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The *n*-th coprime probability and its graph for some dihedral groups

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Abstract. The coprime probability and graph have been studied for various groups by many researchers focusing on the generalization of the probability part. For the coprime graph, the types and properties of the graph have been investigated and the patterns that can be found within a group are analysed. The coprime probability of a group is defined as the probability that the order of a random pair of elements in the group are relatively prime or coprime. Meanwhile, the coprime graph can be explained as a graph whose vertices are elements of a group and two distinct vertices are adjacent if and only if the greatest common divisor of the order of the first vertex and order of the second vertex is equal to one. It was unfortunate that the exploration of probabilities and graphs of groups have not considered both the *n*-th coprime probability and its graph that ultimately became the target in this research. Hence, the newly defined terms are then used to find the generalizations of the *n*-th coprime probability and the *n*-th coprime graphs for some dihedral groups. The types and properties of the graphs are also discussed in this research.

1. Introduction

Graph theory is the study of graphs in which it is formed through vertices or nodes joined by edges. Also, graph theory has its own significance and can benefit many areas of research. For example, the Konisberg bridge problem is solved by using the concept of planar and Eulerian graph. Another example of the application of graph theory is from the study of Natarajan and Balaci [1] where graph theory is used to find the shortest routes in online network services by setting up the variances involved in the research as vertices and edges. Studies on graphs with different approaches have been conducted over the years and in this research, the focus is on the extension of the prime graph.

The prime graph of a group was first introduced by Williams [2] in 1981 as a graph having the prime numbers dividing the order of G as its vertices and two vertices x and y are joined by an edge if and only if G contains an element of order xy. The prime graph has attracted the attention of many researchers that they began to explore and extend the study under different scopes of groups such as dihedral groups, p-group, and nonabelian metabelian groups.

Ma *et al.* [3] extended the study of prime graph to the coprime graph of a group and it is defined as a graph whose vertices are elements of G and two distinct vertices are adjacent if and only if the greatest common divisor of their order is equal to one. In the paper, they determined the types and the properties of the graph which include the diameter, planarity, partition, and clique number. Later, Dorbidi [4] continued the research by proving that the chromatic number of the coprime graph of G is equal to its clique number.

Subsequently, non-coprime graph of a group G with vertex set $G \setminus \{e\}$ has also been introduced in [5] and it is defined as follows: two distinct vertices are adjacent whenever their orders are relatively non-coprime. In the paper, the properties of a graph of a group are obtained and at the same time, they also discussed on the relation between the non-coprime graph and the prime graph. Later on, another graph has been introduced named as the relative coprime graph. Abd Rhani *et al.* [6] defined the relative coprime graph of a group as a graph whose vertices are elements of G and two distinct vertices x and y are joined by an edge if and only if their orders are coprime and any of x or y is in H where $H \lhd G$.

Inspired by the research of the extension of prime graph, later, it was discovered that among those studies, no published work has been done related to probability. So, researchers took the opportunity to explore on the coprime probability for some finite groups. Hence, in 2018, Abd Rhani [7] introduced a probability named as the coprime probability of a group. In the paper, the author generalized the coprime probability for dihedral group, D_n where n is odd and p-group where p is prime. The definition and the results are stated as follows:

Definition 1 [7] Coprime Probability of a Group
Let G be a finite group. For any $x, y \in G$ the coprime probability of G, denoted as $P_{copr}(G)$, is defined as

$$P_{copr}(G) = \frac{\left|\left\{(x,y) \in G \times G : (|x|,|y|) = 1\right\}\right|}{|G|^2}.$$

Note that $(x, y) = 1$ is equivalently as $\gcd(x, y) = 1$.

Proposition 1 [7]
Let G be a finite p-group of order p^n where $n \geq 1$. Then

$$P_{copr}(G) = \frac{2p^n - 1}{p^{2n}}.$$

Proposition 2 [7]
Let D_n be a dihedral group of order $2n$, where $n \geq 3$ and n is odd. Then

$$P_{copr}(G) = \frac{2n^2 + 2n - 1}{4n^2}.$$

Later on, Zulkifli and Mohd Ali [8] continued the study of coprime probability by emphasizing on the nonabelian metabelian groups of order less than 24. They also published another paper as in [9] by introducing the relative coprime probability which is also an extension from the coprime probability. In the paper, their scope of group was also on the nonabelian metabelian groups of order less than 24. Definition 2 describes the relative coprime probability of a group.

