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Complex Uncertainty of Surface Data Modeling via the Type-2 Fuzzy B-Spline Model

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Abstract: This paper discusses the construction of a type-2 fuzzy B-spline model to model complex uncertainty of surface data. To construct this model, the type-2 fuzzy set theory, which includes type-2 fuzzy number concepts and type-2 fuzzy relation, is used to define the complex uncertainty of surface data in type-2 fuzzy data/control points. These type-2 fuzzy data/control points are blended with the B-spline surface function to produce the proposed model, which can be visualized and analyzed further. Various processes, namely fuzzification, type-reduction and defuzzification are defined to achieve a crisp, type-2 fuzzy B-spline surface, representing uncertainty complex surface data. This paper ends with a numerical example of terrain modeling, which shows the effectiveness of handling the uncertainty complex data.

1. Introduction

Data points are the representation of the visible object in a digital system. The shape modification of items is carried out by tweaking the data points as the designer desires. However, the data points representing an object are not necessarily precise due to their characteristics, which are uncertain, leading to complexity. The complexities that are found in the data are due to various possible reasons, e.g., inappropriate analysis, human perception and logical assumption [1,2], limitations of tool accuracy, and the nature of data collection itself [3,4] with different types of uncertainty being based on BIPM (*Bureau International des Poids et Mesures)* concepts. The specific meaning of 'complex' in complex uncertainty data is the stack of the uncertainty of two arguments of the collected data points. It is impossible to accurately model the complex uncertainty data using an appropriate standard curve or surface function, such as the B-spline function, unless we formulate a new definition of the B-spline function with the complex uncertainty meaning.

To make the complex uncertainty data useful in modeling, we define complex uncertainty data in this paper. The complex uncertainty data can be explained by using type-2 fuzzy number (T2FN) concepts [5–7], which are defined by type-2 fuzzy set theory (T2FST) [8–10], especially interval type-2 fuzzy number (IT2FN) concepts [5,11]. The IT2FN concept is implemented to define the complex uncertainty data of real numbers subtly transformed into type-2 fuzzy data points (T2FDP), in line with the definition of type-2 fuzzy relation (T2FR). This is then followed by representing T2FDP as a type-2 fuzzy control point (T2FCP), which can be used for modeling the B-spline surface function [12–14]. Hence, we end up with a standard definition denoted as the type-2 fuzzy B-spline surface.

The interval type-2 fuzzy set (IT2FS) concept has been widely used to model higher-order uncertainty, which has been proven to be more suitable compared to the interval type-

1 fuzzy set. This concept is used when higher-order uncertainty exists in the measurement, and has been used for various applications [5,15–20]. This study employs the concept of the interval type-2 fuzzy set (IT2FN) to deal with the real data problem of complex uncertainty. The advantage of IT2FN is that it can define both complex uncertainty and data uncertainty for modeling. For the uncertainty case, it will first be reduced to become an interval type-1 fuzzy number. For the interval type-1 fuzzy number, this concept only deals with data uncertainty, but is not applicable when dealing with the complex uncertainty or a higher degree of uncertainty. Then, the IT2FN can define the complex uncertainty data or higher-level uncertainty data than the type-1 fuzzy number.

Therefore, the proposed approach in modeling the complex uncertainty data becomes important to obtain a better result instead of just modeling the perfect data, which does do not have complex uncertainty properties. Then, by excluding the complex uncertainty data, the main focus of generating a surface model based on all data is unreasonable and inaccurate. This is why we need an appropriate theory, such as T2FST, or more specifically, IT2FN, to define the complex uncertainty data and model those data to obtain a perfect model for better analysis, predictions and conclusions.

The interpolation method is usually used to produce the desired design. In Computer-Aided Geometric Design (CAGD), the interpolation method performs a pivotal role in modeling various techniques. Therefore, this method is practiced along with the surface data to find the control points as a reference data point. These control points are then represented with a surface function to produce the specific object's interpolation surface. With the complex uncertainty, the procedure in modeling the perfect interpolation surface using the B-spline surface function can be carried out using the type-2 fuzzy interpolation of the B-spline surface.

The remainder of this paper is organized as follows. Section 2 discusses the previous research about uncertainty data modeling via B-spline surface functions. Section 3 discusses the definition of T2FST, T2FN and T2FR. In this section, the definition of T2FDPs is developed along with the definitions of fuzzification, type-reduction and defuzzification processes. Section 4 discusses defining the type-2 fuzzy B-spline model, which uses the interpolation method for both the curve and surface. This section also elaborates on the construction of the type-2 fuzzy interpolation of the B-spline surface. Section 5 discusses the implementation of the type-2 fuzzy interpolation of the B-spline surface in seabed modeling. The alpha-cut operation of the fuzzification process with various alpha values is also discussed to indicate the influence of other alpha values concerning fuzzification, type-reduction, and the defuzzification surface. Section 5 discusses the accuracy of this developed model to sufficiently show the effectiveness of the type-2 fuzzy interpolation B-spline surface in modeling the uncertainty complex data.

2. Previous Work

The T2FST is useful in defining the uncertainty complex data to create a type-2 fuzzy curve and surface using the B-spline curve and surface functions. Regarding the designers' requirements in modeling the complex uncertainty data, the designers cannot choose and decide which data points are essential among the collectives of complex uncertainty data in modeling curves and surfaces. Therefore, we need the T2FST to define the complex uncertainty data and then model them through the B-spline curve and surface functions. This approach will make the complex uncertainty data become T2FDP/T2FCP, which can be modeled after integrating with the B-spline curve and surface functions.

Many types of research are carried out using the type-1 fuzzy set theory to model surface in dealing with uncertainty issues. Examples include the surface model proposed by Gallo et al. [21,22] of Mount Etna, Zakaria et al. [4] of lLakebed's modeling of Kenyir Lake, and Sarwar and Akram [23] proposed the fuzzy tensor product of Bezier surface. Note that the uncertainty level of these models is at level one. Suppose that the level of uncertainty increases due to specific errors, in particular the uncertainty at level two (complex uncertainty data). In that case, the current proposed approach for level one

uncertainty data is not appropriate. The difference between uncertainty and error is the uncertainty defined as a range or interval where the actual value lies in this interval. An error is most likely the actual value obtained, which can be corrected by adding or subtracting the correction factor. Therefore, T2FST is proposed to treat the complex uncertainty data.

The expansion of modeling surfaces through T2FST is relatively new and it is at an early stage. Du proposed the example of T2FST in surface modeling, and Du and Zhu [24] had discussed about the modeling of spatial vagueness based on type-2 fuzzy set and was implemented in the Geographic Information System (GIS). However, this new method has been successfully developed, the surface results as a type-2 fuzzy condition which does not represent a type-2 fuzzy crisp surface. Furthermore, it does not deal with the fuzzification, type-reduction and defuzzification processes of the spatial vagueness modeling through the type-2 fuzzy set. In this article, the processes of fuzzification, type-reduction and defuzzification are discussed in detail based on [25]. The practicality of the proposed model is illustrated by the seabed modeling of Mengabang Telipot Beach.

3. Method: Type-2 Fuzzy Data Points

This section defines the complex uncertainty data with the definition of T2FST, IT2FN and T2FR [25].

Definition 1 ([25,26]). *A type-2 fuzzy set (T2FS), denoted as $\widetilde{A}$ is characterized by a type-2 membership function $\mu_{\overleftrightarrow{A}}(x, u)$, where $x \in X$ and $\forall u \in [0, 1]$, that is,*

$$\widetilde{A} = \left\{ \left((x, u), \mu_{\widetilde{A}}(x, u) \right) \middle| \forall x \in X, \forall u \in [0, 1] \right\} \tag{1}$$

in which $0 \leq \mu_{\widetilde{A}}(x, u) \leq 1$.

