



# Ordinary Muon Capture for Double Beta Decay and Anti-Neutrino Nuclear Responses

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This is a brief review on ordinary muon capture (OMC) experiments at Research Center for Nuclear Physics (RCNP) Osaka University relevant for the study of double beta decays (DBDs) and astro anti-neutrinos (neutrino) nuclear responses. OMC usually leaves the nucleus in highly excited unbound state. OMC is a charge exchange reaction via the charged weak boson as given by  $(\mu, \nu_\mu)$  reactions with  $\mu$  and  $\nu_\mu$  being the muon and muon neutrino. Subjects discussed include 1) unique features of OMC for studying DBDs and astro anti-neutrino (neutrino) nuclear responses, 2) experiments of OMCs on  $^{100}\text{Mo}$  and  $^{\text{nat}}\text{Mo}$  to study neutrino nuclear responses for DBDs and astro anti-neutrinos, 3) impact of the OMC results on neutrino nuclear responses for DBDs and astro anti-neutrinos. Remarks and perspectives on OMC experiments for neutrino nuclear responses are briefly described.

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## 1 INTRODUCTION

Double beta decays (DBDs) and astro (solar and supernova) neutrinos and anti-neutrinos are of current interest. Neutrino-less DBDs are used to explore the fundamental neutrino properties such as the Majorana nature, the absolute mass scale, the mass hierarchy and the CP phases beyond the standard electroweak model. The zero-neutrino ( $0\nu$ ) DBD rate is given by  $R^{0\nu} = G^{0\nu} \times |M^{0\nu}|^2 \times |m^{\text{eff}}|^2$ , where  $G^{0\nu}$  is the phase space volume,  $m^{\text{eff}}$  is the effective neutrino mass and  $M^{0\nu}$  is the zero-neutrino DBD nuclear matrix element, whose squared value is the nuclear response, i.e:  $B^{0\nu} = |M^{0\nu}|^2$ . Here we consider the ground state ( $0^+$ ) to ground state ( $0^+$ ) transition  ${}^A_Z X \rightarrow {}^A_{Z+2} X$ . The DBD nuclear matrix element (NME)  $M^{0\nu}$  is given by the coherent sum of the individual matrix elements  $M_i^{0\nu}$  which connect the initial and final ground states *via* the *i*th intermediate nucleus  ${}^A_{Z+1} X_i$ , where these extend up to about 100 MeV.

The supernova nucleosynthesis rates induced by neutrinos and anti-neutrinos are proportional to the respective nuclear responses

$$B^{v,\bar{v}} = \frac{1}{2J_A + 1} |M^{v,\bar{v}}|^2 \quad (1)$$

where  $M^{v,\bar{v}}$  are the NMEs, and  $(2J_A + 1)$  is the spin factor for the initial nucleus. Note that the NMEs  $M^{v,\bar{v}}$  are the coherent sums of individual NMEs  $M_i^{v,\bar{v}}$

$$M^{v,\bar{v}} = \sum_i M_i^{v,\bar{v}} \quad (2)$$

for each intermediate state ( $i$ ) and in the respective isospin directions  $\tau^-$  for  $\nu$  or  $\tau^+$  for  $\bar{\nu}$ . Note that the  $\nu$  and  $\bar{\nu}$  NMEs for the  $i$ th intermediate state are given by the  $\tau^-$  and  $\tau^+$  NMEs of  $M_i^-$  and  $M_i^+$ , respectively. Accordingly one needs the  $B^{0\nu}$  response to derive the effective neutrino mass  $m^{\text{eff}}$  from the neutrino-less DBD rate and the astro neutrino (anti-neutrino) response  $B^V$  ( $B^{\bar{V}}$ ) to derive the synthesis rate from the neutrino (anti-neutrino) flux. The neutrino nuclear responses are evaluated theoretically. However, the theoretical calculations for the neutrino nuclear responses for DBD and astro neutrino (anti-neutrino) depend much on the nuclear models and the nuclear parameters used. The neutrino nuclear responses and DBDs are extensively discussed in review articles and references therein (Ejiri, 2000; Vergados et al., 2012).