Definition 2 [9] Relative Coprime Probability of a Group
Let G be a finite group and H be a subgroup of G. The relative coprime probability of G is defined as follows;

$$P_{copr}\left(H,G\right)=\frac{\left|\left\{\left(h,y\right)\in H\times G:\left(\left|h\right|,\left|y\right|\right)=1\right\}\right|}{\left|H\right|\left|G\right|}.$$

The results from [9] showed that if the groups of nonabelian metabelian groups of order less than 24 have the same order then their relative coprime probability are the same but still this is only applicable to certain orders only.

Therefore, this paper concentrates on the extension of the coprime probability and the extension of the coprime graphs where the n-th coprime probability and n-th coprime graphs are formulated. Then, the generalization of the n-th coprime probability for dihedral group, D_p where p is prime is determined. Besides, the types and properties of the n-th coprime graphs which include the diameter, chromatic number, domination number, and the independence number are also obtained.

Hence, this paper is structured as follows: the first part discusses on the introduction of this research while the second part states the basic concepts, definitions and proposition for both groups and graphs theory that are useful throughout this research. In the third and fourth sections, the main results and the conclusion of this paper for both the $n-th$ coprime probability and the n-th coprime graphs for dihedral group, D_p where p is prime are discussed.

2. Preliminaries

In this section, some basic concepts on groups and graphs are stated.

Definition 3 [10] Dihedral Groups of Degree n

For each $n\in\mathbb{Z}$ and $n\geq 3$, D_n denoted as the set of symmetries of a regular n-gon. Furthermore, the order of D_n is $2n$. The dihedral groups, D_n can be represented in a form of generators and relations given in the following representation:

$$D_n=\left\langle a,b:a^n=b^2=e,ba=a^{-1}b\right\rangle.$$

Definition 4 [11] Star Graph

A star graph, $K_{1,n}$, is a graph which consists of a single vertex with n neighbors.

Definition 5 [12] Complete Tripartite Graph

The complete tripartite graph, $K_{r,s,t}$, consists of three sets (of sizes r,s,t), with edges joining two vertices if and only if they lie in different set.

Definition 6 [13] The Diameter

The diameter, $diam\left(\Gamma\right)$, of a connected graph Γ is the greatest distance between all pairs of the vertices of Γ.

Definition 7 [13] The Chromatic Number

The chromatic number of Γ, $\chi\left(\Gamma\right)$, is the smallest number of colours needed to colour the vertices of Γ such that no two adjacent vertices get the same colour.

Definition 8 [13] The Domination Number

The dominating set $X\subseteq V(\Gamma)$ is a set where for each v outside X, there exists $x\in X$ such that v is adjacent to x. The minimum size of X is called the dominating number and it is denoted by $\gamma\left(\Gamma\right)$.

Definition 9 [13] The Independence Number

A non-empty set S of $V(\Gamma)$ is called an independent set of Γ if there is no adjacent between two elements of S in Γ. Thus, the independent number is the number of vertices in the maximum independent set and it is denoted as $\alpha(\Gamma)$.

3. *n*-th Coprime Probability and *n*-th Coprime Graph

A new probability named the n-th coprime probability, denoted as $P_{ncopr}(G)$, is introduced in this paper which is also an extension from the coprime probability. The definition for this probability is stated and for further understanding of this definition, D_3 is used as an example to describe the n-th coprime probability.

Definition 10 n-th Coprime Probability of a Group
Let G be a finite group. For any $x, y \in G$, the n-th coprime probability of G is defined as

$$P_{ncopr}(G) = \frac{\left|\left\{(x,y) \in G \times G : \left(\left|x^n\right|, |y|\right) = 1\right\}\right|}{|G|^2},$$

where $1 \le n \le |G|$.

Remark 1.

