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Relative Commutativity Degree of Nonabelian Metabelian Groups of Order 32

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Abstract. A metabelian group is a group whose commutator subgroup is abelian. Similarly, a group G is metabelian if and only if there exists an abelian normal subgroup, A, such that the quotient group, G/A, is abelian. The scope of this research is only for nonabelian metabelian groups of order 32. The commutativity degree of a group G is the probability that two elements of the group G (chosen randomly with replacement) commute. This probability can be used to measure how close a group is to be abelian. This concept has been extended to the co-prime probability which is defined as the probability of a random pair of elements x and y in G for which the greatest common divisor for the order of x and order of y is equal to one. Furthermore, the study of relative commutativity degree of a subgroup H of a group G which is the probability of an element in H commutes with an element in G is included in this research. Previous researchers have determined the commutativity degree of nonabelian metabelian groups of order at most 32. Meanwhile, the co-prime probability and the relative commutativity degree of both cyclic and noncyclic subgroups H are obtained for nonabelian metabelian groups of order at most 30. Since there is no nonabelian group of order 31, thus in this research the co-prime probability and the relative commutativity degree of cyclic subgroups for nonabelian metabelian groups of order 32 are determined.

1. Introduction

A metabelian group is a group whose commutator subgroup is abelian. Similarly, a group $\boldsymbol{G}$ is metabelian if and only if there exists an abelian normal subgroup $\boldsymbol{A}$ such that the quotient group $\boldsymbol{G}/\boldsymbol{A}$ is abelian. A metabelian group can be considered as a group that is close to be abelian, in the sense that every abelian group is metabelian, but not all metabelian groups are abelian. The metabelian groups of order at most 24 have been listed by Abdul Rahman [1] in 2010. The total number of metabelian groups of order at most 24 is 72 which are 59 groups of order less than 24 and 13 groups of order 24. It has been shown that only 25 groups of order less than 24 and 10 groups of order 24 are nonabelian. From these groups, only 35 groups are nonabelian. Meanwhile, in 2018, Simon [2] has determined that all groups of order 32 are metabelian and all those groups are listed in the second column of Table 1.

The relative commutativity degree of a subgroup, H, of a group, G, denoted by $P(H,G)$ was defined by Erfanian et al. in [3]. It can be written as:

$$P(H,G) = \frac{|(h,g) \in H \times G | hg = gh|}{|H||G|}.$$

Hassan [4] has obtained the relative commutativity degree for cyclic subgroups of all nonabelian metabelian groups of order at most 24 while the relative commutativity degree for noncyclic subgroups of all nonabelian metabelian groups of order at most 14 and less than 24 has been determined by Abu Bakar et al. [5] in 2017 and Abu Bakar [6] in 2017, respectively. In addition, Abdul Shukor [7] computed the relative commutativity degree of cyclic subgroups of nonabelian metabelian groups of order 26 to 30 while Jamari [8] has computed the noncyclic subgroups of all nonabelian metabelian groups of order 24 to 30.

Hence, this paper is structured as follows: the first part discusses the introduction of this research while the second part states the basic concepts, definitions and results that are useful throughout this research. In the third and fourth sections, the main results of this paper on both the cyclic subgroups of nonabelian metabelian groups of order 32 and its relative commutativity degree are discussed.

2. Preliminaries

In this section, some basic concepts on groups are stated.

Definition 2.1 [9] Metabelian Group
A group G is metabelian if there exists a normal subgroup A of G such that both A and G/A are abelian.

Theorem 2.1 [3] Let G be finite nonabelian metabelian group and H be a subgroup of G. If $H = \langle 1 \rangle$, then $P(H, G) = 1$.

Definition 2.2 [9] Left and Right Cosets
Let H be a subgroup of G. The subset $aH = \{ah | h \in H\}$ of G is the left coset of H containing a, while the subset $Ha = \{ha | h \in H\}$ is the right coset of H containing a.

Theorem 2.2 [3] Let G be nonabelian and H a subgroup of G.
(i). If $H \subseteq Z(G)$, then $P(H, G) = 1$.
(ii). If $H \nsubseteq Z(G)$ and H is abelian, then $P(H, G) \leq \frac{3}{4}$.
(iii). If $H \nsubseteq Z(G)$ and H is not abelian, then $P(H, G) \leq \frac{5}{8}$.

Definition 2.3 [9] Normal Subgroup
A subgroup H of a group G is normal if its left and right cosets coincide, that is, if $gH = Hg$ for all $g \in G$.

Definition 2.4 [9] Cyclic Subgroup
Suppose that G is a finite group and H is a subgroup of G. The subgroup H is called a cyclic subgroup of G if there is an element a in G such that $H = \langle a^n | n \in \mathbb{Z} \rangle$. Such an element a is called a generator of H and it is denoted as $H = \langle a \rangle$.

Definition 2.5 [3] Centre of a Group
The centre $Z(G)$ of a group G is the subset of elements in G that commute with every element of G. In symbols, $Z(G) = \{a \in G | ax = xa, \forall x \in G\}$.

Definition 2.6 [9] Factor Group
Let H be a normal subgroup of G. Then the cosets of H form a group G/H under the binary operation $(aH)(bH) = (ab)H$. The group G/H is the factor group (or quotient group) of G by H.

Definition 2.7 [3] Relative Commutativity Degree
The relative commutativity degree of a subgroup H of a group G is the probability of an element in H commutes with an element in G and mathematically given as follows:

$$P(H,G)=\frac{|(h,g)\in H \text{ x } G \mid hg=gh|}{|H||G|}.$$

In the next section, the main results of this research are discussed.

3. Results and Discussions

This section is divided into two parts. The first part consists of the list of cyclic subgroups of nonabelian metabelian groups of order 32. Then, the second part consists of the relative commutativity degree for cyclic subgroups of nonabelian metabelian groups of order 32.

3.1 Cyclic Subgroups

There are 44 groups of nonabelian metabelian groups of order 32 that are listed in the second column of Table 1 and these groups are taken from [2]. The cyclic subgroups of those groups are also given in the following table (Table 1).