Experimental studies for the neutrino nuclear responses are interesting to provide the nuclear parameters to be used for theoretical calculations. Charge exchange reactions (CERs) using light ions have been used to study the neutrino nuclear responses. In particular, high energy-resolution ( $^3\text{He}$ ,  $t$ ) CERs have been used to study the neutrino nuclear responses for nuclei of DBD and astro neutrino interests in the broad energy and momentum regions. The responses studied by these light-ion CERs are the  $\tau^-$ -side ones, and the NMEs  $M_i^-$  are derived from CERs. On the other hand, the  $\tau^+$ -side responses and the NMEs  $M_i^+$  are not well studied since the ( $t$ ,  $^3\text{He}$ ) CERs to be used for studying the  $\tau^+$ -side responses require the radioactive  $t$  beam, and thus high precision measurements are difficult. Medium energy-resolution measurements are made by using ( $d$ ,  $^2\text{He}$ ) reactions. Neutrino nuclear responses studied by these light-ion CERs are discussed in the recent review articles and references therein (Ejiri, 2000; Vergados et al., 2012; Frekers and Alanssari, 2018; Ejiri et al., 2019; Ejiri, 2020).

Recently ordinary muon capture reactions (OMC), where a negative muon in an atomic orbit is captured into the nucleus has been shown useful for studying the  $\tau^+$ -side responses, and the  $M_i^+$  NMEs at Research Center for Nuclear Physics (RCNP) Osaka University (Hashim, 2015; Hashim et al., 2018; Hashim and Ejiri, 2019). The present report is a brief mini-review on the OMC studies for the DBD and astro anti-neutrinos responses and related subjects on nuclear isotope productions at RCNP.

## 2 UNIQUE FEATURES OF OMCS FOR NEUTRINO NUCLEAR RESPONSES

In OMC some of the unique feature are as follows:

- (1) OMC can be used to study the  $\tau^+$ -responses of proton ( $p$ )  $\rightarrow$  neutron( $n$ ) in the nucleus, and the NME  $M_i^+$  for DBD and astro anti-neutrino.
- (2) OMC transfers energy between 0–50 MeV and a momentum up to 100 MeV/c to the nucleus that are similar to those involved in neutrino-less DBD and supernova anti-neutrino.
- (3) A negative muon is finally captured into the nucleus *via* the weak interaction. The capture probability in the medium and heavy nuclei is around 95% after a mean

lifetime of about 100 ns in the atomic orbit. Low momentum (a few 10 MeV/c) beam muons at intensities of  $10^3$ – $10^4$  muons/s are used for OMC studies.