If x^n is one of the elements in G, then $P_{ncopr}(G) = P_{copr}(G) = \dfrac{\left|\left\{(x,y) \in G \times G : \left(|x|, |y|\right) = 1\right\}\right|}{|G|^2}$.

Example 1

Let $G = D_3 = \{e, a, a^2, b, ab, a^2b\}$. The Cayley table of D_3 is as follows:

Table 1. Cayley table of D_3.

	$\boldsymbol{e}$	$\boldsymbol{a}$	$\boldsymbol{a^2}$	$\boldsymbol{b}$	$\boldsymbol{ab}$	$\boldsymbol{a^2b}$
$\boldsymbol{e}$	e	a	a^2	b	ab	a^2b
$\boldsymbol{a}$	a	a^2	e	ab	a^2b	b
$\boldsymbol{a^2}$	a^2	e	a	a^2b	b	ab
$\boldsymbol{b}$	b	a^2b	ab	e	a^2	a
$\boldsymbol{ab}$	ab	b	a^2b	a	e	a^2
$\boldsymbol{a^2b}$	a^2b	ab	b	a^2	a	e

The order for each element in D_3 is as follows; $|e| = 1$, $|a| = |a^2| = 3$ and $|b| = |ab| = |a^2b| = 2$. Firstly, for all x in D_3, x^n where $1 \le n \le |D_3|$ are obtained. Table 2 states the elements of x^n where $1 \le n \le |D_3|$.

Table 2. x^n where $n = 1,2,3,4,5$ and 6 for all x in D_3.

$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
$(e)^1 = e$	$(e)^2 = e$	$(e)^3 = e$	$(e)^4 = e$	$(e)^5 = e$	$(e)^6 = e$
$(a)^1 = a$	$(a)^2 = a^2$	$(a)^3 = e$	$(a)^4 = a$	$(a)^5 = a^2$	$(a)^6 = e$
$(a^2)^1 = a^2$	$(a^2)^2 = a$	$(a^2)^3 = e$	$(a^2)^4 = a^2$	$(a^2)^5 = a$	$(a^2)^6 = e$
$(b)^1 = b$	$(b)^2 = e$	$(b)^3 = b$	$(b)^4 = e$	$(b)^5 = b$	$(b)^6 = e$
$(ab)^1 = ab$	$(ab)^2 = e$	$(ab)^3 = ab$	$(ab)^4 = e$	$(ab)^5 = ab$	$(ab)^6 = e$
$(a^2b)^1 = a^2b$	$(a^2b)^2 = e$	$(a^2b)^3 = a^2b$	$(a^2b)^4 = e$	$(a^2b)^5 = a^2b$	$(a^2b)^6 = e$

From the above table, for all x in D_3, x^1 and x^5 are equal to one of the elements in D_3. Secondly, x^2 and x^4 are equal to e, a or a^2. Lastly, x^3 is equal to e, b, ab or a^2b and x^6 is equal to identity. Let $a_i \in x^n$ where $1 \le i \le |x^n|$. In order to find the n-th coprime probability of D_3, the results are explained next.

For $n = 1$, according to Table 2, $x^1 = x^5 = e, a, a^2, b, ab$ or a^2b. Since D_3 is a dihedral group where 3 is prime, then by Proposition 2, $P_{ncopr}(G) = P_{copr}(G) = \frac{23}{36}$.

Now, for $n = 2$, let $a_i \in x^2$ where $|a_i| = 1$ or 3. If $|a_i| = 1$, then $a_i = e$. Thus e is coprime to each element y in G such that $(|e|,|y|) = 1$. Hence, $W_1 = \{(x,y) \in G \times G : (|e|,|y|) = 1\}$. If $|a_i| = 3$, then $a_i = \{a, a^2\}$. Thus, $(|a_i|,|y|) = 1$ if $|y| = 1$ or 2, i.e. $y = e, b, ab$ or a^2b. Hence, $W_2 = \{(x,y) \in G \times G :$ $(|e|,|y|) = 1\}$. Therefore,

$$
\begin{aligned}
P_{ncopr}(G) &= \frac{\sum\{(x,y) \in G \times G : (|e|,|y|) = 1\}}{|G|^2} \\
&= \frac{W_1 + W_2}{|G|^2} \\
&= \frac{7}{9}.
\end{aligned}
$$

The same steps and results are obtained for $n = 4$.