Definition 2 ([5]). *A T2FN is broadly defined as a type-2 fuzzy set (T2FS) that has a numerical domain. An interval of T2FS is defined using the following four constraints, where $\widetilde{A}_{\alpha} = \{ [a^{\alpha}, b^{\alpha}], [c^{\alpha}, d^{\alpha}] \}$, $\forall \alpha \in [0, 1]$, $\forall a^{\alpha}, b^{\alpha}, c^{\alpha}, d^{\alpha} \in \mathbb{R}$ (Figure 1):*

1. $a^{\alpha} \leq b^{\alpha} \leq c^{\alpha} \leq d^{\alpha}$.
2. $[a^{\alpha}, d^{\alpha}]$ *and* $[b^{\alpha}, c^{\alpha}]$ *generate a function that is convex and* $[a^{\alpha}, d^{\alpha}]$ *generate a normal function.*
3. $\forall \alpha_1, \alpha_2 \in [0, 1] : (\alpha_2 > \alpha_1) \Rightarrow ([a^{\alpha_1}, c^{\alpha_1}] \supset [a^{\alpha_2}, c^{\alpha_2}], [b^{\alpha_1}, d^{\alpha_1}] \supset [b^{\alpha_2}, d^{\alpha_2}])$, *for* $c^{\alpha_2} \geq b^{\alpha_2}$.
4. *If the maximum of the membership function generated by* $[b^{\alpha}, c^{\alpha}]$ *is the level* α_m, *that is* $[b^{\alpha_m}, c^{\alpha_m}]$, *then* $[b^{\alpha_m}, c^{\alpha_m}] \subset [a^{\alpha=1}, d^{\alpha=1}]$.
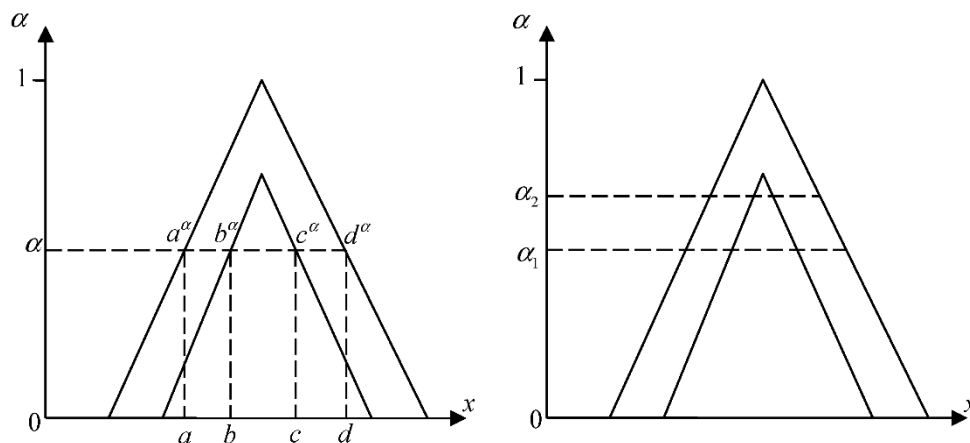

Figure 1. Definition of an IT2FN.

Definition 3 ([25]). *Let* $X, Y \subseteq R$, $\widetilde{A} = \{((x,u), \mu_{\widetilde{A}}(x,u)) | \forall x \in X, \forall u \in [0,1]\}$ *and* $\widetilde{B} = \{((y,v), \mu_{\widetilde{B}}(y,v)) | \forall y \in Y, \forall v \in [0,1]\}$ *are two T2FSs. Then,* $\widetilde{R} = \{(((x,u),(y,v)), \mu_{\widetilde{R}}(\mu_{\widetilde{A}}(x,u), \mu_{\widetilde{B}}(y,v))) | (\forall x \in X, \forall u \in [0,1]) \times (\forall y \in Y, \forall u \in [0,1])\}$ *is a type-2 fuzzy relation (T2FR) on* $\widetilde{A}$ *and* $\widetilde{B}$ *if* $\mu_{\widetilde{R}}(\mu_{\widetilde{A}}(x,u), \mu_{\widetilde{B}}(y,v)) \leq \mu_{\widetilde{A}}(x,u)$, $\forall ((x,u),(y,v)) \in (\forall x \in X, \forall u \in [0,1]) \times (\forall y \in Y, \forall v \in [0,1])$.

The following definition is T2FDP which is defined using the previous definitions as stated before [25].

Definition 4. *Let* $P = \{x | x \text{ type} - 2 \text{ fuzzy point}\}$ *and* $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}} = \{P_i | P_i \text{ data point}\}$ *be the set of the type-2 fuzzy data point with* $P_i \in P \subset X$, *where* X *is a universal set and* $\mu_P(P_i) : P \to [0,1]$ *is the membership function defined as* $\mu_P(P_i) = 1$ *and formulated as* $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}} = \{(P_i, \mu_P(P_i)) | P_i \in \mathbb{R}, \}$ $\{ i = 0, 1, 2, \ldots, n\}$. *Therefore,*

$$\mu_P(P_i) = \begin{cases} 0 & \text{if } P_i \notin X \\ c \in (0,1) & \text{if } P_i \overset{\leftrightarrow}{\in} X \\ 1 & \text{if } P_i \in X \end{cases} \tag{2}$$

with $\mu_P(P_i) = \left\langle \mu_P(\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i^{\leftarrow}), \mu_P(P_i), \mu_P(\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i^{\rightarrow}) \right\rangle$ *which* $\mu_P(\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i^{\leftarrow})$ *and* $\mu_P(\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i^{\rightarrow})$ *are left and right foot-print of membership values with* $\mu_P(\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i^{\leftarrow}) = \langle \mu_P(a_i), \mu_P(e_i), \mu_P(b_i) \rangle$ *where* $\mu_P(a_i)$, $\mu_P(e_i)$ *and* $\mu_P(b_i)$ *are left–left, left, right–left membership grade values and* $\mu_P(\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i^{\rightarrow}) = \langle \mu_P(c_i), \mu_P(f_i), \mu_P(d_i) \rangle$ $\mu_P(d_i)$, $\mu_P(f_i)$ *and* $\mu_P(c_i)$ *are right–right, right, left–right membership grade values, which can be written as follows:*

$$\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}} = \left\{ \overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i : i = 0, 1, 2, \ldots, n \right\} \tag{3}$$

For every i, $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i = \left\langle \overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i^{\leftarrow}, P_i, \overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i^{\rightarrow} \right\rangle$ *with* $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i^{\leftarrow} = \langle a_i, e_i, b_i \rangle$ *where* a_i, e_i *and* b_i *are left–left, left and right–left T2FDPs and* $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}{}_i^{\rightarrow} = \langle c_i, f_i, d_i \rangle$ *where* c_i, f_i *and* d_i *are left–right, right and right–right T2FDPs, respectively. This can be illustrated as shown in Figure* 2.
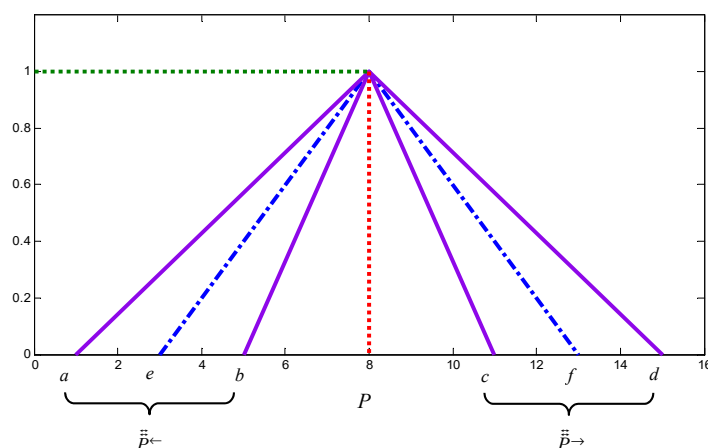

Figure 2. T2FDP around eight.

From Figure 2, T1FDP becomes the primary membership function bounded by upper bound, $[a, d]$ and lower bound, $[b, c]$ respectively.

After T2FDP has been defined, the next procedure is the fuzzification process that applies the alpha-cut of IT2FN [25]. This definition is determined based on the fuzzification process of type-1 fuzzy data points discussed in [4]. Therefore, the description of the fuzzification process against T2FDP can be given through Definition 5, as follows.