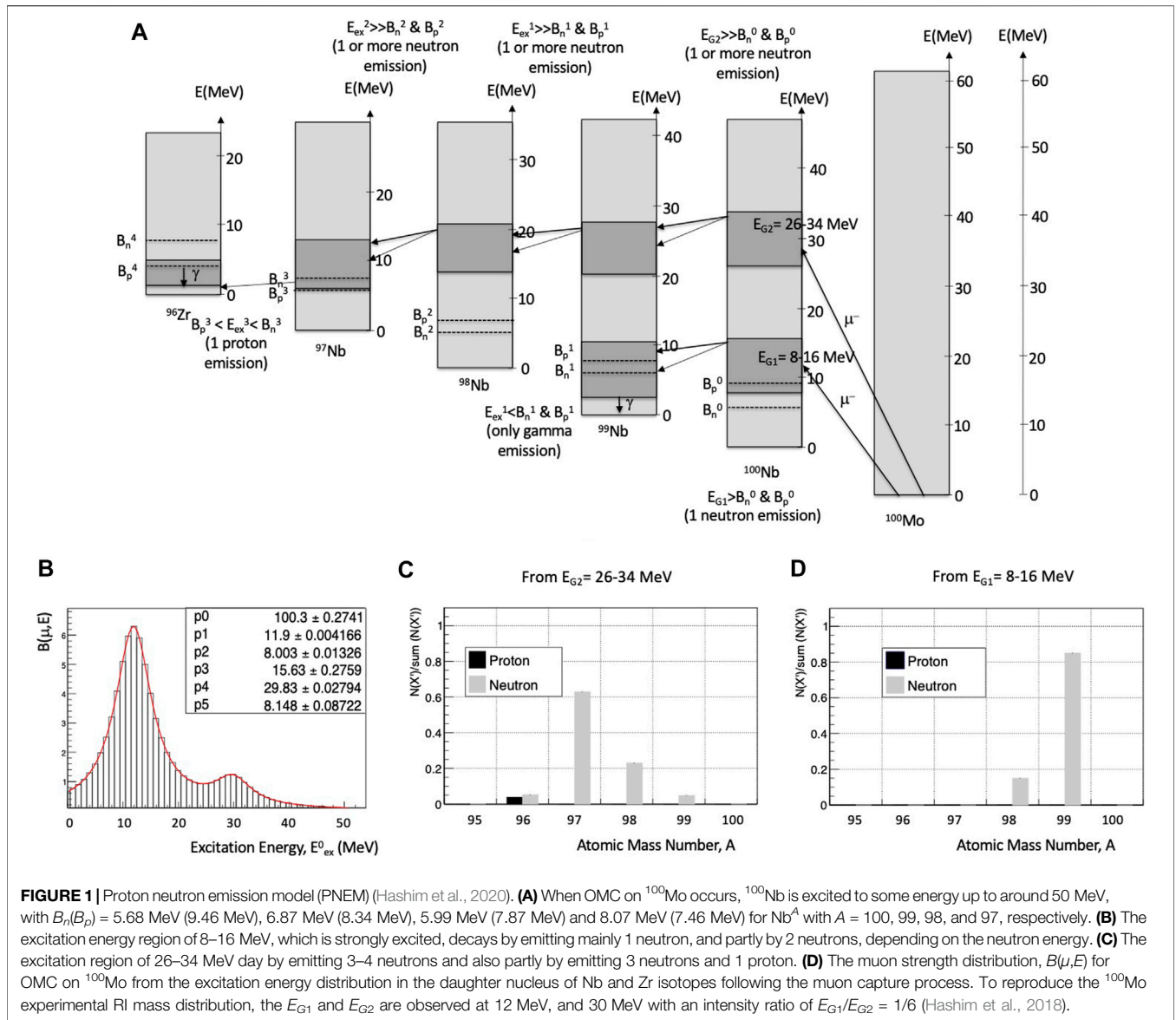
- (4) The OMC on  $^A_Z X$ , where  $A$  and  $Z$  are the mass and the atomic numbers, excites the nucleus  $^A_{Z-1} X$  up to around 50 MeV, which decays by emitting a number ( $x$ ) of mostly neutrons and gamma rays to the ground state of the residual nucleus  $^A_{Z-1} X$ . The number of neutrons reflects the excitation energy. Accordingly, the relative strength as a function of the excitation energy is evaluated by measuring the number ( $x$ ) of the neutrons, i.e. the mass distribution  $A-x$  of the residual nuclei.
- (5) Absolute OMC rate is obtained from the measured lifetime of the trapped muons, and thus the neutrino nuclear responses are derived from the OMC rate.

## 3 NEUTRINO NUCLEAR RESPONSES FOR MO ISOTOPES BY OMC

The intense 400 MeV proton beam with an intensity around  $1 \mu\text{A}$  from the RCNP cyclotron is used to produce pions. The momentum ( $p \approx 30$ – $50$  MeV/c) negative muons produced by the  $\pi$ -decays are guided by the MuSIC beamline to the target port. They are stopped in the target and are trapped in the inner orbit of the atom. Then, after around 100 ns, the muon is mainly (around 95%) captured into the nucleus via the OMC. The  $\mu$ -beam spot and the muon beam intensity at the target port are around  $6 \text{ cm} \times 6 \text{ cm}$  and  $5 \times 10^3$  per second (Hino et al., 2014). The  $\mu$  capture rate is derived by measuring the electrons' time signal from the weak decay of the  $\mu$  trapped in the atomic orbit by plastic scintillation detectors. The residual isotopes are identified by measuring characteristic prompt  $\gamma$ -rays of the residual nucleus by HPGe detectors online and delayed  $\gamma$ -rays from  $\beta$ -decays of residual isotopes by HPGe detectors offline.

