

Estimation of pore volumes to breakthrough number in limestone cores by derivation of an empirical model



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ABSTRACT

Acidizing in carbonate formations is an inevitable stimulation treatment method for oil and gas wells. In the limestone, acidizing stimulation makes capillary wormholes to increase fluids flow reservoir production. The pore volume to breakthrough number is one of the main indexes for recognizing the wormhole structure. Therefore, finding the pore volume to breakthrough number is one of the main goals in the limestone acidizing. Obtaining this number is always required for experimental works, which needs time, energy and cost. The purpose of this research is to develop an empirical method to estimate an acceptable result for this number merely by implementing limestone core and acid properties without any experimental work. In order to create a wormhole, an empirical method is developed using the law of conservation of mass considering that the core of limestone as an isolated package and the overall mass is constant in this package in the acidizing period. Also, to develop the mathematical section, the Damköhler number is used. Since this number must be calculated experimentally, a constant number is created in the model to eliminate the Damköhler number. An average accuracy of 92.31% is obtained for the developed empirical model by comparing the results obtained from the other three experimental and numerical works. This study conclusively provides a thoroughly empirical method for estimating a high accuracy of the pore volume to breakthrough number by only using known physical properties limestone core and acid.

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1. Introduction

This empirical modelling involves estimating the scale of the acid injection flow rate through the wormholes formed inside the limestone core that is called pore volume to breakthrough number (PV_{BT}) based on mass balance law (Volkenstein, 2009). The compressibility of pore volumes in the cores is related to the pore aspect ratio, porosity, and the proportion of pore volumes (Sui et al., 2020). The empirical method maintains the relative of mass transport and chemical reaction between limestone core and injected acid. Grounded on this technique, during the acidizing process and wormhole creation, the overall mass in the limestone core as a closed system is considered to be constant. Also, to determine the PV_{BT} , the shape of the wormhole is considered to be a

cylindrical capillary tube. According to mass balance law, the injected acid to the limestone core as an input mass minus the product fluid came out of it as an output mass shows the created wormhole volume inside the core as a change of masses. To develop the mathematical section, the Damköhler number (N_{Da}) is used (Fredd, 2000; Fredd & Fogler, 1998a, 1998b, 1998c, 1999; Fredd et al., 1997; Gdanski, 1999; Talbot and Gdanski, 2008). This dimensionless number is defined as the time of fluid flow to the time of chemical reaction (Fogler, 1999). Since experimental works must calculate this number, the rate of core dissolution is defined in the model. This factor shows the wormhole growing rate in the core (Inger, 2001). Also, a coefficient constant number which is specific for each type of limestone core and acid is added to the model to eliminate the N_{Da} . The added coefficient number has a linear relative with the N_{Da} .

To enhance oil recovery technologies by using chemical fluids, the characterization of reservoirs is an important aspect (Fang et al., 2018; Qi et al., 2018; Sun et al., 2018). The optimum acid injection

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rate depends on the acid concentration, the core properties, and the temperature; and among all, rock properties have the most influence on the optimum acid rate. Comparing to limestone, in slow dolomite reactions, a significant rise in acid mass and a meaningful decrease in rate are required. Also, the temperature rise increases the reaction rate of dolomite. Therefore, the results of the acid injection in dolomite at high temperatures are aligned with the acid injection in limestone (Akanni & Nasr-El-Din, 2015; Dong, 2018; Dong et al., 2016; Economides and Nolte, 1989; ; Glasbergen et al., 2005; Glasbergen et al., 2009a,b; Liu et al., 2012; Mahmoud, Nasr-El-Din, De Wolf and LePage, 2010; Sidaoui et al., 2018).

Concentration changes by using chemical balance equation rules in the limestone core and acid during acidizing are used for the chemical section of the empirical model. The injected acid cannot solve the entire limestone core since the weight mass of injected acid is less than weight masses of limestone core this phenomenon known as limiting reagent law. “The limiting reagent or limiting reactant in a chemical reaction is the substance that is totally consumed when the chemical reaction is complete” (Zumdahl, 1995).