Definition 5. *Let* $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}$ *be the set of T2FDPs with* $\overset{\leftrightarrow}{P}_i \in \overset{\leftrightarrow}{P}$ *where* $i = 0, 1, \ldots, n-1$. *Then* $\overset{\leftrightarrow}{P}_{i_\alpha}$ *is the alpha-cut operation of T2FDPs with* $i = 0, 1, 2, \ldots, n$ *which is given as follows.*

$$
\begin{aligned}
\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{i_\alpha} &= \left\langle \overset{\leftrightarrow\leftarrow}{\overset{\leftrightarrow}{P}}_{i_\alpha}, P_i, \overset{\leftrightarrow\rightarrow}{\overset{\leftrightarrow}{P}}_{i_\alpha} \right\rangle \\
&= \left\langle \left\langle a_i^\alpha; e_i^\alpha; b_i^\alpha \right\rangle, P_i, \left\langle c_i^\alpha; f_i^\alpha; d_i^\alpha \right\rangle \right\rangle \\
&= \left\langle \left[(P_i - \left\langle a_i; e_i; b_i \right\rangle)\alpha + \left\langle a_i; e_i; b_i \right\rangle \right], P_i, \right. \\
&\quad \left. \left[-(\left\langle c_i; f_i; d_i \right\rangle - P_i)\alpha + \left\langle c_i; f_i; d_i \right\rangle \right] \right\rangle
\end{aligned}
\tag{4}
$$

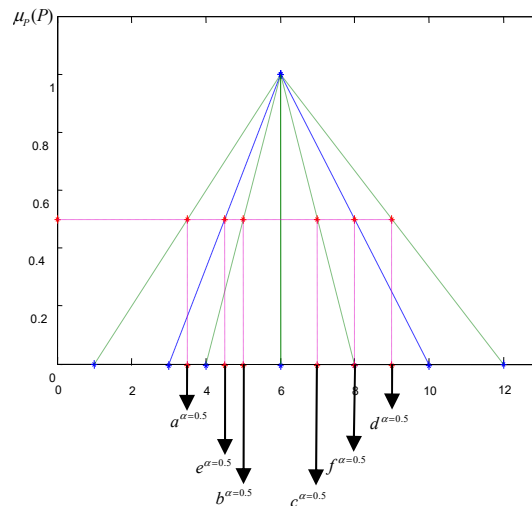*This definition is illustrated in Figure 3.*

Figure 3. The alpha-cut operation toward T2FDP.

Figure 3 shows the implication of the alpha-cut operation against T2FDP, which is the fuzzification process with the specific value of alpha (membership value). The alpha value of this operation is 0.5. If the alpha value increases to one, then the crisp data point is obtained. This can be illustrated by Figure 4 as follows.

After performing the fuzzification process, then the following process is type-reduction. Type-reduction is defined, then used against T2FDP to allow the defuzzification of the type-1 fuzzy set. The type-reduction is an approach to simplify type-2 defuzzification. The first type-reduction had been proposed by Nie and Tan [27]. On the other hand, type-reduction on discretized interval type-2 fuzzy sets have been discussed in [28]. The consistent linear and quadratic type-reduction methods have been introduced by [29].

The type-2 defuzzification has been already discussed, such as the Karnik–Mendel algorithm [30]. Subsequently, type-2 defuzzification with explicit models of the uncertainty was proposed in [31]. Moreover, the various methods of type-2 defuzzification have different mathematical properties discussed by [31].

In this paper, the proposed methods for type-reduction and type-2 defuzzification against T2FDP are based on the centroid method based on Definition 6 and Definition 7, respectively.
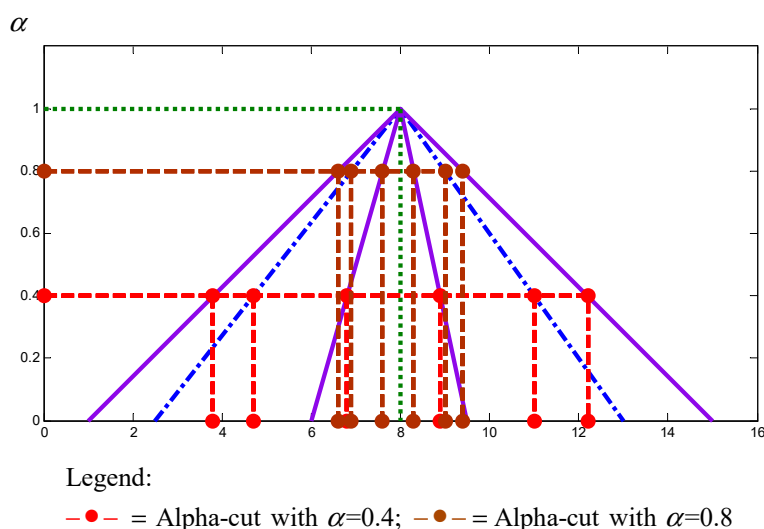
Figure 4. The illustration of the correlation between alpha values and T2FDPs.

Definition 6. *Let $\overset{\leftrightarrow}{P}_i$ be a set T2FDP and $\overset{\leftrightarrow}{P}_{i_\alpha}$ are the set of T2FDP after the fuzzification process for $i = 0, 1, 2, \ldots, n$, then the type-reduction of $\overset{\leftrightarrow}{P}_{i_\alpha}$ which is represented as $\overline{\overset{\leftrightarrow}{P}}_\alpha$ can be defined as follows:*

$$\overline{\overset{\leftrightarrow}{P}}_\alpha = \left\{ \overline{\overset{\leftrightarrow}{P}}_{i_\alpha} = \left\langle \overline{\overset{\leftrightarrow}{P}}_{i_\alpha}^{\leftarrow}, P_i, \overline{\overset{\leftrightarrow}{P}}_{i_\alpha}^{\rightarrow} \right\rangle ; i = 0, 1, 2, \ldots, n \right\} \tag{5}$$

where P_i is crisp data points and $\overline{\overset{\leftrightarrow}{P}}_{i_\alpha}^{\leftarrow}$ and $\overline{\overset{\leftrightarrow}{P}}_{i_\alpha}^{\rightarrow}$ are left and right fuzzified type-reduction T2FDP respectively with their formulation given by the following:

$$\begin{aligned} \overline{\overset{\leftrightarrow}{P}}_{i_\alpha}^{\leftarrow} &= \frac{1}{3} \sum_{i=0,1,\ldots,n} \left\langle a_i^\alpha + e_i^\alpha + b_i^\alpha \right\rangle \\ \overline{\overset{\leftrightarrow}{P}}_{i_\alpha}^{\rightarrow} &= \frac{1}{3} \sum_{i=0,1,\ldots,n} \left\langle c_i^\alpha + f_i^\alpha + d_i^\alpha \right\rangle \end{aligned} \tag{6}$$

Through the implementation of Definition 6, the fuzzified type-reduction T2FDP is obtained. Then, the next procedure to get the crisp T2FDP is the defuzzification process of type-1. The defuzzification process has been defined in [4,32,33] and described as follows.

Definition 7. *Let $\overline{\overset{\leftrightarrow}{P}}_{i_\alpha}$ be the fuzzified type-reduction T2FDP with $i = 0, 1, 2, \ldots, n$. Then, $\overline{\overline{P}}_{i_\alpha}$ is the defuzzification process of $\overline{\overset{\leftrightarrow}{P}}_{i_\alpha}$ if for every $\overline{\overset{\leftrightarrow}{P}}_{i_\alpha} \in \overline{\overset{\leftrightarrow}{P}}_\alpha$,*

$$\overline{\overline{P}}_\alpha = \left\{ \overline{\overline{P}}_{i_\alpha} \middle| i = 0, 1, 2, \ldots, n \right\} \tag{7}$$

where each $\overline{\overline{P}}_{i_\alpha}$ can be formalized as:

$$\overline{\overline{P}}_{i_\alpha} = \frac{1}{3} \sum_{i=0,1,\ldots,n} \left\langle \overline{\overset{\leftrightarrow}{P}}_{i_\alpha}^{\leftarrow}, P_i, \overline{\overset{\leftrightarrow}{P}}_{i_\alpha}^{\rightarrow} \right\rangle \tag{8}$$