Table 1: Cyclic Subgroups of Nonabelian Metabelian Groups of order 32

No.	Group	Cyclic Subgroups
1.	$\mathbb{Z}_2 \bullet \mathbb{Z}_4^2$	$\langle e\rangle,\langle a\rangle,\langle b^2\rangle,\langle ab^2\rangle,\langle c^2\rangle,\langle ac^2\rangle,\langle b^2c^2\rangle,\langle ab^2c^2\rangle,\langle b\rangle,\langle ab\rangle,\langle c\rangle,\langle ac\rangle,\langle bc\rangle,\langle abc\rangle,\langle b^2c\rangle,\langle ab^2c\rangle,\langle b^3c\rangle,\langle ab^3c\rangle,\langle bc^2\rangle,\langle abc^2\rangle$
2.	$\mathbb{Z}_4 \rtimes \mathbb{Z}_4$	$\langle e\rangle,\langle a^2\rangle,\langle a^4\rangle,\langle b^2\rangle,\langle a^2b^2\rangle,\langle a^4b^2\rangle,\langle a\rangle,\langle a^5\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle a^4b\rangle,\langle a^6b\rangle$
3.	$\mathbb{Z}_2^2 \rtimes \mathbb{Z}_8$	$\langle e\rangle,\langle a^2\rangle,\langle a^4\rangle,\langle b\rangle,\langle a^2b\rangle,\langle a^4b\rangle,\langle a\rangle,\langle ab\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle abc\rangle,\langle a^4c\rangle,\langle bc\rangle,\langle a^2bc\rangle,\langle a^4bc\rangle$
4.	$\mathbb{Z}_2^3 \rtimes \mathbb{Z}_4$	$\langle e\rangle,\langle c\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle ac\rangle,\langle bc\rangle,\langle abc\rangle,\langle d\rangle,\langle ad\rangle,\langle bd\rangle,\langle abd\rangle,\langle cd\rangle,\langle acd\rangle,\langle bcd\rangle,\langle abcd\rangle,\langle d^2\rangle,\langle ad^2\rangle,\langle bd^2\rangle,\langle abd^2\rangle,\langle cd^2\rangle,\langle bcd^2\rangle$
5.	$\mathbb{Z}_2 \bullet D_4$	$\langle e\rangle,\langle a^4\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^4b\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,\langle a^4c\rangle,\langle a^6c\rangle,\langle bc\rangle,\langle a^2bc\rangle,\langle a^4bc\rangle,\langle a^6bc\rangle$
6.	$\mathbb{Z}_4 \dot{}_{10} D_4$	$\langle e\rangle,\langle a^4\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle aba\rangle,\langle a^3bab\rangle,\langle a^3ba\rangle,\langle bab\rangle,\langle a^7bab\rangle,\langle abab\rangle$
7.	$D_4 \rtimes \mathbb{Z}_4$	$\langle e\rangle,\langle a^4\rangle,\langle a^2b\rangle,\langle a^6b\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,\langle a^4c\rangle,\langle a^5c\rangle,\langle a^6c\rangle,\langle a^7c\rangle,\langle bc\rangle,\langle a^2bc\rangle,\langle a^4bc\rangle,\langle a^6bc\rangle$
8.	$Q_8 \rtimes \mathbb{Z}_4$	$\langle e\rangle,\langle a^4\rangle,\langle a^2b\rangle,\langle a^6b\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,\langle a^4c\rangle,\langle a^5c\rangle,\langle a^6c\rangle,\langle a^7c\rangle,\langle bc\rangle,\langle a^2bc\rangle,\langle a^4bc\rangle,\langle a^6bc\rangle$
9.	$\mathbb{Z}_4 \times \mathbb{Z}_2$	$\langle e\rangle,\langle a^2\rangle,\langle ab^2\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle b^2\rangle,\langle a^2b^2\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,\langle bc\rangle,\langle abc\rangle,\langle b^2c\rangle,\langle ab^2c\rangle$