The constant equations for different kinds of acids such as mineral, organic, complex are used for derivation of the empirical model. The final equation for the model that can calculate the PV_{BT} number contains only physical limestone core and injected acid properties, and this empirical model can estimate the PV_{BT} number with acceptable results without any experimental works. Using the same method for complex acids and limestone cores shows the accuracy of 81.36% and, the accuracy of 95.98% for hydrochloric acid and carbonate cores (Lohrasb & Junin, 2019a, 2019b). On the other hand, an empirical model that can estimate this number has not yet been developed for hydrochloric acid and limestone cores. Therefore, in this study, an empirical model capable of estimating PV_{BT} based on the properties of acids and carbonate formations without any experimental work was developed.

2. Methodology

Three categories of experimental, semiempirical and numerical works were used to produce and test the model for the data collected in this empirical model. The first category is experimental data and was collected from Fredd and Fogler (1998b) work. A single type of Indiana limestone core and five varieties of acids have been collected. Indiana limestone with an average porosity of 20%, a diameter of 3.81 cm and a length of 10.16 cm. Regarding the injected acids, hydrochloric acid (HCl) with a concentration of 0.5 M, acetic acid (HAc) with a concentration of 0.5 M, Ethylenediaminetetraacetic acid (EDTA) with a concentration of 0.25 M, Cyclohexylenedinitrilotetraacetic acid (CDTA) with a concentration of 0.25 M, and Diethylenetriaminepentaacetic acid (DTPA) with a concentration of 0.25 M was used. The second category is semiempirical data is one type of limestone core and one type of acid with two different temperatures and collected from Buijse and Glasbergen (2005), the core is Indiana limestone with a porosity of 20%, diameter of 5.08 cm, and length of 20 cm and the injected acid is HCl with a concentration of 1.5 M in 25 °C and 60 °C. Furthermore, for the third category, one type of limestone core and one type of acid were obtained from Maheshwari and Balakotaiah (2013); limestone with a porosity of 20%, diameter of 3.81 cm, and length of 10.16 cm. The injected acid is HCl, with a concentration of 0.5 was used.

For data processing, the method is focused on the maintenance of mass law for the empirical part of the model, and the entire core was defined to be a close system. Therefore, the total mass in the process should be constant throughout the process of fluid injection into the core and wormhole creation. Also, it is presumed that

the wormhole has a cylinder shape so that its volume can be measured. By using this method, the mass entry at the one side minus output mass at the other side of carbonate core as a system shows the occurred changes in the system that creates the volume of the wormhole. The dimensionless Damköhler number (N_{Da}) is used for the mathematical section of model development. This number is defined as the flow time to chemical time scale ratio. A coefficient constant was utilized in this empirical model to eliminate N_{Da} number, which is basically calculated experimentally.

3. Model development

For model development, the wormhole is considered as a capillary tube with the cross section of a with length of l and diameter of d created with acid injection flow rate of q_1 and concentration of C_1 inside the limestone core with the length of X and cross section of A . The initial equation based on the mass balance law is presented in Fig. 1 and Equation (1) and the initial equation is expanded step by step as below.

The empirical model is initiated by expanding the Equation (1) using the following known Equations (3)–(5) to determine PV_{BT} number.

$$C_1 = \frac{m_a}{V_i} \tag{3}$$

$$q_a = \frac{\pi dlk}{N_{Da}} \tag{4}$$

$$V_w = \frac{\pi d^2 l}{4} \tag{5}$$

In the mentioned equations, m_a is the mass weight of injected acid, V_i is the volume of injected acid, q_a is acid injected rate presented by Fredd and Fogler (1998b), π is equal to 3.14, k is acid and core dissolution rate constant and V_w is wormhole volume. The limestone core and acid reaction time or time differential (Δt) is the

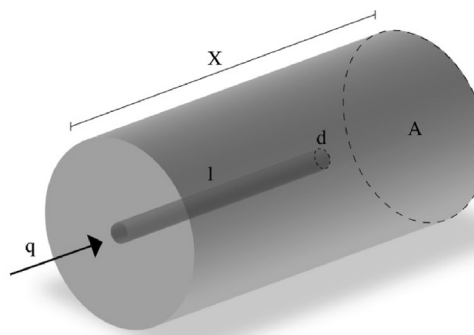


Fig. 1. Capillary tube wormhole created inside a limestone core.