The process of obtaining T2FDP defuzzified from defining T2FDP, including the fuzzification, type-reduction and defuzzification processes can be summarized and illustrated in the following Figure 5.
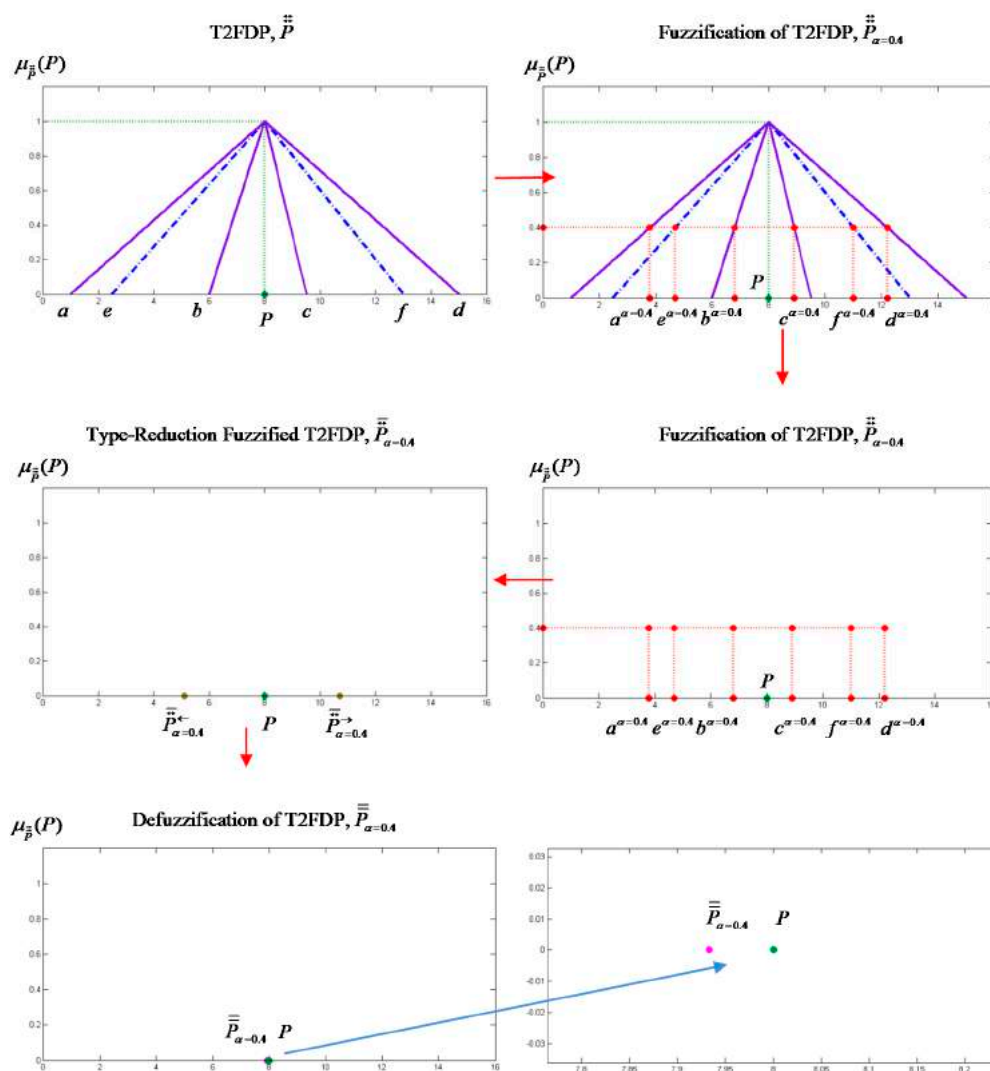
Figure 5. The processes of defining, fuzzification, type-reduction and defuzzification towards T2FDP.

4. Results: Type-2 Fuzzy B-Spline Model

This section will discussing type-2 fuzzy B-spline model which specifically used the interpolation method. When creating a type-2 fuzzy curve and surface, the T2FDP are integrated into the B-spline curve and surface function where the end result is known as a type-2 fuzzy B-spline curve and surface. This type-2 fuzzy model meets the processes of fuzzification, type-reduction and defuzzification to obtain the crisp type-2 fuzzy curve and surface solution (single curve and surface solution).

The construction of the type-2 fuzzy B-spline model is based on the studies carried out by Zakaria et al. [4], Zakaria and Wahab [32], Wahab et al. [33], and Karim et al. [34]. These studies discussed the construction of the type-1 fuzzy B-spline model. The type-2 fuzzy interpolation B-spline model [25] is defined in Definition 8 and illustrated by Figure 6.

Definition 8. *Let $\overset{\leftrightarrow}{D_i} \in R$ be a list of T2FDP with $i = 0, 1, 2, \ldots, n$, then the type-2 fuzzy interpolation B-spline curve (T2FIBsC) can be defined as follows:*

$$\overset{\leftrightarrow}{BsC}(t) = \sum_{i=0}^{k+h-1} \overset{\leftrightarrow}{P_i} N_{i,h}(t) \text{ which } \overset{\leftrightarrow}{BsC}(t_i) = \overset{\leftrightarrow}{D_i} \tag{9}$$

where $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_i$ are T2FCP, $N_{i,h}(t)$ is a basic function of B-spline and t is crisp knot sequences $t_1, t_2, \ldots, t_{m=d+n+1}$ in which d represents the degree of B-spline function and n represents the numbers of control points.
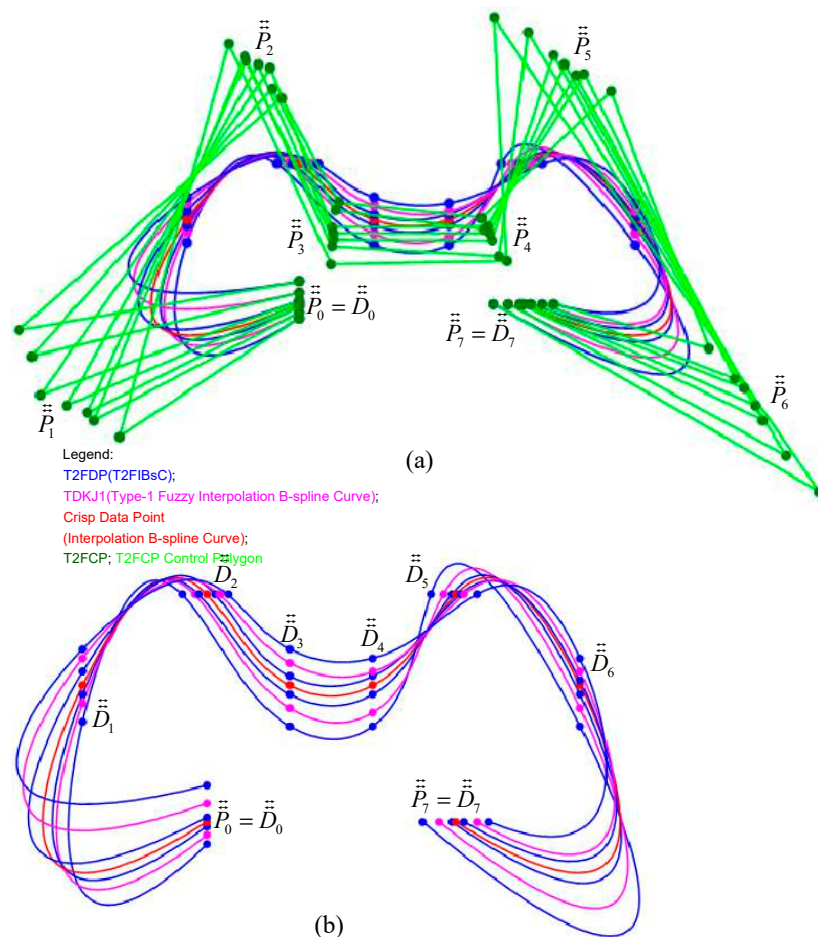

Figure 6. The example of T2FIBsC model: (**a**) with T2FCP; (**b**) without T2FCP.