10.	$\mathbb{Z}_4\times\mathbb{Z}_8$	$\langle e\rangle,\langle a^2\rangle,\langle b^2\rangle,\langle a^2b^2\rangle,\langle b^4\rangle,\langle a^2b^4\rangle,\langle b^6\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle ab^2\rangle,$ $\langle a^3b^2\rangle,\langle ab^4\rangle$
11.	$\mathbb{Z}_4\times Q_8$	$\langle e\rangle,\langle a^4\rangle,\langle b^2\rangle,\langle a^4b^2\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle a^4b\rangle,\langle a^5b\rangle,\langle a^6b\rangle,\langle a^7b\rangle,$ $\langle ab^2\rangle,\langle a^2b^2\rangle$
12.	$\mathbb{Z}_2\times D_8$	$\langle e\rangle,\langle a^4\rangle,\langle b^2\rangle,\langle a^4b^2\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle a^4b\rangle,\langle a^5b\rangle,\langle a^6b\rangle,\langle a^7b\rangle,$ $\langle ab^2\rangle,\langle a^2b^2\rangle$
13.	$\mathbb{Z}_8\bullet\mathbb{Z}_4$	$\langle e\rangle,\langle a^2\rangle,\langle a^4\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle ba\rangle,\langle b^2\rangle,\langle ab^2\rangle,\langle a^2b^2\rangle,\langle a^6b^2\rangle$
14.	$M_5(2)$	$\langle e\rangle,\langle a^2\rangle,\langle a^4\rangle,\langle a^8\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^4b\rangle,\langle a^8b\rangle$
15.	D_{16}	$\langle e\rangle,\langle a^8\rangle,\langle a\rangle,\langle a^2\rangle,\langle a^4\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle a^4b\rangle,\langle a^5b\rangle,\langle a^6b\rangle,\langle a^7b\rangle,$ $\langle a^8b\rangle,\langle a^9b\rangle,\langle a^{10}b\rangle,\langle a^{11}b\rangle,\langle a^{12}b\rangle,\langle a^{13}b\rangle,\langle a^{14}b\rangle,\langle a^{15}b\rangle$
16.	$\mathbb{S}D_{32}$	$\langle e\rangle,\langle a^8\rangle,\langle a\rangle,\langle a^2\rangle,\langle a^4\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle a^4b\rangle,\langle a^5b\rangle,\langle a^6b\rangle,\langle a^7b\rangle,\langle a^8b\rangle,\langle a^{10}b\rangle,$ $\langle a^{12}b\rangle,\langle a^{14}b\rangle$
17.	Q_{32}	$\langle e\rangle,\langle a^8\rangle,\langle a\rangle,\langle a^2\rangle,\langle a^4\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle a^4b\rangle,\langle a^5b\rangle,\langle a^6b\rangle,\langle a^7b\rangle$
18.	$\mathbb{Z}_2\times(\mathbb{Z}_2^2\rtimes\mathbb{Z}_4)$	$\langle e\rangle,\langle a\rangle,\langle c^2\rangle,\langle ac^2\rangle,\langle d\rangle,\langle ad\rangle,\langle c^2d\rangle,\langle ac^2d\rangle,\langle b\rangle,\langle ab\rangle,\langle c\rangle,\langle ac\rangle,\langle bc\rangle,\langle abc\rangle,$ $\langle bc^2\rangle,\langle abc^2\rangle,\langle bd\rangle,\langle abd\rangle,\langle cd\rangle,\langle acd\rangle,\langle abcd\rangle,\langle bc^2d\rangle,\langle abc^2d\rangle,\langle bcd\rangle$
19.	$\mathbb{Z}_2\times(\mathbb{Z}_4\rtimes\mathbb{Z}_4)$	$\langle e\rangle,\langle a^2\rangle,\langle b^2\rangle,\langle a^2b^2\rangle,\langle c\rangle,\langle a^2c\rangle,\langle b^2c\rangle,\langle a^2b^2c\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,$ $\langle ab^2\rangle,\langle ac\rangle,\langle bc\rangle,\langle abc\rangle,\langle a^2bc\rangle,\langle a^3bc\rangle,\langle ab^2c\rangle$
20.	$\mathbb{Z}_4^2\rtimes\mathbb{Z}_2$	$\langle e\rangle,\langle a\rangle,\langle a^2\rangle,\langle c^2\rangle,\langle ac^2\rangle,\langle a^2c^2\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,\langle bc\rangle,$ $\langle abc\rangle,\langle a^2bc\rangle,\langle a^3bc\rangle,\langle bc^2\rangle,\langle abc^2\rangle,\langle a^2bc^2\rangle$
21.	$\mathbb{Z}_4\times D_4$	$\langle e\rangle,\langle a^2\rangle,\langle c\rangle,\langle a^2c\rangle,\langle c^2\rangle,\langle a^2c^2\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle ac\rangle,\langle a^3c\rangle,\langle bc\rangle,$ $\langle abc\rangle,\langle a^2bc\rangle,\langle a^3bc\rangle,\langle ac^2\rangle,\langle bc^2\rangle,\langle abc^2\rangle,\langle a^2bc^2\rangle,\langle a^3bc^2\rangle$
22.	$\mathbb{Z}_4\times Q_8$	$\langle e\rangle,\langle a^2\rangle,\langle c\rangle,\langle a^2c\rangle,\langle c^2\rangle,\langle a^2c^2\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle ac\rangle,\langle a^3c\rangle,\langle bc\rangle,\langle abc\rangle,\langle a^2bc\rangle,$ $\langle a^3bc\rangle,\langle ac^2\rangle,\langle bc^2\rangle,\langle abc^2\rangle$
23.	$\mathbb{Z}_2^2\rtimes\mathbb{Z}_2$	$\langle e\rangle,\langle acac\rangle,\langle bcbc\rangle,\langle abcabc\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle c\rangle,\langle ac\rangle,\langle bc\rangle,\langle cac\rangle,\langle abc\rangle,\langle aca\rangle,$ $\langle bca\rangle\langle abca\rangle,\langle bcb\rangle,\langle abcb\rangle,\langle abcab\rangle,\langle bcac\rangle,\langle cbc\rangle,\langle abcac\rangle,\langle acbc\rangle,\langle abcbc\rangle,$ $\langle cabc\rangle,\langle acabc\rangle,\langle bcabc\rangle$
24.	$\mathbb{Z}_4\rtimes D_4$	$\langle e\rangle,\langle a^2\rangle,\langle bcbc\rangle,\langle a^2bcbc\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,\langle bc\rangle,\langle abc\rangle,$ $\langle a^2bc\rangle,\langle a^3bc\rangle,\langle bcb\rangle,\langle abcb\rangle,\langle a^2bcb\rangle,\langle cbc\rangle,\langle a^3bcb\rangle,\langle acbc\rangle,\langle a^2cbc\rangle,\langle abcbc\rangle$
25.	$\mathbb{Z}_4^2\rtimes Q_8$	$\langle e\rangle,\langle a^2\rangle,\langle bcbc\rangle,\langle a^2bcbc\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle bc\rangle,\langle abc\rangle,\langle a^2bc\rangle,$ $\langle a^3bc\rangle,\langle bcb\rangle,\langle abcb\rangle,\langle a^2bcb\rangle,\langle cbc\rangle,\langle acbc\rangle,\langle abcbc\rangle$