$$\left\{ \begin{array}{l} \text{mass of acid} \\ \text{into the core} \end{array} \right\} - \left\{ \begin{array}{l} \text{mass of product fluid} \\ \text{out of the core} \end{array} \right\} = \left\{ \begin{array}{l} \text{accumulation and rate of} \\ \text{chages of mass inside the core} \end{array} \right\} \tag{1}$$

$$\frac{\{q_1 C_1\}_{in} - \{q_2 C_2\}_{out}}{\Delta C} = \frac{dl}{\Delta t} \tag{2}$$

same as wormhole creation time that is considered as T in this model. Therefore, the empirical model is developed by the definition of PV_{BT} number. “The number of pore volumes to break through is defined as the ratio of the volume of fluid injected to achieve channel breakthrough to the volume of the pore space in the core” (Fredd and Fogler, 1999). Assuming that the injected acid flow will be stopped somewhere in the middle of the limestone core and, on the other side of the core there is no fluid flow rate and q_2 is equal to zero, the initial equation is derived step by step as Equations (6)–(10).

$$\frac{q_1 C_1 - q_2 C_2}{\Delta C} = \frac{al}{\Delta t} \quad (6)$$

$$\frac{q_1 C_1 - q_2 C_2}{\Delta C} = \frac{\pi d^2 l}{4T} \quad (7)$$

$$q \frac{m_a}{V_i} = \frac{\pi d^2 l \Delta C}{4T} \quad (8)$$

$$\frac{V_c \emptyset}{V_i} = \frac{\pi d^2 l \Delta C V_c \emptyset}{4 q m_a T} \quad (9)$$

$$PV_{BT} = \frac{4 q m_a T}{\pi d^2 l \Delta C V_c \emptyset} \quad (10)$$

Where C_2 is product fluid concentration, ΔC is fluid concentration changes, V_c is limestone core volume and \emptyset is the porosity of the core, the dissolution rate (D_R) and N_{Da} equations is used to expand the PV_{BT} number equation as below.

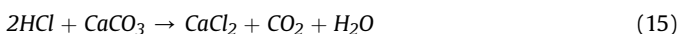
$$\pi d l k = \frac{V_w}{T} = D_R \quad (11)$$

$$q = \frac{V_w/T}{N_{Da}} = \frac{D_R}{N_{Da}} \quad (12)$$

$$PV_{BT} = \frac{q m_a T}{V_w V_c \Delta C \emptyset} \quad (13)$$

$$PV_{BT} = \frac{m_a}{N_{Da} V_c \Delta C \emptyset} \quad (14)$$

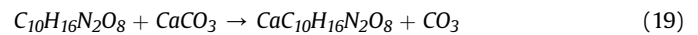
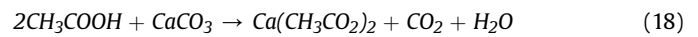
According to the objective of this study, the main purpose is finding the PV_{BT} number without any experimental work and just by physical properties of limestone core and acid. But as shown in Equation (14), the N_{Da} number must be calculated experimentally, and concentration changes is not known. To find the concentration changes, the chemical balance equation law was used. For example, the mass concentration of product fluid in the reaction of hydrochloric acid (HCl) with the molarity of 0.5 mol per liter and a limestone core was calculated and shown in Table 1 and the concentration changes could be calculated by Equations (16) and (17).



$$C_2 = \frac{\sum m}{V_i} \quad (16)$$

$$\Delta C = C_2 - C_1 \quad (17)$$

Now the only parameter that still needs to be calculated experimentally is N_{Da} number. To solve this problem, N_{Da} numbers from Fredd and Fogler (1998b) experimental work were used in the Equation (14) (primary equation). Equation 18, and 19 shows the chemical reaction of Acetic acid (HAc), and Ethylenediaminetetraacetic acid ($EDTA$) with limestone cores; and Fig. 2 shows the calculated PV_{BT} based on primary equation and Fredd and Fogler (1998b) experimental results for these three acids (HCl , HAc , $EDTA$). Where the vertical axis shows PV_{BT} and the horizontal axis shows acid injection rate (cm^3/min).