Definition 9. *Let* $\overset{\leftrightarrow}{\overset{\leftrightarrow}{D}}_{a,b}$ *be given as a set of T2FDP with* $a = 0, 1, \ldots, m$ *and* $b = 0, 1, \ldots, n$ *which has degree p and q. Then, the type-2 fuzzy interpolation B-spline surface (T2FIBsS) model with degree p and q can be given as follows:*

$$\overset{\leftrightarrow}{\overset{\leftrightarrow}{BsSi}}(s_a, t_b) = \sum_{l=0}^{n} \sum_{k=0}^{m} N_{k,p}(s_a) N_{l,q}(t_b) \overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{k,l} = \overset{\leftrightarrow}{\overset{\leftrightarrow}{D}}_{a,b} \tag{10}$$

where $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{k,l}$ *is the set of T2FCP which is the unknown value.*

Equation (10) can be rewritten as

$$\overset{\leftrightarrow}{\overset{\leftrightarrow}{D}}_{a,b} = \sum_{l=0}^{n} N_{k,p}(s_a) \left[\sum_{k=0}^{m} N_{l,q}(t_b) \overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{k,l} \right] = \sum_{l=0}^{n} N_{k,p}(s_a) \overset{\leftrightarrow}{\overset{\leftrightarrow}{R}}_{k,b} \tag{11}$$

with

$$\overset{\leftrightarrow}{\overset{\leftrightarrow}{R}}_{k,b} = \sum_{k=0}^{m} N_{l,q}(t_b) \overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{k,l} \tag{12}$$

Therefore, Equation (12) is resolved first which used $\overset{\leftrightarrow}{\overset{\leftrightarrow}{D}}_{a,b}$ before $N_{k,p}(s_a)$. It is followed by solving $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{k,l}$ using Equation (10). Therefore, we obtain T2FCP values that allow the surface to interpolate the T2FDP. Thus, the illustration of the T2FIBsS model can be illustrated in Figure 7 as follows.
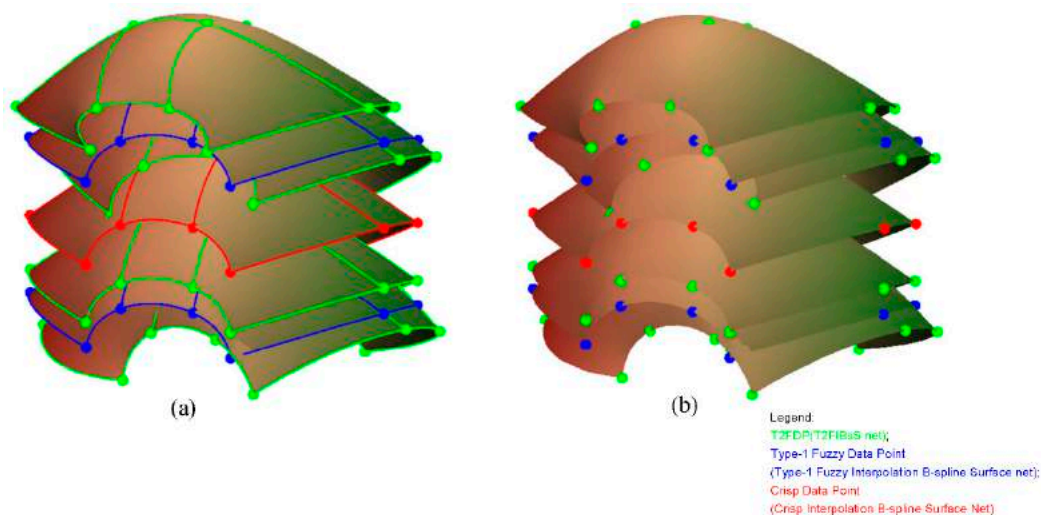

Figure 7. The example T2FIBsS model: with (**a**) and; without (**b**) type-2 fuzzy data net.

Figure 7 shows that the T2FIBsS model in bicubic shapes was constructed based on the T2FCP net obtained using Equation (10). The T2FDP net of T2FIBsS of Figure 7a illustrates the first impression of the against the type-2 fuzzy surface.

The next process is the fuzzification process. This fuzzification is performed against the T2FDPs before being integrated with the basic B-spline surface function. Therefore, the fuzzification of T2FIBsS can be defined as the following equation

$$\overset{\leftrightarrow}{\overset{\leftrightarrow}{BsSi}}_{\alpha_j}(s_a, t_b) = \sum_{l=0}^{n} \sum_{k=0}^{m} N_{k,p}(s_a) N_{l.q}(t_b) \overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{(k,l)_{\alpha_j}} = \overset{\leftrightarrow}{\overset{\leftrightarrow}{D}}_{(a,b)_{\alpha_j}} \tag{13}$$

where α_j is the value of alpha-cut operation of the type-2 triangular fuzzy number represents the fuzzification process with $\alpha \in (0,1]$, $j = 1, 2, \ldots, y$. Therefore, based on Figure 7, the illustration after fuzzification process can be shown in Figure 8.

After the fuzzification process has been applied, the next step is the type-reduction process. Therefore, the reduction of fuzzified T2FIBsS can be formalized as follows, based on Definition 6.

$$\overline{\overset{\leftrightarrow}{BsSi}}_{\alpha_j}(s_a, t_b) = \sum_{l=0}^{n} \sum_{k=0}^{m} N_{k,p}(s_a) N_{l.q}(t_b) \overline{\overset{\leftrightarrow}{P}}_{(k,l)_{\alpha_j}} = \overline{\overset{\leftrightarrow}{D}}_{(a,b)_{\alpha_j}} \tag{14}$$

The defuzzification of type-1 fuzzy then follows it to obtain the final result as the crisp T2FIBsS. Therefore, the defuzzification of type-reduced fuzzified T2FIBsS is given by the following equation.

$$\overline{\overline{BsSi}}_{\alpha_j}(s_a, t_b) = \sum_{l=0}^{n} \sum_{k=0}^{m} N_{k,p}(s_a) N_{l.q}(t_b) \overline{\overline{P}}_{(k,l)_{\alpha_j}} = \overline{\overline{D}}_{(a,b)_{\alpha_j}} \tag{15}$$
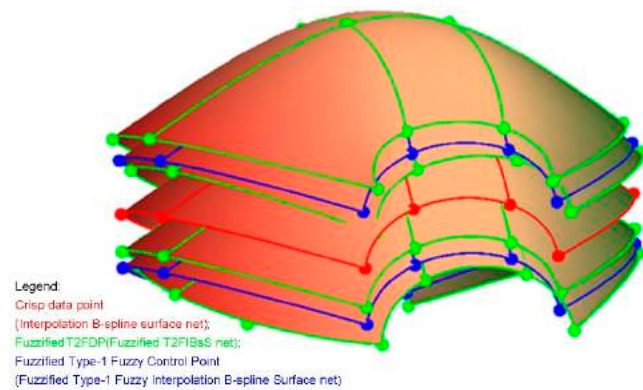
Figure 8. The example fuzzified T2FIBsS model with fuzzified type-2 fuzzy data net.

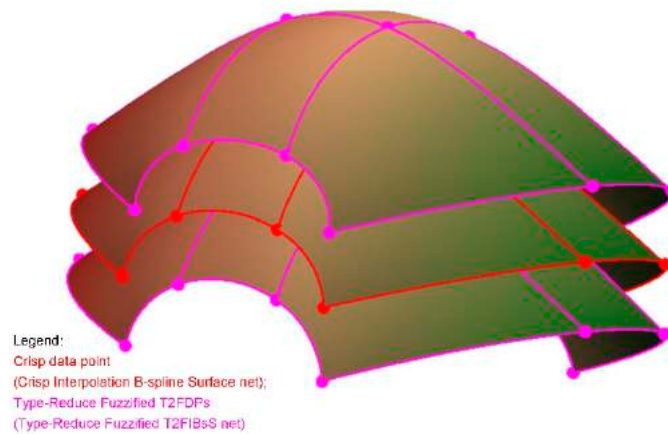Then, the illustration of this type-reduction process can be illustrated in Figure 9 as follows.

Figure 9. The example of type-reduced fuzzified T2FIBsS together with type-reduced fuzzified T2FDPs net.

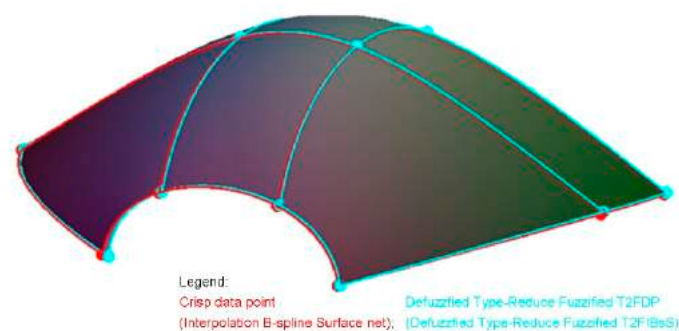The illustration of the defuzzification process against the type-reduced fuzzified T2FIBsS is shown in Figure 10.

Figure 10. The example of defuzzification-reduced T2FIBsS with defuzzification-reduced T2FDPs.

Figure 10 shows that the defuzzification-reduced T2FIBsS model along with crisp interpolation B-spline surface. Both surfaces were built by finding the control points that interpolates data. The crisp data points were marked by red and the defuzzfication-reduced T2FDPs were marked by cyan.