Recently OMCS were studied on  $^{\text{nat}}\text{Mo}$  and  $^{100}\text{Mo}$ , which are interesting for DBD and astro neutrino studies (Ejiri et al., 2019). The OMC on  $^A_{42}\text{Mo}$  isotopes with the mass-number  $A = 92$ – $100$  produces  $^A_{41}\text{Nb}$  with the excitation energy  $E \approx 0$ – $50$  MeV. The excited Nb isotope decays mostly by emitting a number  $10(x)$  of neutrons until the final state gets particle-bound. Then it decays by emitting prompt  $\gamma$  rays to the ground state of  $^{A-x}\text{Nb}$ , which is followed by beta decay with comparatively long half-lives.

The OMC strength distribution as a function of the excitation energy  $E$  is derived from the mass-number ( $A-x$ ) distribution ( $x$  distribution) through the particle cascade model (Hashim et al., 2018; Hashim & Ejiri, 2019; Hashim et al., 2020). The excited states in Nb isotopes decay by emitting mostly neutrons since the Coulomb barrier much suppresses proton emission. Thus, the neutron cascade emission model (NEM) (Hashim et al., 2017; Hashim et al., 2018) was developed to deduce the excitation energy in the initial isotope of  $^A\text{Nb}$  from the number  $10(x)$  of the emitted neutrons, i.e., the mass-number  $A-x$  of the residual isotope  $^{A-x}\text{Nb}$  after the  $x$  neutron emission. The neutron energy spectrum for the first neutron consists of the pre-equilibrium (PEQ) and equilibrium (EQ) stages (Ejiri and de Voight, 1989). It is given as



$$S(E_{n1}) = k \left[ E_{n1} \exp\left(-\frac{E_{n1}}{T_{EQ}(E)}\right) + p E_{n1} \exp\left(-\frac{E_{n1}}{T_{PEQ}(E)}\right) \right] \quad (3)$$

where  $T_{EQ}(E)$  and  $T_{PEQ}(E)$  are the EQ and PEQ nuclear temperatures, respectively.  $T_{EQ}(E)$  is given as a function of excitation energy  $E$  (Ejiri and de Voight, 1989). The ratio of  $T_{PEQ}(E)/T_{EQ}(E) = 3$  for the medium excitation  $10 < E < 40$  MeV. After one neutron emission, the emission takes place only via the EQ stage. The OMC strength distribution for  $^{100}\text{Nb}$  is derived from the observed mass-number ( $A-x$ ) distribution for the OMC on  $^{100}\text{Mo}$ .