As it is clear in the Fig. 2, the shape of curves in the empirical model and experimental results are almost the same but they have a huge deviation. To eliminate this deviation and calibrate Equation (14), a specific dimensionless coefficient number ($Co.$) for each PV_{BT} number was multiplied to the empirical model equation as shown in Table 2, and by analyzing the results; it was found that the $Co.$ have a linear and direct relation with N_{Da} for each acid as shown in Fig. 3.

$$PV_{BT} = \frac{Co. m_a}{N_{Da} V_c \Delta C \emptyset} \quad (20)$$

Therefore, the main empirical equation of PV_{BT} for mentioned acids were presented by Equations (21)–(23). And the developed empirical model was evaluated with others' works as below.

$$\text{For } HCl \rightarrow Co. = 1.3791 N_{Da} \rightarrow PV_{BT} = \frac{1.3791 m_a}{V_c \Delta C \emptyset} \quad (21)$$

$$\text{For } HAc \rightarrow Co. = 0.8333 N_{Da} \rightarrow PV_{BT} = \frac{0.8333 m_a}{V_c \Delta C \emptyset} \quad (22)$$

$$\text{For } EDTA \rightarrow Co. = 0.3454 N_{Da} \rightarrow PV_{BT} = \frac{0.3454 m_a}{V_c \Delta C \emptyset} \quad (23)$$

4. Evaluation of empirical model equations

To evaluate the accuracy and deviation of the empirical model equations, calculated results were compared to the PV_{BT} results presented by Fredd and Fogler (1998b), Buijse and Glasbergen (2005), and Maheshwari and Balakotaiah (2013) (as reviewed in Table 3); and evaluation of empirical model were measured by three statistic methods including Standard Deviation (SD), Mean

Table 1
Mass concentration for limestone core and 0.5 M HCl .

	Constant	Compound	Molar concentration (mol/lit)	Molar mass (g/mol)	Weight Concentration (g/lit)	Mass Concentration (g/cm ³)
Injected acid	2	HCl	0.5	36.46094	18.2305	0.0182
Product fluid	1	$CaCl_2$	0.25	110.984	27.746	0.0277
	1	CO_2	0.25	44.0095	11.0024	0.011
	1	H_2O	0.25	18.01528	4.5038	0.0045

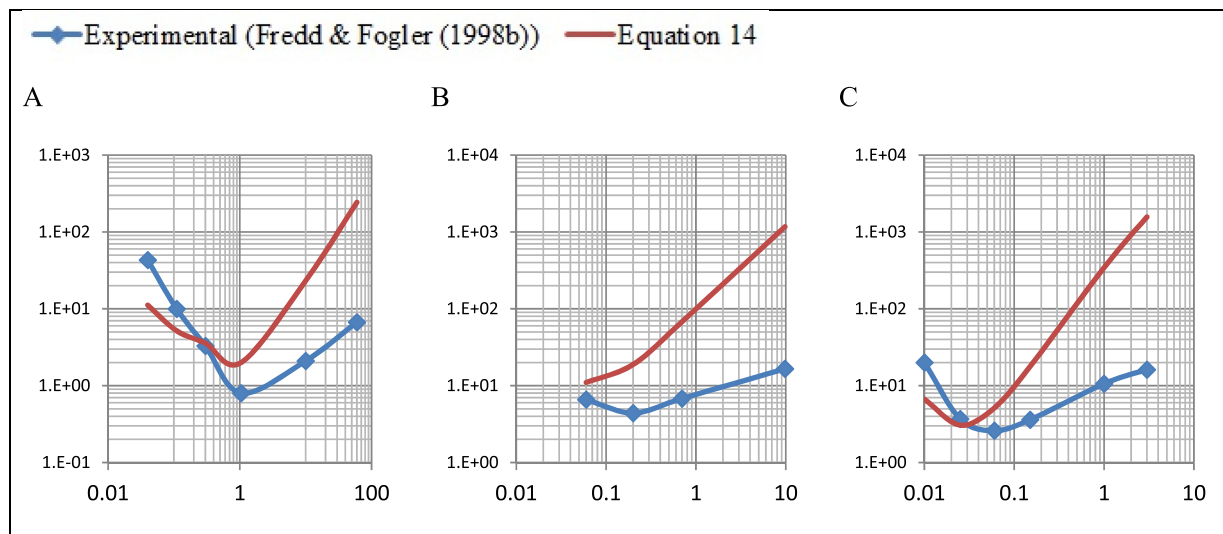


Fig. 2. Comparison of the results obtained by Equation (14) and experimental results given by Fredd and Fogler (1998b) for three acids and limestone cores. A) 0.5 M HCl; B) 0.5 M HAc; C) 0.25 M EDTA.

Table 2
Calculated coefficient numbers by Equation 15.