5. Application: Seabed Modeling

This section discusses the practical application of the T2FIBsS model of seabed modeling. In the seabed modeling, multiple uncertainties or errors occur due to the nature of collecting data points, i.e., the wavy water surface and the collector's uncertain perception and truth level. Thus, the errors of the data collected are bounded by two uncertainties. This scenario is illustrated in Figure 11, which is also the extension of uncertainty data for lakebed modeling [4].
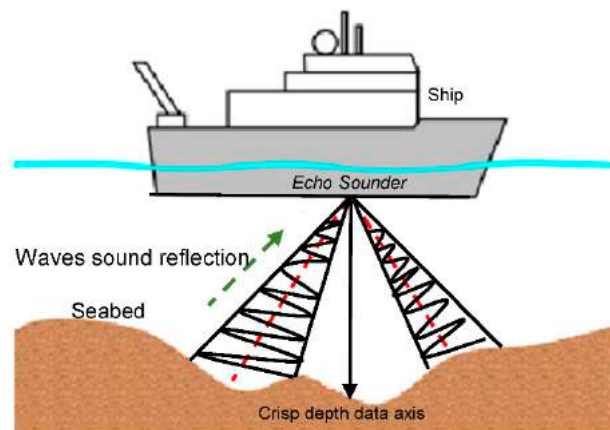

Figure 11. Illustration of procedure in taking depth data point (in meter) of the seabed, which consists of uncertainty complex data.

Figure 11 shows that obtaining the depth sea by means of an echo sounder where the data points obtained have the complex uncertainty properties. Therefore, it necessary to use the T2FIBsS method to model those complex uncertainty data after the data points are defined through the T2FST.

The following algorithm shows a step-by-step process of defining uncertainty complex data until the final stage of defuzzification-reduced T2FDP B-spline surface interpolation function.

The result of Algorithm 1 is illustrated in Figures 12–14 as follows.

Figure 12 shows the implementation of T2FIBsS against the seabed data modeling. In getting the final output named as crisp T2FIBsS, the fuzzifying process involves the output of T2FIBsS for seabed surface from the complex uncertainty data. Then, the fuzzification process is applied to shorten the T2FIBsS interval with the alpha value of 0.2 by using the alpha-cut of a triangular type-2 fuzzy number. After the fuzzification process, the following process applied is the type-reduction process, which reduces the type-2 fuzzy to become type-1 fuzzy to allow the defuzzfication process. Later, the defuzzification process is applied to obtain the crisp type-2 fuzzy solution that shows the distinct surface.

For Figures 13 and 14, getting the crisp T2FIBsS is the same as the processes in Figure 12. The only difference between all these three figures are the alpha values of the fuzzification process. The main focus is on showing the relation between alpha values crisp T2FIBsS output, and crisp B-spline surface seabed modeling, where the crisp T2FIBsS also tends to crisp B-spline surface seabed modeling when the alpha values are used to one side. On the other hand, percentage errors of all three figures have also demonstrated that the errors are smaller as the alpha-values increases.