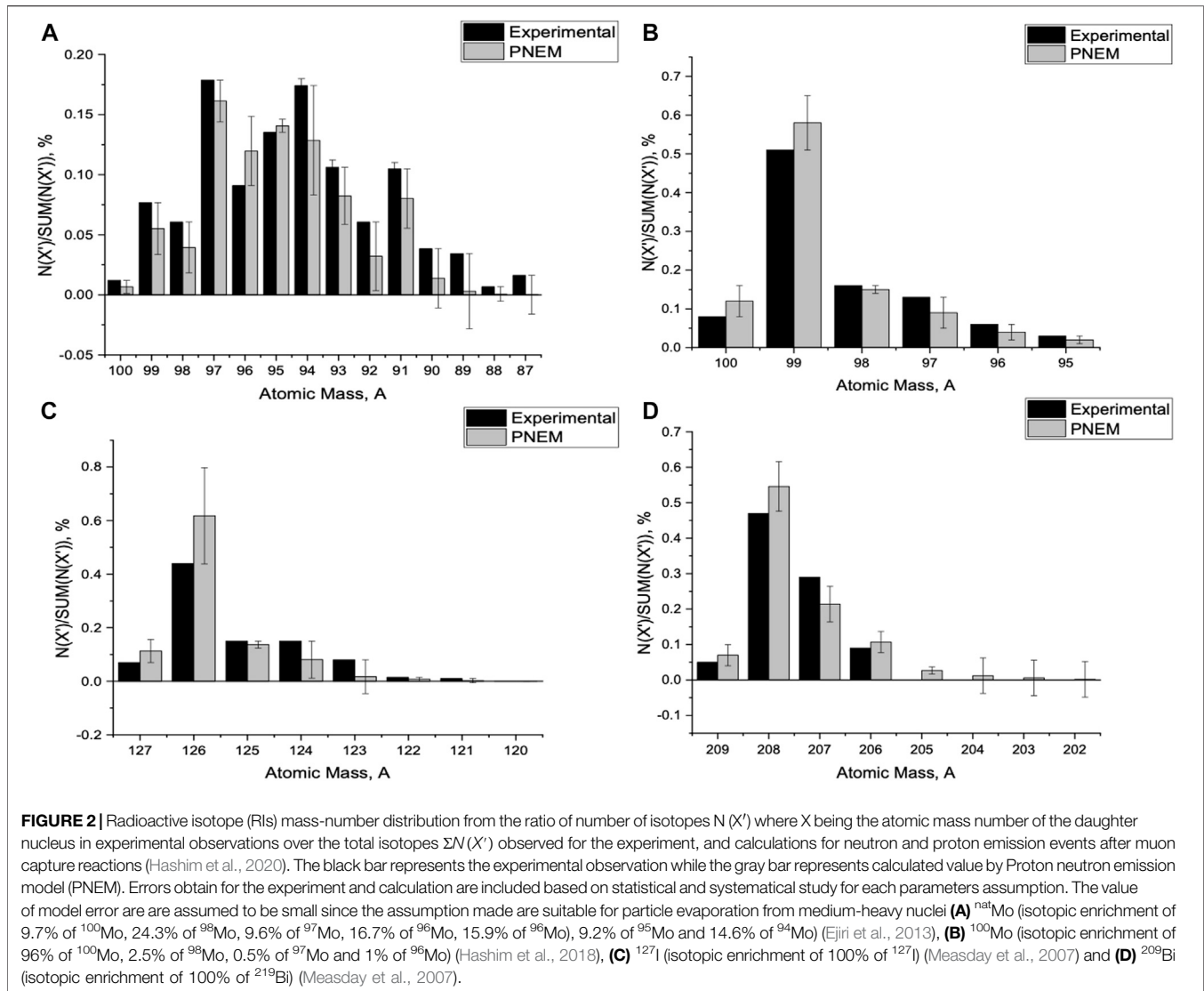
The NEM analysis on the observed mass-number distribution of  $^{100-x}\text{Nb}$  shows preferential excitation (muon giant resonance,  $\mu$ -GR) at 10–14 MeV region and a broad bump at the higher excitation region of 25–40 MeV. The NEM analyses on other medium-heavy nuclei by (Ibrahim, 2018) using experimental data from (Measday et al., 2007b; Measday et al., 2007a) show similar

features of the preferential excitation of the 10–15 MeV. This preferential excitation reflects the large branch of the one neutron ( $x = 1$ ) emission. The NME analyses on the mass-number distributions for OMCs on light nuclei show a preferential excitation around 4–8 MeV region by (Muslim, 2018) using experimental data from (Evans, 1973; Measday et al., 2007c). This observation is consistent with the calculation in (Kortelainen and Suhonen, 2004).