0.5 M HCl			0.5 M HAc			0.25 M EDTA		
q (cm^3/min)	N_{Da}	Co.	q (cm^3/min)	N_{Da}	Co.	q (cm^3/min)	N_{Da}	Co.
0.04	2.8	3.8615	0.06	0.72	0.6000	0.01	8.7	3.0027
0.11	1.4	1.9308	0.2	0.28	0.2333	0.025	3.5	1.2066
0.3	0.67	0.9240	0.7	0.12	0.1000	0.06	1.5	0.5056
1.05	0.29	0.3999	9.8	0.17	0.0142	0.15	0.6	0.2031
10	0.066	0.0910				1	0.09	0.0308
60	0.02	0.0276				3	0.03	0.0103

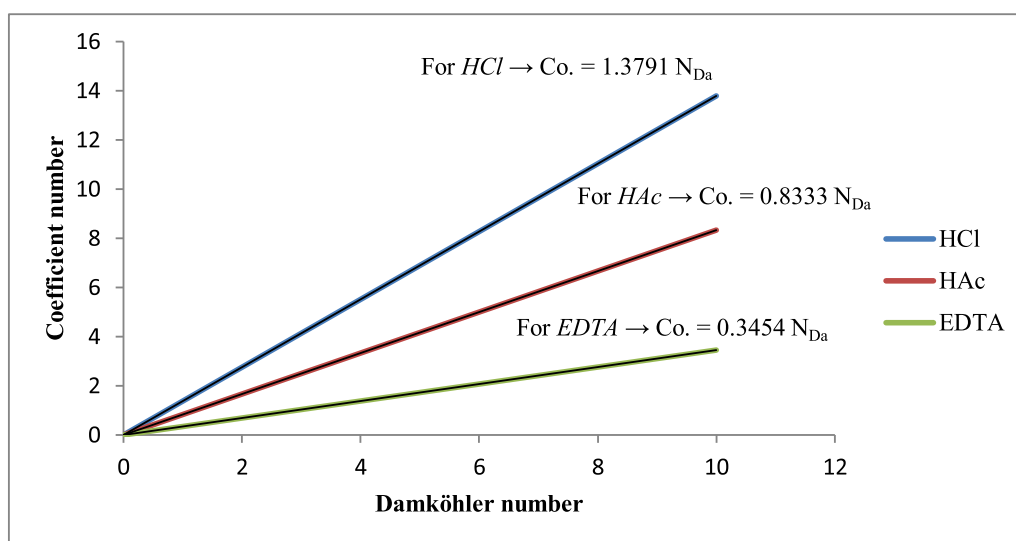


Fig. 3. Coefficient number and Damköhler number relation.

Table 3
Overview of the others works.

Author	Acid	Acid Molarity (mol/lit)	Core	Core porosity (%)	Core length (cm)	Core diameter (cm)
Fredd and Fogler (1998b)	HCl	0.5	limestone	20	10.16	3.81
	HAc	0.5				
	EDTA	0.25				
	CDTA	0.25				
	DTPA	0.25				
Buijse and Glasbergen (2005)	HCl, 25°C	1.5	limestone	20	20	5.08
	HCl, 60°C	1.5				
Maheshwari and Balakotaiah (2013)	HCl	0.5	limestone	20	10.16	3.81