Therefore, the different fuzzification processes involving alpha-cut, 0.2, 0.5 and 0.8 as shown in the figures. The average error percentage for each surface is 0.0018403 m, 0.0011502 m and 0.00046008 m, respectively. This average error percentage shows T2FDPs tends to crisp data points if their alpha values increase and tends to towards 1.
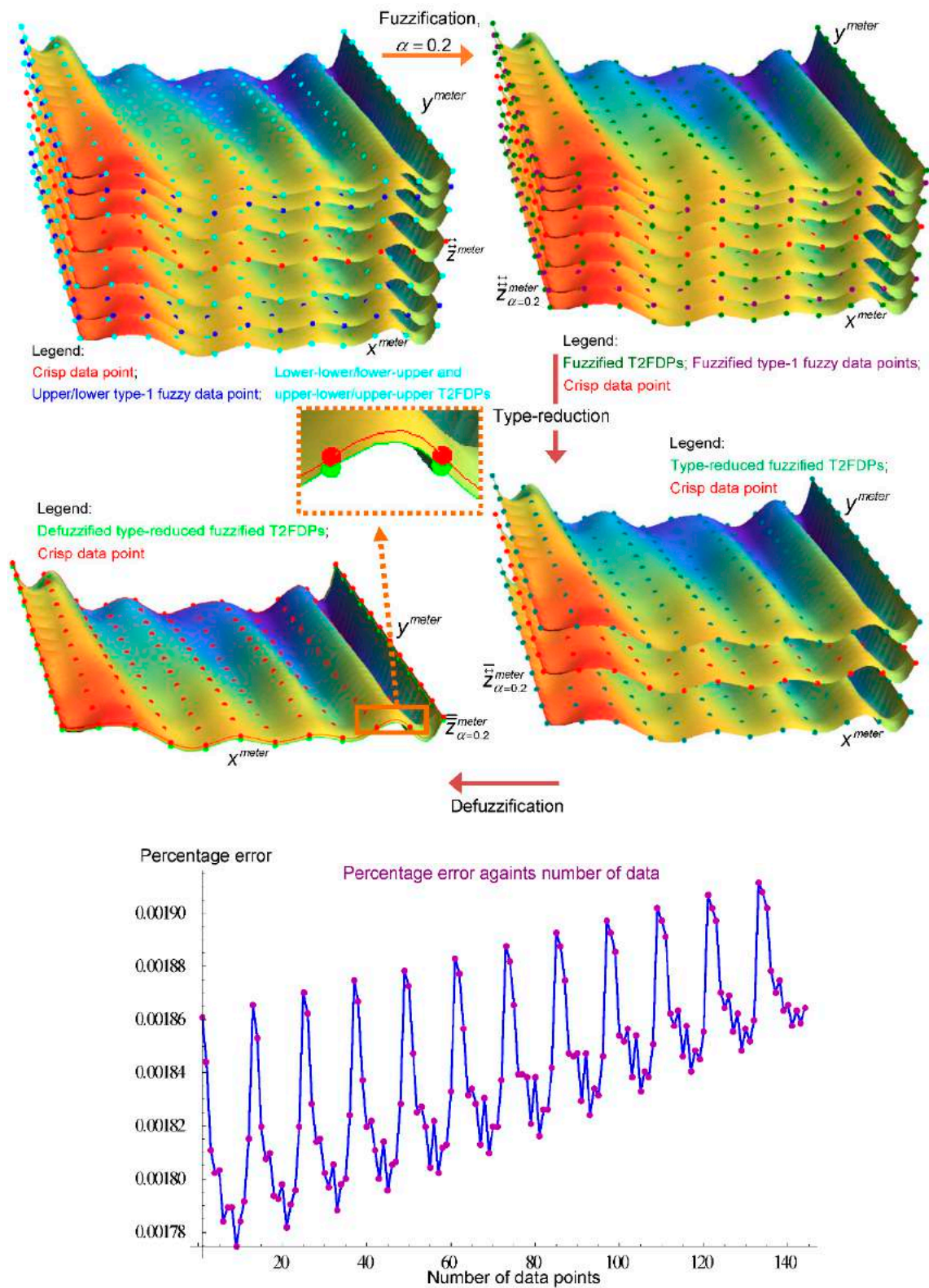
Figure 12. The T2FIBsS modeling through fuzzification until defuzzification processes along with the error plot.
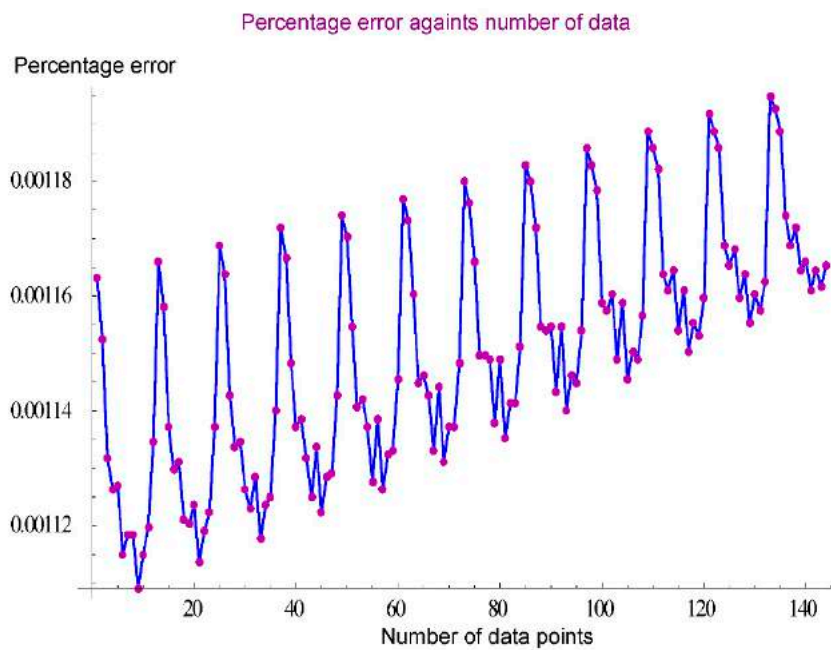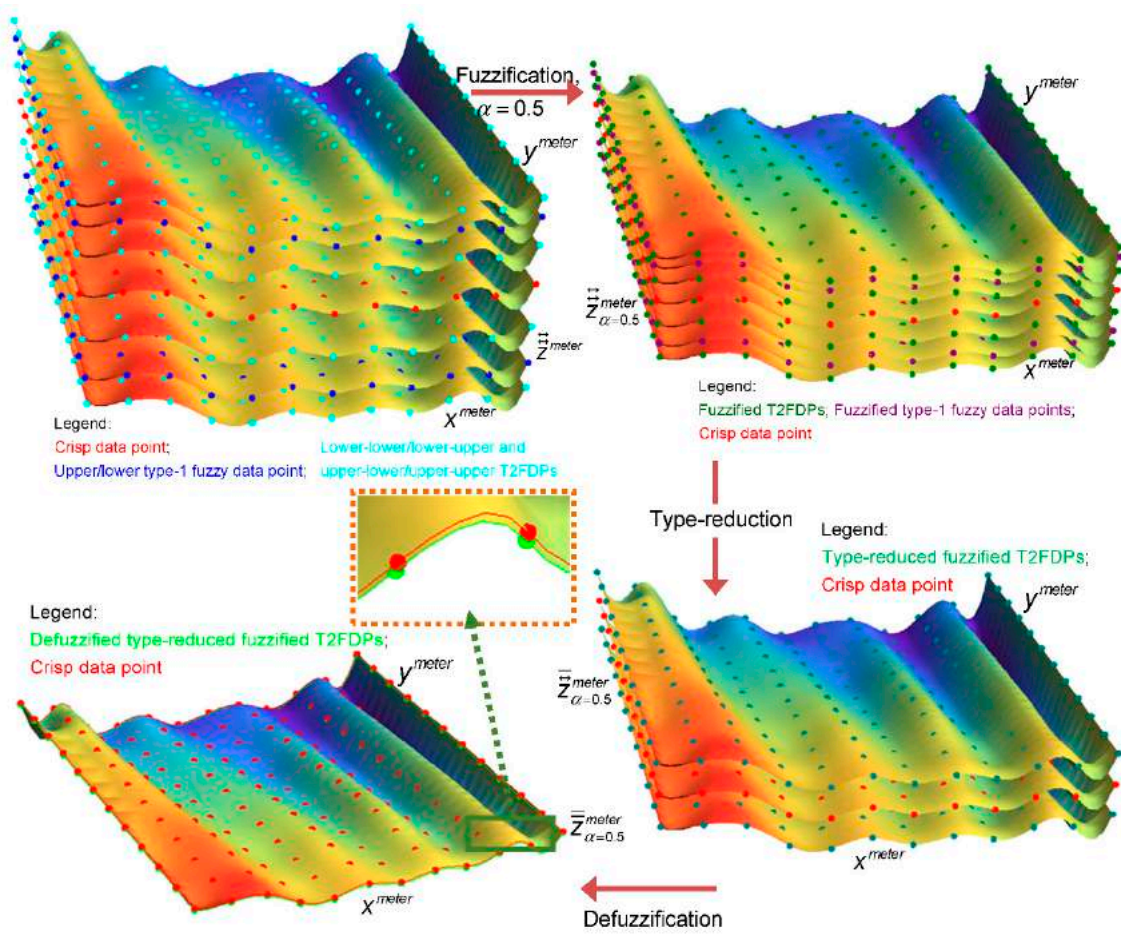
Figure 13. The T2FIBsS modeling through fuzzification until defuzzification processes with $\alpha = 0.5$ along the error plot.
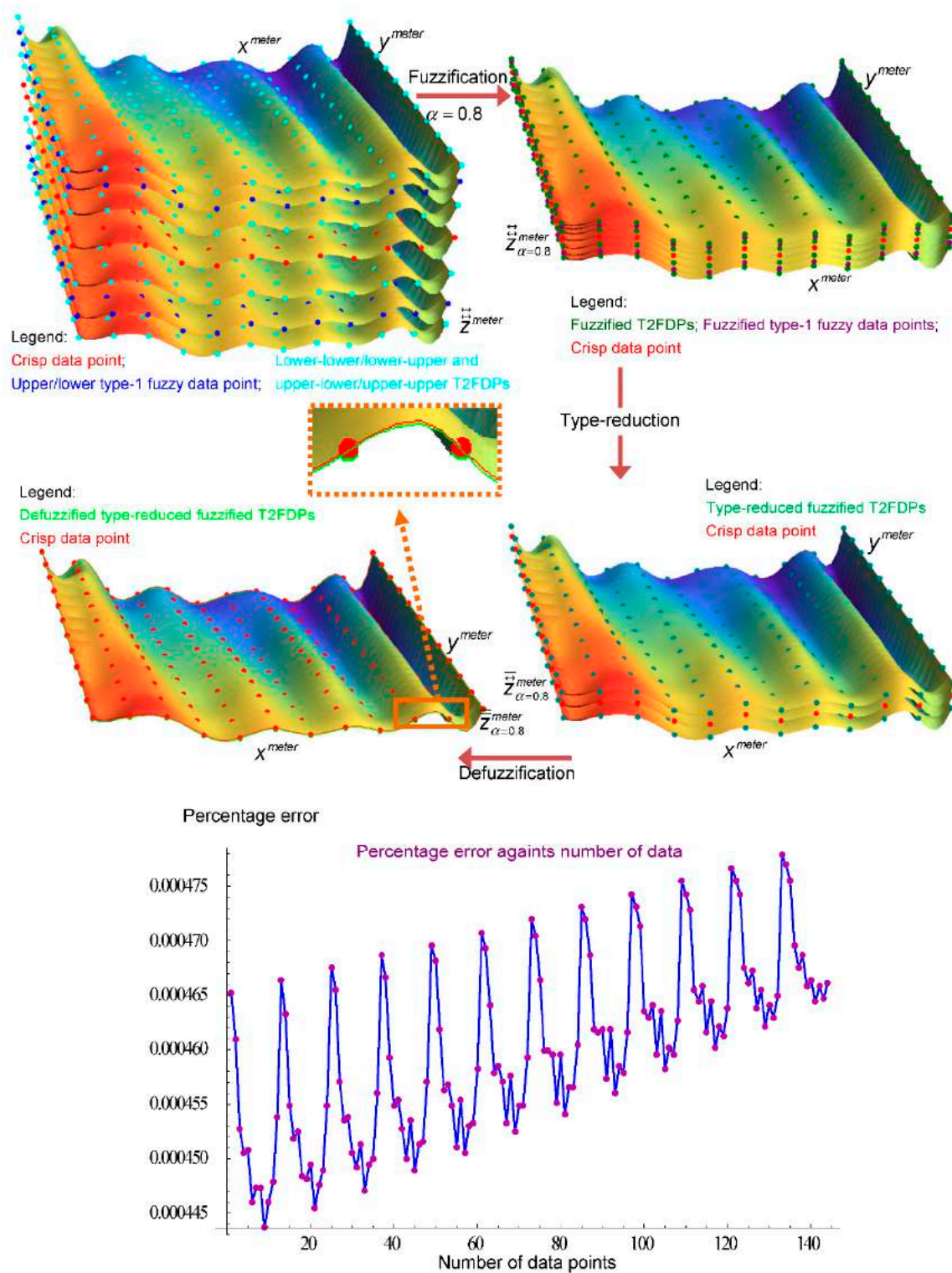
Figure 14. The T2FIBsS modeling through fuzzification until defuzzification processes with $\alpha = 0.8$ along the error plot.

Algorithm 1. Modeling T2FDP using interpolation type-2 B-spline surface.