The strength distribution is fitted by the sum of the  $\mu$ -GR strengths of  $B_1(\mu, E_1)$  and  $B_2(\mu, E_2)$  given by

$$B(\mu, E) = \sum_{i=1,2} B_i(\mu, E_i), B_i(\mu, E_i) = \frac{B_i(\mu)}{(E - E_{Gi})^2 + (\Gamma_i/2)^2} \quad (4)$$

where  $E_{Gi}$  and  $\Gamma_i$  with  $i = 1, 2$  are the resonance energy and the width for the  $i$ th GR, and the constant  $B_i(\mu)$  is given by  $\sigma_i \Gamma_i / (2\pi)$  with  $\sigma_i$  being the total strength integrated over the excitation



energy. The obtained GR energies are 12 MeV for the first GR and 32.5 MeV for the second GR, and the widths for both GRs are 8 MeV, as shown in **Figure 1B**. Here we note the GR2 is not entirely clear beyond the statistical error and maybe a broad bump. The  $\mu$ -GR energy peaks at around 12 MeV, which is lower than the giant dipole resonance (GDR) energy of 14 MeV deduced from a photon-induced reaction (Ejiri et al., 2011). It also features a broader width of about 8 MeV compared to the 5 MeV width of the latter. This is, of course due to the additional multipole components of higher spin states excited by  $\tau^+$ , i. e.: (n, p)-type, muon capture.

Recently a proton-neutron emission model (PNEM) has been developed by considering the probability of proton emission as well (Hashim et al., 2020). The proton energy spectrum is given by  $S(E_p) = m \times \exp(-n \times E_p)$ , where  $m$  and  $n$  are the density parameter constant with the value of 0.832 MeV and  $0.163 \text{ MeV}^{-1}$ . Here, the proton is assumed to be emitted in the case of  $B_p \leq E \leq B_n$  with  $B_p$ ,  $E$  and  $B_n$  being the proton binding energy, the excitation energy and the neutron binding energy,

because the neutron decays are forbidden and gamma decays are much smaller than the proton emission even below the Coulomb barrier. The PNEM is shown schematically in **Figure 1A**. The analysis of the mass-number distribution of  $^{100-x}\text{Nb}$  shows the same GR1 and GR2 as in case of the NEM analysis. The missing Zr isotopes produced by a proton emission in experimental data are predicted by PNEM for both  $^{nat}\text{Mo}$  (Ejiri et al., 2013) and  $^{100}\text{Mo}$  (Hashim et al., 2018). The OMC strength distribution shown in **Figure 1A** reproduces the observed radioactive isotope (RI) mass distribution.

Since then, various calculations have been made for understanding the formation of the GR populated by muon capture reactions in  $98, < A < 209$  (Hashim et al., 2020). These calculations have been compared with previous experimental works by OMC on the nuclei reported in references (Measday, 2001; Measday et al., 2007b; Measday et al., 2007a). The parameters of  $E_{G1}$  and  $E_{G2}$  as a function of  $A$  are given as  $E_{G1} = 30A^{-1/5}$  and  $E_{G2} = 75A^{-1/5}$  for OMC on  $^{nat}\text{Mo}$ ,  $^{100}\text{Mo}$ ,  $^{107}\text{Pd}$ ,  $^{108}\text{Pd}$ ,  $^{127}\text{I}$  and  $^{209}\text{Bi}$ . They

are obtained from a comparison of PNEM calculations with experimental data. The observed mass distributions agree with the predictions as shown in **Figure 2**.

## 4 IMPLICATIONS FOR DOUBLE BETA DECAYS

The experimental OMC rate for  $^{100}\text{Mo}$  as a function of the excitation energy is compared with the theoretical calculation by using proton-neutron quasi-particle random-phase approximation (pn-QRPA) (Jokiniemi et al., 2019). The pn-QRPA well reproduces the GR at around 10–14 MeV with mainly low multipoles of  $J^\pi = 0^\pm, 1^\pm$  and  $2^\pm$ . The observed OMC rate is the same order of magnitude as the empirical value suggested by Primakoff (Suzuki et al., 1987) but is lower by a factor 5 than the pn-QRPA (Jokiniemi et al., 2019; Jokiniemi and Suhonen, 2019). This suggests a quenching coefficient for the axial-vector weak coupling of  $g_A^{\text{eff}}/g_A \approx 0.5$  (Jokiniemi et al., 2019; Jokiniemi and Suhonen, 2019) for  $\mu$ -NMEs, being consistent with the quenching factors for Gamow-Teller (GT) and spin dipole (SD) NMEs (Ejiri and Suhonen, 2015; Ejiri et al., 2019). Here the quenching is common in the wide momentum region (Ejiri, 2019a; Ejiri, 2019b). The quenching effects are also discussed theoretically (Menéndez et al., 2011; Suhonen, 2017; Ejiri et al., 2019).