26.	$\mathbb{Z}_2^2 \bullet D_4$	$\langle e\rangle,\langle b\rangle,\langle c^2\rangle,\langle bc^2\rangle,\langle a\rangle,\langle ab\rangle,\langle c\rangle,\langle ac\rangle,\langle bc\rangle,\langle abc\rangle,\langle ac^2\rangle,\langle abc^2\rangle,\langle d\rangle,\langle ad\rangle,$ $\langle bd\rangle,\langle cd\rangle,\langle acd\rangle,\langle c^2d\rangle,\langle ac^2d\rangle,\langle bc^2d\rangle,\langle c^3d\rangle,\langle ac^3d\rangle$
27.	$\mathbb{Z}_{4\cdot 4}D_4$	$\langle e\rangle,\langle a^2\rangle,\langle b^2\rangle,\langle a^2b^3\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle ab^2\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,$ $\langle bc\rangle,\langle abc\rangle,\langle b^2c\rangle,\langle ab^2c\rangle,\langle a^2b^2c\rangle,\langle a^3b^2c\rangle,\langle b^3c\rangle,\langle ab^3c\rangle$
28.	$\mathbb{Z}_4^2 \bullet \mathbb{Z}_2$	$\langle e\rangle,\langle a^2\rangle,\langle b^2\rangle,\langle a^2b^2\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle ab^2\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,$ $\langle bc\rangle,\langle abc\rangle,\langle a^2bc\rangle,\langle a^3bc\rangle$
29.	$\mathbb{Z}_4^2 \rtimes_2 \mathbb{Z}_2$	$\langle e\rangle,\langle a^2\rangle,\langle b^2\rangle,\langle a^2b^2\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle ab^2\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,$ $\langle bc\rangle,\langle a^2bc\rangle,\langle abc\rangle,\langle b^2c\rangle,\langle a^2b^2c\rangle,\langle ab^3c\rangle$
30.	$\mathbb{Z}_4 \times_2 D_4$	$\langle e\rangle,\langle a^2\rangle,\langle a^3\rangle,\langle a^2b^2\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle ab^2\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,\langle bc\rangle,$ $\langle abc\rangle,\langle a^2bc\rangle,\langle a^3bc\rangle,\langle b^2c\rangle,\langle ab^2c\rangle,\langle a^2b^2c\rangle,\langle a^3b^2c\rangle,\langle b^3c\rangle,\langle ab^3c\rangle,\langle a^2b^3c\rangle,\langle a^3b^3c\rangle$
31.	$\mathbb{Z}_4 \rtimes Q_8$	$\langle e\rangle,\langle a^2\rangle,\langle b^2\rangle,\langle a^2b^2\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle ab^2\rangle,\langle c\rangle,\langle ac\rangle,\langle bc\rangle,\langle abc\rangle,$ $\langle b^2c\rangle,\langle ab^2c\rangle,\langle b^3c\rangle,\langle ab^3c\rangle$
32.	$\mathbb{Z}_2 \rtimes M_4(2)$	$\langle e\rangle,\langle a^2\rangle,\langle a^4\rangle,\langle c\rangle,\langle a^2c\rangle,\langle a^4c\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^4b\rangle,\langle ac\rangle,\langle bc\rangle,\langle abc\rangle,$ $\langle a^2bc\rangle,\langle a^4bc\rangle$
33.	$\mathbb{Z}_8 \circ D_4$	$\langle e\rangle,\langle a^2\rangle,\langle a^4\rangle,\langle ac\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^4b\rangle,\langle c\rangle,\langle a^2c\rangle,\langle a^4c\rangle,\langle bc\rangle,\langle abc\rangle,$ $\langle a^2bc\rangle,\langle a^6bc\rangle$
34.	$\mathbb{Z}_2 \times D_8$	$\langle e\rangle,\langle a^4\rangle,\langle c\rangle,\langle a^4c\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle a^4b\rangle,\langle a^5b\rangle,\langle a^6b\rangle,\langle a^7b\rangle,$ $\langle ac\rangle,\langle a^2c\rangle,\langle bc\rangle,\langle abc\rangle,\langle a^2bc\rangle,\langle a^3bc\rangle,\langle a^4bc\rangle,\langle a^5bc\rangle,\langle a^6bc\rangle,\langle a^7bc\rangle$
35.	$\mathbb{Z}_2 \times \mathbb{SD}_{16}$	$\langle e\rangle,\langle a^4\rangle,\langle c\rangle,\langle a^4c\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle a^4b\rangle,\langle a^6b\rangle,\langle ac\rangle,\langle a^2c\rangle,$ $\langle bc\rangle,\langle abc\rangle,\langle a^2bc\rangle,\langle a^3bc\rangle,\langle a^4bc\rangle,\langle a^6bc\rangle$
36.	$\mathbb{Z}_2 \times Q_{16}$	$\langle e\rangle,\langle a^4\rangle,\langle c\rangle,\langle a^4c\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle bc\rangle,\langle abc\rangle,$ $\langle a^2bc\rangle,\langle a^3bc\rangle$
37.	$\mathbb{Z}_4 \circ D_8$	$\langle e\rangle,\langle a^4\rangle,\langle b\rangle,\langle a\rangle,\langle a^2\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^6b\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^3c\rangle,\langle a^4c\rangle,\langle a^5c\rangle,$ $\langle a^6c\rangle,\langle a^7c\rangle,\langle bc\rangle,\langle abc\rangle,\langle a^2bc\rangle,\langle a^3bc\rangle$
38.	$\mathbb{Z}_8 \rtimes \mathbb{Z}_2^2$	$\langle e\rangle,\langle a^4\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle a^4b\rangle,\langle a^5b\rangle,\langle a^6b\rangle,\langle a^7b\rangle,\langle c\rangle,\langle ac\rangle,$ $\langle a^2c\rangle,\langle a^3c\rangle,\langle a^4c\rangle,\langle a^6c\rangle,\langle bc\rangle,\langle abc\rangle,\langle a^2bc\rangle,\langle a^4bc\rangle$
39.	$\mathbb{Z}_8 \bullet \mathbb{Z}_2^2$	$\langle e\rangle,\langle a^4\rangle,\langle a\rangle,\langle a^2\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle a^4c\rangle,\langle bc\rangle,\langle abc\rangle,$ $\langle a^2bc\rangle,\langle a^3bc\rangle,\langle a^5bc\rangle,\langle a^7bc\rangle$

40.	$\mathbb{Z}_2^3 \times D_4$	$\langle e\rangle,\langle a^2\rangle,\langle c\rangle,\langle a^2c\rangle,\langle d\rangle,\langle a^2d\rangle,\langle cd\rangle,\langle a^2cd\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle ac\rangle,$ $\langle bc\rangle,\langle abc\rangle,\langle a^2bc\rangle,\langle a^3bc\rangle,\langle ad\rangle,\langle bd\rangle,\langle abd\rangle,\langle a^2bd\rangle,\langle a^3bd\rangle,\langle acd\rangle,\langle bcd\rangle,$ $\langle abcd\rangle,\langle a^2bcd\rangle$
41.	$\mathbb{Z}_2^2 \times Q_8$	$\langle e\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle c^2\rangle,\langle ac^2\rangle,\langle bc^2\rangle,\langle abc^2\rangle,\langle c\rangle,\langle ac\rangle,\langle bc\rangle,\langle abc\rangle,\langle d\rangle,\langle ad\rangle,$ $\langle bd\rangle,\langle abd\rangle,\langle cd\rangle,\langle acd\rangle,\langle bcd\rangle,\langle abcd\rangle$
42.	$\mathbb{Z}_2 \times \mathbb{Z}_4 \circ D_4$	$\langle e\rangle,\langle a\rangle,\langle a^2\rangle,\langle d\rangle,\langle ad\rangle,\langle a^2d\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle c\rangle,\langle ac\rangle,\langle a^2c\rangle,\langle bc\rangle,\langle abc\rangle,$ $\langle a^3bc\rangle,\langle bd\rangle,\langle abd\rangle,\langle a^2bd\rangle,\langle cd\rangle,\langle acd\rangle,\langle a^2cd\rangle,\langle bcd\rangle,\langle abcd\rangle,\langle a^3bcd\rangle$
43.	2_+^{I+4}	$\langle e\rangle,\langle a^2\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle c\rangle,\langle ac\rangle,\langle a^3c\rangle,\langle bc\rangle,\langle abc\rangle,\langle d\rangle,\langle ad\rangle,\langle a^2d\rangle,$ $\langle bd\rangle,\langle abd\rangle,\langle a^2bd\rangle,\langle a^3bd\rangle,\langle cd\rangle,\langle acd\rangle,\langle a^2cd\rangle,\langle bcd\rangle,\langle abcd\rangle,\langle a^2bcd\rangle,\langle a^3bcd\rangle$
44.	2_-^{I+4}	$\langle e\rangle,\langle a^2\rangle,\langle a\rangle,\langle b\rangle,\langle ab\rangle,\langle a^2b\rangle,\langle a^3b\rangle,\langle c\rangle,\langle ac\rangle,\langle a^3c\rangle,\langle bc\rangle,\langle abc\rangle,\langle d\rangle,\langle ad\rangle,\langle a^3d\rangle,$ $\langle bd\rangle,\langle abd\rangle,\langle cd\rangle,\langle acd\rangle,\langle a^3cd\rangle,\langle bcd\rangle,\langle abcd\rangle$