Next, for the case of $n = 3$, let $a_i \in x^3$ where $|a_i| = 1$ or 2. If $|a_i| = 1$, then $a_i = e$. Thus e is coprime to each element y in G such that $(|e|,|y|) = 1$. Hence, $W_3 = \{(x,y) \in G \times G : (|e|,|y|) = 1\}$. If $|a_i| = 2$, then $a_i = \{b, ab, a^2b\}$. Therefore, $(|a_i|,|y|) = 1$ if $|y| = 1$ or 3, i.e. $y = e, a$ or a^2. Hence, $W_4 =$ $\{(x,y) \in G \times G : (|e|,|y|) = 1\}$. Thus,

$$
\begin{aligned}
P_{ncopr}(G) &= \frac{\sum\{(x,y)\in G\times G:(|e|,|y|)=1\}}{|G|^2} \\
&= \frac{W_3+W_4}{|G|^2} \\
&= \frac{5}{8}.
\end{aligned}
$$

Finally, for $n=6,$ let $a_i \in x^6$ where $|a_i|=1$. If $|a_i|=1,$ then $a_i=e.$ Thus e is coprime to each element y in G such that $(|e|,|y|)=1$. Hence, $W_5=\{(x,y)\in G\times G:(|e|,|y|)=1\}$. Therefore,

$$
\begin{aligned}
P_{ncopr}(G) &= \frac{\sum\{(x,y)\in G\times G:(|e|,|y|)=1\}}{|G|^2} \\
&= \frac{W_5}{|G|^2} \\
&= 1.
\end{aligned}
$$

Next, a new graph is also introduced named as the n-th coprime graph which is also an extension from the study of the prime graph. The definition is given below and D_3 is used an example to illustrate the definition.

Definition 11 n-th Coprime Graph of a Group
The n-th coprime graph of $G,$ denoted as $\Gamma_{ncopr}(G),$ is a graph whose vertices are elements of G and two distinct vertices x and y in G are adjacent if and only if $(|x^n|,|y|)=1$ where $1\le n\le |G|$.

Example 2
This example illustrates the n-th coprime graph of D_3, $\Gamma_{ncopr}(D_3)$. From Definition 11, by referring to Table 1 and 2, below are the graphs that can be formed. When $n=1,2,3,4,5,$ a complete tripartite graph is constructed whereas when $n=6,$ a star graph is formed.

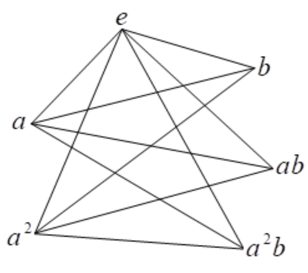

Figure 1. When $n=1,2,3,4,5.$

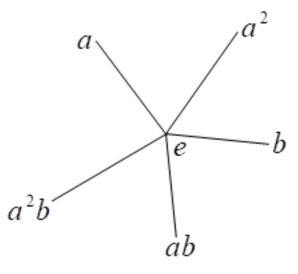

Figure 2. When $n=6.$

In the next section, the main results of this research are discussed.

4. Results and Discussions

This section is divided into two parts. The first part discussed the generalization of the n-th coprime probability which stated in Theorem 1. Then, the second part discussed the types of the n-th coprime graphs with the properties of their graph. Throughout this research, the study on both n-th coprime probability and its graphs are determined for dihedral group, D_p where p is prime.