Step 1: *Define the uncertainty complex data of seabed by using Definition 4.*

Step 2: *Use T2FDP ($\overset{\leftrightarrow}{\overset{\leftrightarrow}{D}}_{a,b}$) of seabed to solve Equation (12).*

Step 3: *Find the values of $N_{k,p}(s_a)$ after Equation (12) has been solved.*

Step 4: *Find the points of $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{k,l}$ through Equation (10) by using Equation (12).*

Step 5: *Plot the $\overset{\leftrightarrow}{\overset{\leftrightarrow}{D}}_{(a,b)}$ and then model $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{k,l}$ via Equation (10).*

Step 6: *Fuzzification process: Apply Equation (4), then apply step 2 until step 4 and model $\overset{\leftrightarrow}{\overset{\leftrightarrow}{P}}_{(k,l)_{\alpha_j}}$ as in Equation (13).*

Step 7: *Type-reduction process: Apply Equation (6), then apply step 2 until step 4 and model $\overline{\overset{\leftrightarrow}{P}}_{(k,l)_{\alpha_j}}$ as in Equation (14).*

Step 8: *Defuzzification process: Apply Equation (8), then apply step 2 until step 4 and model $\overline{\overline{P}}_{(k,l)_{\alpha_j}}$ by Equation (15).*

Step 9: *Find and plot the error between defuzzification-reduced T2FDPs and crisp data points of seabed depth data using the following equation:*

$$\frac{\sum\limits_{k=0,\dots,n}^{n} {}^{L}D_{k_e}}{\sum\limits_{k=0,\dots,n} n({}^{L}D_{k_e})} where \quad {}^{L}D_{k_e} = \frac{{}^{L}\overline{\overline{D}}_k - {}^{L}D_k}{{}^{L}D_k} \quad with \quad k = 0,1,2,\dots,n. \tag{16}$$

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6. Discussion and Conclusions

This paper proposed a new method for defining complex uncertainty data, and modeled it on the hybrid model constructed called T2FIBsS. The complex uncertainty data was defined through IT2FN concepts obtaining T2FDPs and integrated with the B-spline surface function to produce the T2FIBsS model. This model has an advantage in modeling complex uncertainty data, as shown in seabed modeling. Meanwhile, the error and the percentage error have been calculated between defuzzification-reduced T2FDPs and crisp data points demonstrating the feasibility of the proposed model.

This developed model as T2FIBsS also deals with uncertainty modeling, which type-1 fuzzy modeling can do when the T2FIBsS first is reduced to the type-1 fuzzy model. However, the type-1 fuzzy model, which has been discussed in the literature, cannot be used in defining and modeling the complex uncertainty data. Therefore, T2FIBsS can be used to define and model the uncertainty and complex uncertainty compared to the type-1 fuzzy model, which only can be used to define and model the uncertainty data, but not applicable for complex uncertainty data.

The limitation of this study is the properties of the data that we want to model. The data that we obtained is secondary data, which had been filtered from raw data. Next is the surface function, which generates a desired surface based on the properties of the data. This function generates the surface entirety, rather than as a suitable surface patch. This surface that is generated entirety will make the surface sometimes unreasonable due to the data point position.

This research can be improved in the future by taking into account the complex uncertainty data, which has more uncertainty based on decision making or perceptions. This research can be expanded by using a complicated surface function, such as the Non-Uniform Rational B-spline (NURBS) surface, for modeling parts. The NURBS surface function has the advantage of local control on the surface other than the control points and

knots, but the added value as weight in the NURBS formulation can modify the surface locally.

Notations Index

Notations	Explanation
$\widetilde{A}$	Type-2 fuzzy set (T2FS)
$\widetilde{A}_\alpha$	Interval T2FS
$\widetilde{R}$	Type-2 fuzzy relation
P	Data point
$\overset{\leftrightarrow}{P}$	Type-2 fuzzy data point (T2FDP)
$\mu_P(P_i)$	Membership function of T2FDP
$\mu_P(\overset{\leftrightarrow}{P}_{i}{}^{\leftarrow})$	Left footprint of membership values
$\mu_P(\overset{\leftrightarrow}{P}_{i}{}^{\rightarrow})$	Right footprint of membership values
$\mu_P(a_i)$	Left-left membership grade value
$\mu_P(e_i)$	Left membership grade value
$\mu_P(b_i)$	Right-left membership grade value
$\mu_P(c_i)$	Left-right membership grade value
$\mu_P(f_i)$	Right membership grade value
$\mu_P(d_i)$	Right-right membership grade value
$\overset{\leftrightarrow}{P}_{i_\alpha}$	Fuzzification process (alpha-cut process) against ith T2FDP
$\overset{\leftrightarrow}{P}_{i_\alpha}{}^{\leftarrow}$	Left interval fuzzification process (alpha-cut process) against ith T2FDP
$\overset{\leftrightarrow}{P}_{i_\alpha}{}^{\rightarrow}$	Right interval fuzzification process (alpha-cut process) against ith T2FDP
a_i^α	Left-left fuzzification process (alpha-cut process) against ith T2FDP
e_i^α	Left fuzzification process (alpha-cut process) against ith T2FDP
b_i^α	Right-left fuzzification process (alpha-cut process) against ith T2FDP
c_i^α	Left-right fuzzification process (alpha-cut process) against ith T2FDP
f_i^α	Right fuzzification process (alpha-cut process) against ith T2FDP
d_i^α	Right-right fuzzification process (alpha-cut process) against ith T2FDP
$\overset{\leftrightarrow}{P}_\alpha$	Type-reduction of T2FDP after fuzzification process
$\overset{\leftrightarrow}{P}_{i_\alpha}{}^{\leftarrow}$	Left type-reduction of ith T2FDP after fuzzification process
$\overset{\leftrightarrow}{P}_{i_\alpha}{}^{\rightarrow}$	Right type-reduction of ith T2FDP after fuzzification process
$\overline{\overset{\leftrightarrow}{P}}_{i_\alpha}$	Defuzzification of ith T2FDP after type-reduction process
$\overset{\leftrightarrow}{D}_i$	ith T2FDP of type-2 fuzzy interpolation B-spline curve (T2FIBsC)

Notations	Explanation
$\overset{\leftrightarrow}{BsC}(t)$	T2FIBsC model
$\overset{\leftrightarrow}{P}_i$	ith type-2 fuzzy control point (T2FCP) of T2FIBsS
$\overset{\leftrightarrow}{D}_{a,b}$	T2FDP of Type -2 fuzzy interpolation B-spline surface (T2FIBsS)
$\overset{\leftrightarrow}{BsSi}_{\alpha_j}(s_a, t_b)$	T2FIBsS function modeling after fuzzification process
$\overset{\leftrightarrow}{D}_{(a,b)_{\alpha_j}}$	Fuzzification (alpha-cut operation) against T2FDP of T2FIBsS with $\alpha \in (0,1]$
$\overset{\leftrightarrow}{P}_{(k,l)_{\alpha_j}}$	Fuzzification (alpha-cut operation) against T2FCP of T2FIBsS with $\alpha \in (0,1]$
$\overline{\overset{\leftrightarrow}{BsSi}}_{\alpha_j}(s_a, t_b)$	T2FIBsS function modeling after type-reduction process
$\overline{\overset{\leftrightarrow}{D}}_{(a,b)_{\alpha_j}}$	Type reduction process against T2FDP of T2FIBsS after fuzzification process
$\overline{\overset{\leftrightarrow}{P}}_{(k,l)_{\alpha_j}}$	Type reduction process against T2FCP of T2FIBsS after fuzzification process
$\overline{\overline{BsSi}}_{\alpha_j}(s_a, t_b)$	Defuzzify modeling of T2FIBsS after type-reduction process
$\overline{\overline{D}}_{(a,b)_{\alpha_j}}$	T2FDP defuzzification of T2FIBsS
$\overline{\overline{P}}_{(k,l)_{\alpha_j}}$	T2FCP defuzzification of T2FIBsS
$\overset{\leftrightarrow meter}{z}$	T2FIBsS model of Seabed data collection (depth) in meter
$\overset{\leftrightarrow meter}{z}_{\alpha=0.2}$	T2FIBsS fuzzification model with the alpha value is 0.2
$\overline{\overset{\leftrightarrow meter}{z}}_{\alpha=0.2}$	T2FIBsS type-reduction model after fuzzification process
$\overline{\overline{z}}^{meter}_{\alpha=0.2}$	T2FIBsS defuzzification model after type-reduction process

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