Remarks: $A \rtimes B$ represent a semidirect product of group *A* with group *B*, $A \times B$ represent a direct product of group *A* with group *B*, $A^2 \times B$ represent a direct product of 2 copies of group *A* with group *B*. Meanwhile $\mathbb{Z}_n$ is a cyclic groups of order *n*, D_n is a dihedral group of order 2*n* and Q_n is a quaternion group of order *n*.

3.2 Relative Commutativity Degree

In this section, the relative commutativity degree for cyclic subgroups of nonabelian metabelian groups of order 32 are computed by using Cayley Table and 0-1 Table. The Cayley table for $\mathbb{Z}_2 \cdot \mathbb{Z}_4^2$ can be found as in Table 2 and this Cayley table is determined by using Maple software.

Proposition 3.1 Let $G = \mathbb{Z}_2 \cdot \mathbb{Z}_4^2$ and H be a cyclic subgroup of G. Then

$$P(H,G) = \begin{cases} 1, \text{ if } H = \langle e\rangle \text{ or } H = Z(G) \\ \dfrac{3}{4}, \text{ otherwise} \end{cases}$$

Before the proof of Proposition 3.1 is given, the following Cayley table (for $\mathbb{Z}_2 \cdot \mathbb{Z}_4^2$) is needed.

Table 2: The Cayley table of $\mathbb{Z}_2 \cdot \mathbb{Z}_4^2$

	1	a	b^2	ab^2	c^2	ac^2	b^2c^2	ab^2c^2	b	ab	b^3	ab^3	c	ac	bc	abc	b^2c	ab^2c	b^3c	ab^3c	bc^2	abc^2	b^3c^2	ab^3c^2	c^3	ac^3	bc^3	abc^3	b^2c^3	ab^2c^3	b^3c^3	ab^3c^3
1	1	a	b^2	ab^2	c^2	ac^2	b^2c^2	ab^2c^2	b	ab	b^3	ab^3	c	ac	bc	abc	b^2c	ab^2c	b^3c	ab^3c	bc^2	abc^2	b^3c^2	ab^3c^2	c^3	ac^3	bc^3	abc^3	b^2c^3	ab^2c^3	b^3c^3	ab^3c^3
a	a	1	ab^2	b^2	ac^2	c^2	ab^2c^2	b^2c^2	ab	b	ab^3	b^3	ac	c	abc	bc	ab^2c	b^2c	ab^3c	b^3c	abc^2	bc^2	ab^3c^2	b^3c^2	ac^3	c^3	abc^3	bc^3	ab^2c^3	b^2c^3	ab^3c^3	b^3c^3
b^2	b^2	ab^2	1	a	b^2c^2	ab^2c^2	c^2	ac^2	b^3	ab^3	b	ab	b^2c	ab^2c	b^3c	ab^3c	c	ac	bc	abc	b^3c^2	ab^3c^2	bc^2	abc^2	b^2c^3	ab^2c^3	b^3c^3	ab^3c^3	c^3	ac^3	bc^3	abc^3
ab^2	ab^2	b^2	a	1	ab^2c^2	b^2c^2	ac^2	c^2	ab^3	b^3	ab	b	ab^2c	b^2c	ab^3c	b^3c	ac	c	abc	bc	ab^3c^2	b^3c^2	abc^2	bc^2	ab^2c^3	b^2c^3	ab^3c^3	b^3c^3	ac^3	c^3	abc^3	bc^3
c^2	c^2	ac^2	b^2c^2	ab^2c^2	1	a	b^2	ab^2	bc^2	abc^2	b^3c^2	ab^3c^2	c^3	ac^3	bc^3	abc^3	b^2c^3	ab^2c^3	b^3c^3	ab^3c^3	b	ab	b^3	ab^3	c	ac	bc	abc	b^2c	ab^2c	b^3c	ab^3c
ac^2	ac^2	c^2	ab^2c^2	b^2c^2	a	1	ab^2	b^2	abc^2	bc^2	ab^3c^2	b^3c^2	ac^3	c^3	abc^3	bc^3	ab^2c^3	b^2c^3	ab^3c^3	b^3c^3	ab	b	ab^3	b^3	ac	c	abc	bc	ab^2c	b^2c	ab^3c	b^3c
b^2c^2	b^2c^2	ab^2c^2	c^2	ac^2	b^2	ab^2	1	a	b^3c^2	ab^3c^2	bc^2	abc^2	b^2c^3	ab^2c^3	b^3c^3	ab^3c^3	c^3	ac^3	bc^3	abc^3	b^3	ab^3	b	ab	b^2c	ab^2c	b^3c	ab^3c	c	ac	bc	abc
ab^2c^2	ab^2c^2	b^2c^2	ac^2	c^2	ab^2	b^2	a	1	ab^3c^2	b^3c^2	abc^2	bc^2	ab^2c^3	b^2c^3	ab^3c^3	b^3c^3	ac^3	c^3	abc^3	bc^3	ab^3	b^3	ab	b	ab^2c	b^2c	ab^3c	b^3c	ac	c	abc	bc
b	b	ab	b^3	ab^3	bc^2	abc^2	b^3c^2	ab^3c^2	b^2	ab^2	1	a	bc	abc	b^2c	ab^2c	b^3c	ab^3c	c	ac	b^2c^2	ab^2c^2	c^2	ac^2	bc^3	abc^3	b^2c^3	ab^2c^3	b^3c^3	ab^3c^3	c^3	ac^3
ab	ab	b	ab^3	b^3	abc^2	bc^2	ab^3c^2	b^3c^2	ab^2	b^2	a	1	abc	bc	ab^2c	b^2c	ab^3c	b^3c	ac	c	ab^2c^2	b^2c^2	ac^2	c^2	abc^3	bc^3	ab^2c^3	b^2c^3	ab^3c^3	b^3c^3	ac^3	c^3
b^3	b^3	ab^3	b	ab	b^3c^2	ab^3c^2	bc^2	abc^2	1	a	b^2	ab^2	b^3c	ab^3c	c	ac	bc	abc	b^2c	ab^2c	c^2	ac^2	b^2c^2	ab^2c^2	b^3c^3	ab^3c^3	c^3	ac^3	bc^3	abc^3	b^2c^3	ab^2c^3
ab^3	ab^3	b^3	ab	b	ab^3c^2	b^3c^2	abc^2	bc^2	a	1	ab^2	b^2	ab^3c	b^3c	ac	c	abc	bc	ab^2c	b^2c	ac^2	c^2	ab^2c^2	b^2c^2	ab^3c^3	b^3c^3	ac^3	c^3	abc^3	bc^3	ab^2c^3	b^2c^3