Theorem 1

Let G be a dihedral group, D_p where p is prime. For any $x, y \in G$, let $(|x^n|, |y|) = 1$ where $1 \le n \le |G|$, then

$$P_{ncopr}(G) = \begin{cases} \dfrac{2p^2+2p-1}{4p^2}, & \text{if } n \text{ is odd,} \\ \dfrac{p^2+2p-1}{2p^2}, & \text{if } n \text{ is even,} \\ \dfrac{p+2}{2(p+1)}, & \text{if } n = p, \\ 1, & \text{if } n = |G|. \end{cases}$$

Proof:

Suppose G is a dihedral group, D_p of order $2p$, where p is prime. If n is odd, then $a_i \in x^n$ where $1 \le i \le |x^n|$ and $|a_i| = 1, 2$ or p. Now, if $|a_i| = 1$, then a_i is coprime to each element y in G such that $(|a_i|, |y|) = 1$. If $|a_i| = 2$ then $(|a_i|, |y|) = 1$ when $|y| = 1$ or p. Finally, if $|a_i| = p$ then $(|a_i|, |y|) = 1$ when $|y| = 1$ or 2. Therefore,

$$P_{ncopr}(G) = \frac{\sum\{(x,y) \in G \times G : (|x^n|, |y|) = 1\}}{|G|^2} = \frac{2p^2+2p-1}{4p^2}.$$

The same steps are applied for the case when n is even, $n = p$ and $n = |G|$. In this case, when n is even, then $|a_i| = 1$ or p, when $n = p$ then $|a_i| = 1$ or 2 and when $n = |G|$ then $|a_i| = 1$.

Next, Proposition 3 to Proposition 8 discussed the n-th coprime graph for dihedral group, D_p where p is prime. The types and some properties of the graph which include the diameter, the chromatic number, the domination number and the independence number are analyzed.

Proposition 3

Let G be a dihedral group, D_p where p is prime. Then

1) $\Gamma_{ncopr}(G)$ is a star graph if and only if $x^n \in G$ where $n = |G|$.
2) $\Gamma_{ncopr}(G)$ is a complete tripartite graph if and only if $x^n \in G$ where $1 \le n < |G|$.

Proof:
$(\Leftarrow)$ Let $x^n \in G$. If $n=|G|$, then $x^n = e$. It is obvious that e is adjacent to all elements in G. Hence, $\Gamma_{ncopr}(G)$ is a star graph.

But if $1 \le n < |G|$, then $|x^n| = 2p$ when n is odd, $|x^n| = p$ when n is even and $|x^n| = p+1$ when $n = p$. Let $a_i \in x^n$ and $|a_i| = 1,2$ or p. If $|a_i| = 1$, then $a_i = e$. So, e is adjacent to all elements in G. Also, other elements such as $|a_i| = 2$ or p, need to be considered as there are also edges between two non-identity elements such that $(|x^n|,|y|) = 1$. Hence, three partition can be made since no edges between elements that have the same order. So, $\Gamma_{ncopr}(G) = K_{1,|G|,|G|-1}$ which is a complete tripartite graph for dihedral group.

$(\Rightarrow)$ If $\Gamma_{ncopr}(G)$ is a star graph then there is only one element that have edges to all other elements in G. In this case, only e can have edges to all other elements in G since $|e| = 1$. Hence, $x^n = e$ when $n = |G|$.

If $\Gamma_{ncopr}(G)$ is a complete tripartite graph then there are three partition exist in this graph and each element in the partition are adjacent to each element in another partitions. Since G is dihedral groups, D_p where p is prime, so, let $a_i \in x^n$ then $|a_i| = 1,2$ or p. By Definition 11, no edges between two vertices that have the same order. Therefore, the partitions are divided by the order of element which are $1,2$ and p. In this case, $|a_i| = 1,2$ or p will be obtained if $|x^n| > 1$. Hence, $x^n \in G$ when $1 \le n < |G|$.

Proposition 4
Let G be a dihedral group, D_p where p is prime. Then $diam(\Gamma_{ncopr}(G)) = 2$.

Proof:
By Proposition 3, $\Gamma_{ncopr}(G)$ is either star or complete tripartite graph. Let $a_i \in x^n$ where $|a_i| = 1,2$ or p. If $a_i = e$, then e is adjacent to all elements in G. If $a_i \in x^n \setminus e$, then there are also edges between the non-identity elements in G. Also, by Definition 11, no edges between two vertices that have the same order. Hence, $diam(\Gamma_{ncopr}(G)) = 2$.

