it was discovered that R3 and R6 resulted in relatively more false alarm. The proposed Nelson's run rules combinations for each of type of pattern is summarized in Table 4 and graphically displayed in Figure 9.

Table 4. Proposed Nelson's run rules combinations for pattern identification

No	Patterns	Rule Combination
1	Shift	*R2,R5,R6,*
2	Trend	*R2,R5,R6*
3	Cyclic	*R3,R5,R6*
4	Systematic	*R4,R5,R8*
5	Stratification	*R7*
6	Normal pattern	none

From the above results, we can notice that for trend pattern R3 is not sensitive. In general, the results suggest that the run rules are not effective for detecting small shift, small trend, and small amplitude cycle and for systematic pattern with small departure value. Rules R5 seems to be the most sensitive rule for most of the patterns. Identification of either shift or trend can be done using combination of R2, R5 and R6. However, to explicitly identify trend or shift patterns is not possible by this combination. This finding is consistent with Guh et al. (2005). The cyclic, stratification and systematic patterns can be reasonably predicted using the run rules combinations as given in Table 4. This study also convinced that why researchers have inclined toward more advanced pattern identification technique using soft computing technology such as artificial neural network and fuzzy sets for control chart pattern recognition (Zaman and Hassan, 2019). From this study it is also confirmed that runs rules can be effectively applied for stability test. If any combination of run rules is triggered more frequently, one may predict that a process is most likely unstable.

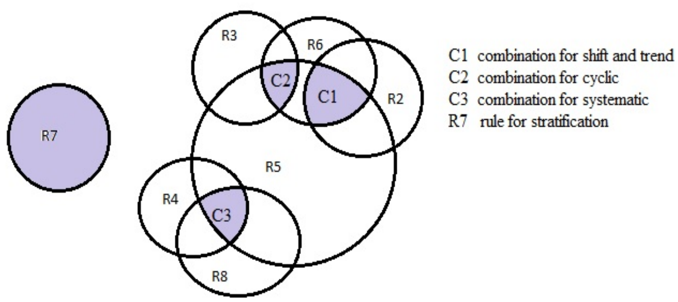

Figure 9. Nelson's Run rules combinations for different pattern identification

6. Conclusions

This paper investigates the effectiveness of Nelson's run rules for identification of different types of abnormal patterns on *X*-bar control chart. The Nelson's run rules sensitivity and patterns recognition capability were evaluated individually and in combinations. Combinations of run rules are proposed for identification of specific unstable patterns. The shift and trend patterns cannot be explicitly differentiated using the Nelson's runs rules combinations. since the same combination of runs rules were sensitive for both shift and trend patterns. The cyclic and systematic patterns can be effectively identified using the proposed run rules combinations. The stratification patterns are more sensitive to R7 individually rather than combination of run rules. There is a need for further investigation to explicitly differentiate between trend and shift patterns. Designing new run rules for small shift, trend and other patterns may be an interesting future research direction especially for application within short run production environment where data driven soft computing pattern recognition techniques may have limited effectiveness.

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Biography
Munawar Zaman earned M.Sc. Industrial Engineering from Universiti Teknologi Malaysia in 2017. Currently he holds a senior position in a manufacturing company in Pakistan.

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