c	c	ac	b^2c	ab^2c	c^3	ac^3	b^2c^3	ab^2c^3	abc	bc	ab^3c	b^3c	c^2	ac^2	abc^2	bc^2	b^2c^2	ab^2c^2	ab^3c^2	b^3c^2	abc^3	bc^3	ab^3c^3	b^3c^3	1	a	ab	b	b^2	ab^2	ab^3	b^3
ac	ac	c	ab^2c	b^2c	ac^3	c^3	ab^2c^3	b^2c^3	bc	abc	b^3c	ab^3c	ac^2	c^2	bc^2	abc^2	ab^2c^2	b^2c^2	b^3c^2	ab^3c^2	bc^3	abc^3	b^3c^3	ab^3c^3	a	1	b	ab	ab^2	b^2	b^3	ab^3
bc	bc	abc	b^3c	ab^3c	bc^3	abc^3	b^3c^3	ab^3c^3	ab^2c	b^2c	ac	c	bc^2	abc^2	ab^2c^2	b^2c^2	b^3c^2	ab^3c^2	ac^2	c^2	ab^2c^3	b^2c^3	ac^3	c^3	b	ab	ab^2	b^2	b^3	ab^3	a	1
abc	abc	bc	ab^3c	b^3c	abc^3	bc^3	ab^3c^3	b^3c^3	b^2c	ab^2c	c	ac	abc^2	bc^2	b^2c^2	ab^2c^2	ab^3c^2	b^3c^2	c^2	ac^2	b^2c^3	ab^2c^3	c^3	ac^3	ab	b	b^2	ab^2	ab^3	b^3	1	a
b^2c	b^2c	ab^2c	c	ac	b^2c^3	ab^2c^3	c^3	ac^3	ab^3c	b^3c	abc	bc	b^2c^2	ab^2c^2	ab^3c^2	b^3c^2	c^2	ac^2	abc^2	bc^2	ab^3c^3	b^3c^3	abc^3	bc^3	b^2	ab^2	ab^3	b^3	1	a	ab	b
ab^2c	ab^2c	b^2c	ac	c	ab^2c^3	b^2c^3	ac^3	c^3	b^3c	ab^3c	bc	abc	ab^2c^2	b^2c^2	b^3c^2	ab^3c^2	ac^2	c^2	bc^2	abc^2	b^3c^3	ab^3c^3	bc^3	abc^3	ab^2	b^2	b^3	ab^3	a	1	b	ab
b^3c	b^3c	ab^3c	bc	abc	b^3c^3	ab^3c^3	bc^3	abc^3	ac	c	ab^2c	b^2c	b^3c^2	ab^3c^2	ac^2	c^2	bc^2	abc^2	ab^2c^2	b^2c^2	ac^3	c^3	ab^2c^3	b^2c^3	b^3	ab^3	a	1	b	ab	ab^2	b^2
ab^3c	ab^3c	b^3c	abc	bc	ab^3c^3	b^3c^3	abc^3	bc^3	c	ac	b^2c	ab^2c	ab^3c^2	b^3c^2	c^2	ac^2	abc^2	bc^2	b^2c^2	ab^2c^2	c^3	ac^3	b^2c^3	ab^2c^3	ab^3	b^3	1	a	ab	b	b^2	ab^2
bc^2	bc^2	abc^2	b^3c^2	ab^3c^2	b	ab	b^3	ab^3	b^2c^2	ab^2c^2	c^2	ac^2	bc^3	abc^3	b^2c^3	ab^2c^3	b^3c^3	ab^3c^3	c^3	ac^3	b^2	ab^2	1	a	bc	abc	b^2c	ab^2c	b^3c	ab^3c	c	ac
abc^2	abc^2	bc^2	ab^3c^2	b^3c^2	ab	b	ab^3	b^3	ab^2c^2	b^2c^2	ac^2	c^2	abc^3	bc^3	ab^2c^3	b^2c^3	ab^3c^3	b^3c^3	ac^3	c^3	ab^2	b^2	a	1	abc	bc	ab^2c	b^2c	ab^3c	b^3c	ac	c
b^3c^2	b^3c^2	ab^3c^2	bc^2	abc^2	b^3	ab^3	b	ab	c^2	ac^2	b^2c^2	ab^2c^2	b^3c^3	ab^3c^3	c^3	ac^3	bc^3	abc^3	b^2c^3	ab^2c^3	1	a	b^2	ab^2	b^3c	ab^3c	c	ac	bc	abc	b^2c	ab^2c
ab^3c^2	ab^3c^2	b^3c^2	abc^2	bc^2	ab^3	b^3	ab	b	ac^2	c^2	ab^2c^2	b^2c^2	ab^3c^3	b^3c^3	ac^3	c^3	abc^3	bc^3	ab^2c^3	b^2c^3	a	1	ab^2	b^2	ab^3c	b^3c	ac	c	abc	bc	ab^2c	b^2c
c^3	c^3	ac^3	b^2c^3	ab^2c^3	c	ac	b^2c	ab^2c	abc^3	bc^3	ab^3c^3	b^3c^3	1	a	ab	b	b^2	ab^2	ab^3	b^3	abc	bc	ab^3c	b^3c	c^2	ac^2	abc^2	bc^2	b^2c^2	ab^2c^2	ab^3c^2	b^3c^2
ac^3	ac^3	c^3	ab^2c^3	b^2c^3	ac	c	ab^2c	b^2c	bc^3	abc^3	b^3c^3	ab^3c^3	a	1	b	ab	ab^2	b^2	b^3	ab^3	bc	abc	b^3c	ab^3c	ac^2	c^2	bc^2	abc^2	ab^2c^2	b^2c^2	b^3c^2	ab^3c^2
bc^3	bc^3	abc^3	b^3c^3	ab^3c^3	bc	abc	b^3c	ab^3c	ab^2c^3	b^2c^3	ac^3	c^3	b	ab	ab^2	b^2	b^3	ab^3	a	1	ab^2c	b^2c	ac	c	bc^2	abc^2	ab^2c^2	b^2c^2	b^3c^2	ab^3c^2	ac^2	c^2
abc^3	abc^3	bc^3	ab^3c^3	b^3c^3	abc	bc	ab^3c	b^3c	b^2c^3	ab^2c^3	c^3	ac^3	ab	b	b^2	ab^2	ab^3	b^3	1	a	b^2c	ab^2c	c	ac	abc^2	bc^2	b^2c^2	ab^2c^2	ab^3c^2	b^3c^2	c^2	ac^2
b^2c^3	b^2c^3	ab^2c^3	c^3	ac^3	b^2c	ab^2c	c	ac	ab^3c^3	b^3c^3	abc^3	bc^3	b^2	ab^2	ab^3	b^3	1	a	ab	b	ab^3c	b^3c	abc	bc	b^2c^2	ab^2c^2	ab^3c^2	b^3c^2	c^2	ac^2	abc^2	bc^2
ab^2c^3	ab^2c^3	b^2c^3	ac^3	c^3	ab^2c	b^2c	ac	c	b^3c^3	ab^3c^3	bc^3	abc^3	ab^2	b^2	b^3	ab^3	a	1	b	ab	b^3c	ab^3c	bc	abc	ab^2c^2	b^2c^2	b^3c^2	ab^3c^2	ac^2	c^2	bc^2	abc^2
b^3c^3	b^3c^3	ab^3c^3	bc^3	abc^3	b^3c	ab^3c	bc	abc	ac^3	c^3	ab^2c^3	b^2c^3	b^3	ab^3	a	1	b	ab	ab^2	b^2	ac	c	ab^2c	b^2c	b^3c^2	ab^3c^2	ac^2	c^2	bc^2	abc^2	ab^2c^2	b^2c^2
ab^3c^3	ab^3c^3	b^3c^3	abc^3	bc^3	ab^3c	b^3c	abc	bc	c^3	ac^3	b^2c^3	ab^2c^3	ab^3	b^3	1	a	ab	b	b^2	ab^2	c	ac	b^2c	ab^2c	ab^3c^2	b^3c^2	c^2	ac^2	abc^2	bc^2	b^2c^2	ab^2c^2