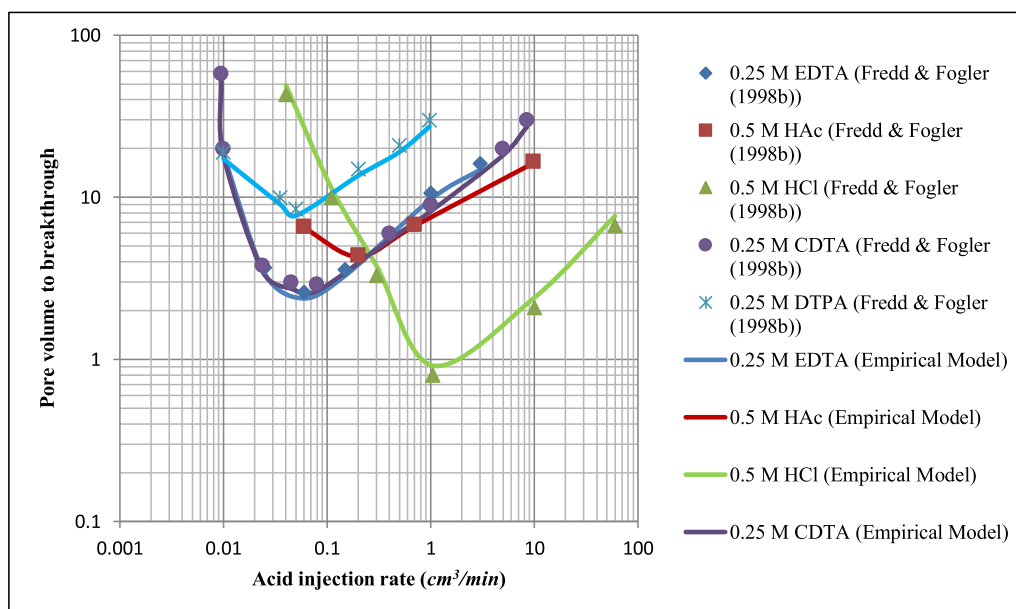


Fig. 4. Evaluation of the empirical model with [Fredd and Fogler \(1998b\)](#) results.

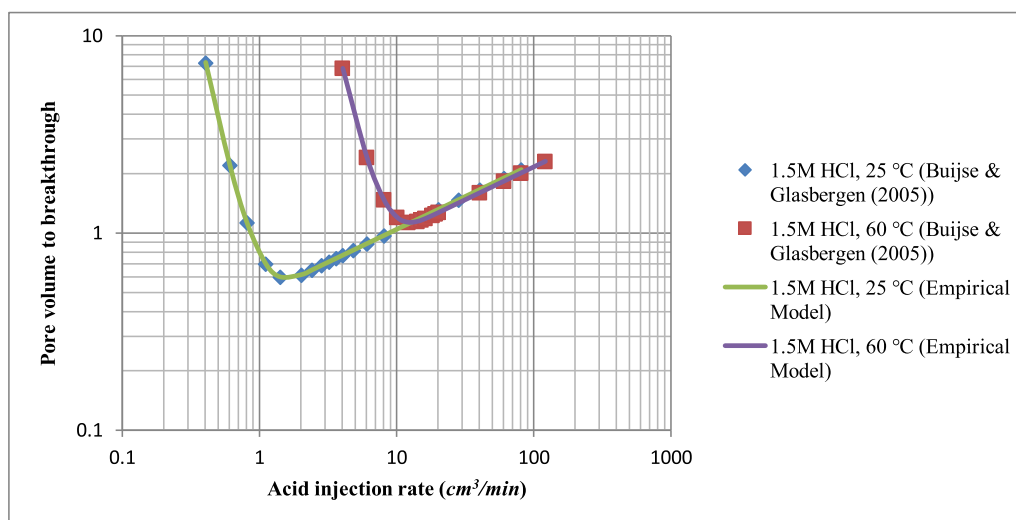


Fig. 5. Evaluation of the empirical model with [Buijse and Glasbergen \(2005\)](#) results.

Absolute Percentage Error (MAPE), and Accuracy percentage (Equation (24)–26).