It is interesting to note that the OMC results together with the ( $^3\text{He}, t$ ) CERs and  $\beta^\pm$  data suggest severe quenching of experimental axial-vector NMEs of  $M_i^-$  and  $M_i^+$  for multipoles of  $J^\pi = 1^+, 2^-, 3^+$  (Jokiniemi et al., 2019; Jokiniemi and Suhonen, 2019). The common quenching coefficients of  $k_{NM} = g_A^{\text{eff}}/g_A$  for them are around 0.4–0.6 with respect to the pn-QRPA. In this case, the axial-vector DBD NMEs are quenched by the coefficient  $k_{NM}^2$  and the axial-vector anti-neutrino NMEs by  $k_{NM}$ . On the other hand, the recent pn-QRPA calculations for OMC rates show no quenching (Šimkovic et al., 2020) or only weak quenching (Ciccarelli et al., 2020). The different quenching coefficients are partly due to the different model space and the different muon-wave functions used in their QRPA calculations (Jokiniemi et al., 2019; Jokiniemi and Suhonen, 2019; Ciccarelli et al., 2020; Šimkovic et al., 2020). Further studies are needed to see how pn-QRPA calculations reproduce both the relative and absolute OMC rates.

## 5 REMARKS AND PERSPECTIVES

The relative strength distribution of OMC on Mo isotopes show the  $\mu$ -GR around  $E \approx 12$  MeV consistent with the pn-QRPA calculation (Jokiniemi et al., 2019). However, the absolute strength derived from the lifetime is much smaller than the

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model value, suggesting a similar quenching of the  $M_i^+$  NMEs as the  $M_i^-$  NMEs derived from light-ion CERs. Another pn-QRPA (Šimkovic et al., 2020) shows no severe quenching. The quenching for  $\mu$  NMEs remains to be studied.

The extensive experimental programs on OMC for other nuclei of DBD and supernova anti-neutrino interests are under progress at RCNP and Paul Scherrer Institute (PSI), Switzerland, by the joint group of Joint Institute for Nuclear Research (JINR), Dubna, RCNP, Osaka and Universiti Teknologi Malaysia (UTM) (Hashim and Ejiri, 2019). The pn-QRPA theoretical calculation is also in progress at Jyväskylä (Jokiniemi et al., 2019).

Finally, it is remarked that muon capture isotope production (MuCIP) is used for producing efficiently nuclear isotopes with the atomic number  $Z-1$ , less by one than the atomic number  $Z$  of the target isotope. By using  $^A_Z X$  target isotopes, isotopes of  $^{A-1}_{Z-1} X$  are preferentially produced, and several isotopes with  $A, A-2, A-3, A-4$  are also produced (Ejiri et al., 2013; Zinatulina et al., 2019). There is potential for using them for basic and applied science. One RI used for medical checks is the  $^{99}\text{Mo}$ , which is well produced by OMC on  $^{100}\text{Mo}$ , as studied at RCNP (Ejiri et al., 2013). MuCIP is complementary to photon capture reactions where isotopes of  $^{A-1}_Z X$  are well produced (Ejiri et al., 2011; Szpunar et al., 2013). Note that ppb-level nuclei (impurities) are identified by measuring gamma rays from OMC, which are characteristic of the nuclei, as explained in (Ejiri et al., 2013).

## AUTHOR CONTRIBUTIONS

IH and HE are equally accounted for the content of the work.

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**Conflict of Interest:** The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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