Proof of Proposition 3.1 Suppose $G = \mathbb{Z}_2 \cdot \mathbb{Z}_4^2$. Thus, G has 32 elements which the group presentation as follows

$$G = \left\langle a, b, c \mid a^2 = b^4 = c^4 = 1, ba = ab, ac = ca, cb = abc \right\rangle$$

Let H be a cyclic subgroup of G,

Case 1 $H_1 = \{e\} = \langle e \rangle$

By Theorem 2.1, $P(H_1, G) = 1$.

Case 2 $H_2 = \{e, a\}, H_3 = \{e, b^2\}, H_4 = \{e, ab^2\}, H_5 = \{e, c^2\}, H_6 = \{e, ac^2\}, H_7 = \{e, b^2c^2\}$, and $H_8 = \{e, ab^2c^2\}$

By Definition 2.5, $Z(G) = \{e, a, b^2, ab^2, c^2, ac^2, b^2c^2, ab^2c^2\}$.

Since $H_2, H_3, H_4, H_5, H_6, H_7, H_8 \subset Z(G)$, and by Theorem 2.2, then $P(H_n, G) = 1$ for $n = 2,3,4,5,6,7$ and 8.

Case 3 $H_9 = \{e, b, b^2, b^3\} = \langle b \rangle = \langle b^3 \rangle$, $H_{10} = \{e, ab, b^2, ab^3\} = \langle ab \rangle = \langle ab^3 \rangle$,
$H_{11} = \{e, c, c^2, c^3\} = \langle c \rangle = \langle c^3 \rangle$, $H_{12} = \{e, ac, c^2, ac^3\} = \langle ac \rangle = \langle ac^3 \rangle$,
$H_{13} = \{e, bc, ab^2c^2, ab^3c^3\} = \langle bc \rangle = \langle ab^3c^3 \rangle$, $H_{14} = \{e, abc, ab^2c^2, b^3c^3\} = \langle abc \rangle = \langle b^3c^3 \rangle$,
$H_{15} = \{e, b^2c, b^2c^3\} = \langle b^2c \rangle = \langle b^2c^3 \rangle$, $H_{16} = \{e, ab^2c, ab^2c^3\} = \langle ab^2c \rangle = \langle ab^2c^3 \rangle$,
$H_{17} = \{e, b^3c, ab^2c^2, abc^3\} = \langle b^3c \rangle = \langle abc^3 \rangle$, $H_{18} = \{e, ab^3c, ab^2c^2, bc^3\} = \langle ab^3c \rangle = \langle bc^3 \rangle$,
$H_{19} = \{e, bc^2, b^2, b^3c^2\} = \langle bc^2 \rangle = \langle b^3c^2 \rangle$, $H_{20} = \{e, abc^2, b^2, ab^3c^2\} = \langle abc^2 \rangle = \langle ab^3c^2 \rangle$

From the Cayley Table of $\mathbb{Z}_2 \cdot \mathbb{Z}_4^2$ (Table 2) and Definition 2.7,

$$P(H_i, G) = \frac{96}{32 \times 4} = \frac{3}{4} \text{ for } i = 9, 10, \ldots, 19.$$

Notes that $P(H,G)$ in Case 3 fulfil Theorem 2.2 (ii). Relative commutativity degree of all 44 nonabelian metabelian groups of order 32 are shown in Table 3.