$$SD = \sqrt{\frac{\sum_{i=1}^N (PV_{actual} - PV_{model})^2}{N}} \tag{24}$$

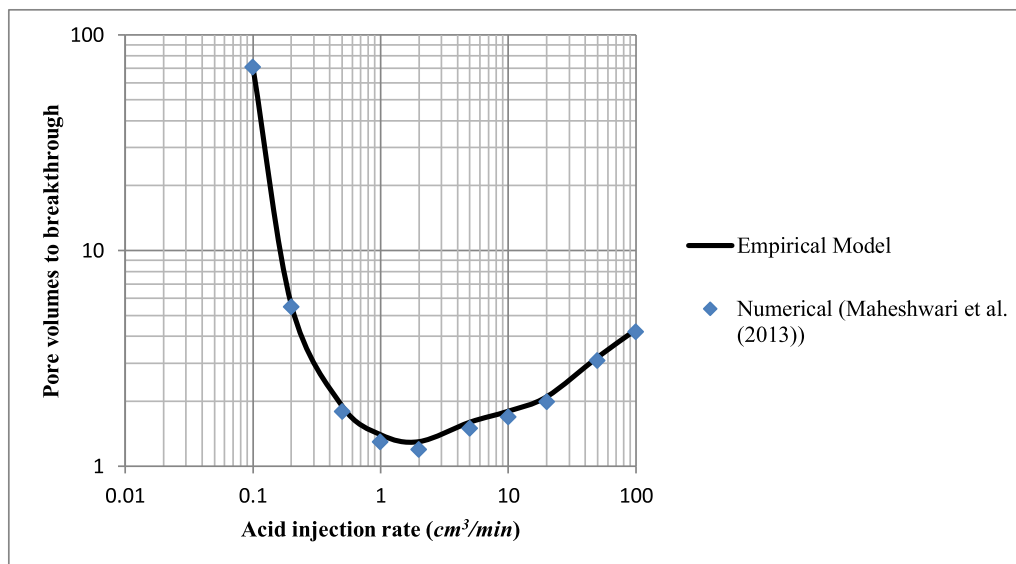


Fig. 6. Evaluation of the empirical model with Maheshwari and Balakotaiah (2013) results.

Table 4
Evaluation of the model.

	Acid type	SD	MAPE (%)	Accuracy (%)
Fredd and Fogler (1998b)	0.5 M HCl	0.0003	0.01	99.99
	0.5 M HAc	0.0004	0.02	99.98
	0.25 M EDTA	0.0597	0.99	99.01
	0.25 M CDTA	2.8159	19.65	80.35
	0.25 M DTPA	6.5973	35.27	64.73
Buijse and Glasbergen (2005)	1.5 M HCl, 25°C	0.1789	0.45	99.55
	1.5 M HCl, 60°C	0.2059	0.45	99.55
Maheshwari and Balakotaiah (2013)	0.5 M HCl	0.0999	4.67	95.33
Average		1.24	7.69	92.31

$$MAPE = \frac{1}{N} \sum_{i=1}^N \frac{|PV_{actual} - PV_{model}|}{PV_{actual}} \times 100 \tag{25}$$

$$\% Accuracy = 100 - MAPE \tag{26}$$

Fig. 4–6 shows the evaluation of the developed empirical model with three different other works.

For the verification, the developed empirical model was evaluated by some statistics methods. Table 4 shows the calculated SD, MAPE and Accuracy percentage for all 76 samples from three different works were used in this paper.

5. Conclusions

The average accuracy for the developed empirical model has been obtained by evaluating the results with 76 samples from three different works, and it is shown to be 92.31% accuracy. Statistical comparison of the first classification of experimental results with empirical results indicates that mineral and organic acids had better results than complex acids in the empirical model. Evaluation of the second classification of the experimental results with empirical model results shows that temperature increase of mineral acid injection inside limestone core has no meaningful effect on the accuracy percentage of the developed empirical model, while the SD shows better results in higher temperatures. Also, a comparison of the third classification proves that higher acid concentration leads to more accurate results for hydrochloric acid.

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Symbols and Abbreviations

ΔC	Fluid concentration changes (gr/cm^3)
A	Cross section of the core (cm^2)
a	Cross section of the wormhole (cm^2)
C_1	Acid concentration before injection (gr/cm^3)
C_2	Fluid concentration after reaction (gr/cm^3)
$CaCO_3$	Limestone (Calcium Carbonate)
$CDTA$	Cyclohexylenedinitrilotetraacetic acid ($C_{14}H_{22}N_2O_8$)
Co	Coefficient number (Dimensionless)
d	Wormhole diameter (cm)
D_R	Dissolution rate constant (cm^3/min)
$DTPA$	Diethylenetriaminepentaacetic acid ($C_{14}H_{23}N_3O_{10}$)
$EDTA$	Ethylenediaminetetraacetic acid ($C_{10}H_{16}N_2O_8$)
HAc	Acetic acid (CH_3COOH)
HCl	Hydrochloric acid (HCl)
k	Overall dissolution rate (cm/min)
l	Wormhole length (cm)
M	Molarity (mol/lit)
m_a	Mass of acid (gr)
$MAPE$	Mean absolute percentage error (%)
N	Number of samples
N_{Da}	Damköhler number (Dimensionless)

\emptyset	Porosity
PV_{BT}	Pore volume to breakthrough (<i>Dimensionless</i>)
q	injection rate (cm^3/min)
SD	Standard deviation
T	Processing time (<i>min</i>)
t	Time (<i>min</i>)
V_c	Core volume (cm^3)
V_i	Volume of injected fluid (cm^3)
V_w	Wormhole volume (cm^3)
X	length of core (<i>cm</i>)
π	Pi number (3.14)

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