Proposition 5
Let G be a dihedral group, D_p where p is prime. Then

$$\chi(\Gamma_{ncopr}(G)) = \begin{cases} 3, & 1 \le n < |G|, \\ 2, & n = |G|. \end{cases}$$

Proof:
By Proposition 3, if $n = |G|$, then $\Gamma_{ncopr}(G)$ is a star graph which consists of one vertex that is adjacent to all other elements in G. Hence, $\chi(\Gamma_{ncopr}(G)) = 2$. If $1 \le n < |G|$, then $a_i \in x^n$ such that $|a_i| = 1,2$ or p.

By Proposition 3, $\Gamma_{ncopr}(G)$ is a complete tripartite graph with three partition. In this case, the partitions are separated by the order of the elements which are $1,2$ or p. Since it is a complete tripartite graph, so each vertex in a partition is connected to each vertex in another partitions. So, $\chi\left(\Gamma_{ncopr}(G)\right)=3$.

Proposition 6
Let G be a dihedral group, D_p where p is prime. Then $\gamma\left(\Gamma_{ncopr}(G)\right)=1$.

Proof:
Let $a_i \in x^n$ where $1 \le n \le |G|$. If $a_i = e$ then $|e|=1$. By Definition 11, e is adjacent to all elements in G such that $(|e|,|y|)=1$. Therefore, by the Definition 8, $\gamma\left(\Gamma_{ncopr}(G)\right)=1$.

Proposition 7
Let G be a dihedral group, D_p where p is prime. Then

$$\alpha\left(\Gamma_{ncopr}(G)\right)=\begin{cases} p, & 1 \le n < |G|, \\ 2p-1, & n=|G|. \end{cases}$$

Proof:
If $n=|G|$, then $x^n = e$ and e is adjacent to all elements in G. It is obvious that the maximum independent set is all element in G except e. Hence, $\alpha\left(\Gamma_{ncopr}(G)\right)=2p-1$. By Theorem 1, the $\Gamma_{ncopr}(G)$ is a complete tripartite graph. There are three partitions separated by the order of the elements and they are $1,2$ or p. By Definition 11, $\left(|x^n|,|y|\right)=1$, so, there is no adjacent between elements that have the same order. Hence, $\{a_1,a_2,\ldots,a_p\}$ is the maximum independent set when $|a_1|=|a_2|=\ldots=|a_p|=2$. Therefore, $\alpha\left(\Gamma_{ncopr}(G)\right)=p$.

Proposition 8
Let G be a dihedral group, D_p where p is prime. Then

$$|E(G)|=\begin{cases} p^2+p-1, & 1 \le n < |G|, \\ p-1, & n=|G|. \end{cases}$$

Proof:
If $1 \le n < |G|$, then $a_i \in x^n$ where $|a_i|=1,2$ or p. So, by Definition 11**,** there are edges between e and other elements in G such that $\gcd(1,2)=\gcd(1,p)=1$ and also, there are edges between two non-identity elements such that $\gcd(2,p)=1$. Hence, $|E(G)|=p^2+p-1$. If $n=|G|$, then $x^n=e$ and $|e|=1$. So, there are edges between e and other elements such that $\gcd(|e|,|y|)=$ $\gcd(1,2)=\gcd(1,p)=1$ and $(1,p)$. Hence, $|E(G)|=p-1$.

5. Conclusion
In this paper, two new definitions are introduced named the n-th coprime probability and the n-th coprime graphs of a group. The generalization of the n-th coprime probability and its graphs are also

discussed in addition to providing the types and the properties of the n-th coprime graph which include the diameter, the chromatic number, the domination number and the independence number of a graph. Throughout this research, the scope of group used in this research is the dihedral group, D_p where p is prime.

It can be concluded that the n-th coprime probability for D_p where p is prime vary for each n where n is even, odd, p and $|G|$. For the n-th coprime graph for D_p where p is prime, the properties of the graphs that have been obtained in this research are the diameter is equal to two and the domination number is equal to one. The independent number, the chromatic number and the number of edges of a graph are separated into two parts that are when $1 \leq n < |G|$ and $n = |G|$. At the same time, two types of graphs are also formed in this research which are the star and the complete tripartite graph for $n = |G|$ and $1 \leq n < |G|$, respectively.

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