Table 3: Relative Commutativity Degree for Cyclic Subgroups of Nonabelian Metabelian Groups of Order 32.

No	Group	$P(H,G)$	No	Group	$P(H,G)$	No	Group	$P(H,G)$
1.	$\mathbb{Z}_2 \bullet \mathbb{Z}_4^2$	1 or $\frac{3}{4}$	16.	$\mathbb{S}D_{32}$	$1, \frac{3}{4}, \frac{5}{8}$ or $\frac{9}{16}$	31.	$\mathbb{Z}_4 \rtimes Q_8$	$1, \frac{3}{4}$ or $\frac{5}{8}$
2.	$\mathbb{Z}_4 \rtimes \mathbb{Z}_4$	1 or $\frac{3}{4}$	17.	Q_{32}	$1, \frac{3}{4}, \frac{5}{8}$ or $\frac{9}{16}$	32.	$\mathbb{Z}_2 \rtimes M_4(2)$	1 or $\frac{3}{4}$
3.	$\mathbb{Z}_2^2 \rtimes \mathbb{Z}_8$	1 or $\frac{3}{4}$	18.	$\mathbb{Z}_2 \times (\mathbb{Z}_2^2 \rtimes \mathbb{Z}_4)$	1 or $\frac{3}{4}$	33.	$\mathbb{Z}_8 \circ D_4$	1 or $\frac{3}{4}$
4.	$\mathbb{Z}_2^3 \rtimes \mathbb{Z}_4$	$1, \frac{3}{4}, \frac{5}{8}$ or $\frac{1}{2}$	19.	$\mathbb{Z}_2 \times (\mathbb{Z}_4 \rtimes \mathbb{Z}_4)$	1 or $\frac{3}{4}$	34.	$\mathbb{Z}_2 \times D_8$	$1, \frac{3}{4}$ or $\frac{5}{8}$
5.	$\mathbb{Z}_2 \bullet D_4$	$1, \frac{3}{4}, \frac{5}{8}$ or $\frac{1}{2}$	20.	$\mathbb{Z}_4^2 \rtimes \mathbb{Z}_2$	1 or $\frac{3}{4}$	35.	$\mathbb{Z}_2 \times \mathbb{S}D_{16}$	$1, \frac{3}{4}$ or $\frac{5}{8}$
6.	$\mathbb{Z}_4 \cdot_{10} D_4$	$1, \frac{3}{4}, \frac{5}{8}$ or $\frac{1}{2}$	21.	$\mathbb{Z}_4 \times D_4$	1 or $\frac{3}{4}$	36.	$\mathbb{Z}_2 \times Q_{16}$	$1, \frac{3}{4}$ or $\frac{5}{8}$
7.	$D_4 \rtimes \mathbb{Z}_4$	$1, \frac{3}{4}$ or $\frac{5}{8}$	22.	$\mathbb{Z}_4 \times Q_8$	1 or $\frac{3}{4}$	37.	$\mathbb{Z}_4 \circ D_8$	$1, \frac{3}{4}$ or $\frac{5}{8}$
8.	$Q_8 \rtimes \mathbb{Z}_4$	$1, \frac{3}{4}$ or $\frac{5}{8}$	23.	$\mathbb{Z}_2^2 \rtimes \mathbb{Z}_2$	$1, \frac{3}{4}$ or $\frac{5}{8}$	38.	$\mathbb{Z}_8 \rtimes \mathbb{Z}_2^2$	$1, \frac{3}{4}, \frac{5}{8}$ or $\frac{1}{2}$

9.	$\mathbb{Z}_4 \times \mathbb{Z}_2$	$1, \frac{3}{4}$ or $\frac{5}{8}$	24.	$\mathbb{Z}_4 \rtimes D_4$	$1, \frac{3}{4}$ or $\frac{5}{8}$	39.	$\mathbb{Z}_8 \bullet \mathbb{Z}_2^2$	$1, \frac{3}{4}, \frac{5}{8}$ or $\frac{1}{2}$
10.	$\mathbb{Z}_4 \times \mathbb{Z}_8$	1 or $\frac{3}{4}$	25.	$\mathbb{Z}_4^2 \rtimes Q_8$	$1, \frac{3}{4}$ or $\frac{5}{8}$	40.	$\mathbb{Z}_2^3 \times D_4$	1 or $\frac{3}{4}$
11.	$\mathbb{Z}_4 \times Q_8$	$1, \frac{3}{4}$ or $\frac{5}{8}$	26.	$\mathbb{Z}_2^2 \bullet D_4$	$1, \frac{3}{4}$ or $\frac{5}{8}$	41.	$\mathbb{Z}_2^2 \times Q_8$	1 or $\frac{3}{4}$
12.	$\mathbb{Z}_2 \times D_8$	$1, \frac{3}{4}$ or $\frac{5}{8}$	27.	$\mathbb{Z}_{4 \cdot 4} D_4$	$1, \frac{3}{4}$ or $\frac{5}{8}$	42.	$\mathbb{Z}_2 \times \mathbb{Z}_4 \circ D_4$	1 or $\frac{3}{4}$
13.	$\mathbb{Z}_8 \bullet \mathbb{Z}_4$	$1, \frac{3}{4}$ or $\frac{5}{8}$	28.	$\mathbb{Z}_4^2 \bullet \mathbb{Z}_2$	$1, \frac{3}{4}$ or $\frac{5}{8}$	43.	2_+^{I+4}	1 or $\frac{3}{4}$
14.	$M_5(2)$	1 or $\frac{3}{4}$	29.	$\mathbb{Z}_4^2 \rtimes_2 \mathbb{Z}_2$	$1, \frac{3}{4}$ or $\frac{5}{8}$	44.	2_-^{I+4}	1 or $\frac{3}{4}$
15.	D_{16}	$1, \frac{3}{4}, \frac{5}{8}$ or $\frac{9}{16}$	30.	$\mathbb{Z}_4 \times_2 D_4$	$1, \frac{3}{4}$ or $\frac{5}{8}$			

4. Conclusion

This research gives the relative commutativity degree for cyclic subgroups of 44 nonabelian metabelian groups of order 32. All groups of nonabelian metabelian groups of order 32 have $\boldsymbol{P(H,G)} = 1, \frac{\mathbf{1}}{\mathbf{2}}, \frac{\mathbf{3}}{\mathbf{4}}, \frac{\mathbf{5}}{\mathbf{8}}$ or $\frac{\mathbf{9}}{\mathbf{16}}$.

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