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Elementary components of electroencephalography signals viewed as prime numbers

To cite this article: Amirul Aizad Ahmad Fuad *et al* 2021 *J. Phys.: Conf. Ser.* **1988** 012073

Elementary components of electroencephalography signals viewed as prime numbers

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Abstract. The disabling aspect of epilepsy disease is that it seems to be erratic and random in nature. Electroencephalogram (EEG) serves as an essential non-invasive tool to diagnose and manage epilepsy, allowing the physiological manifestations of irregular cortical excitability to be demonstrated. Certain prior EEG features related to seizure onset may facilitate seizure pattern predictions through mathematical models. A better understanding of these patterns can positively improve epilepsy management and, in turn, improve the quality of life of epilepsy patients. Thus, the goal of the current paper is to show that elementary EEG signals during a seizure can be perceived as prime numbers. First, the recorded EEG signals are written as a product of their elementary components through the Krohn–Rhodes decomposition technique. Following this, every elementary component of EEG signal is expressed in terms of a summation of their simpler parts via the Jordan–Chevalley decomposition process. Conversely, some prime numbers are decomposed similar to Jordan–Chevalley decomposition of elementary EEG signals and presented as Pseudo–Goldbach Theorem. Finally, the results demonstrate substantial evidence that the EEG signals follow a pattern similar to that of the distribution of prime numbers among positive integers.

1. Introduction

Epilepsy is a common neurological disease that has affected over 65 million people around the globe [1]. It is caused by an excessive electrical charge in certain parts of the brain resulting in loss of control over the patient's own body [2]. Patients with epilepsy can be managed through medicine or treated by surgical treatments. Unfortunately, it has been recorded that 30% of the cases where seizures cannot be managed through the existing treatment regimens (surgical procedures or medicine) [3]. Abnormal electrical activity in the brain during seizure attacks can be conveniently recorded using an electroencephalogram (EEG) [4]. For convenience, EEG signals during a seizure are written as EEG signals unless it is stated otherwise.

For decades, neuroscientists believed that epileptic seizures occur a few seconds before clinical attacks. However, recent studies of EEG recordings from patients being diagnosed for epilepsy surgery showed that seizures are developed minutes or even hours before clinical onset [5]. Clinical evidence across diverse publications indicates that epileptic seizures can be predicted [6–9]. Seizure prediction will not only facilitate the diagnosis of epilepsy but, more importantly, improve the patients suffering epilepsy to live a better life.

The goal of this paper is to demonstrate that the recorded EEG signals resemble a similar behavior of that prime numbers' distribution among positive integers and vice versa. Beyond the introduction, this paper consists of five sections. In Section 2, the recorded EEG signals are composed as a set of square matrices. In Section 3, it is shown that the elementary EEG signals contain a similar structure to that of prime numbers. Contrariwise, the prime numbers are shown to resemble the characteristic of elementary EEG signals in Section 4. Finally, the concluding remarks and possible future studies are given in Section 5.

2. Square matrix of EEG signals

The recorded EEG data of an epileptic seizure can be digitized at 256 samples per second using NicoletOne EEG software [10] and written as a set of square matrices [11]. Figure 1 illustrates a recorded EEG data on Patient A at time $t = 1$ and tabulated in Table 1.

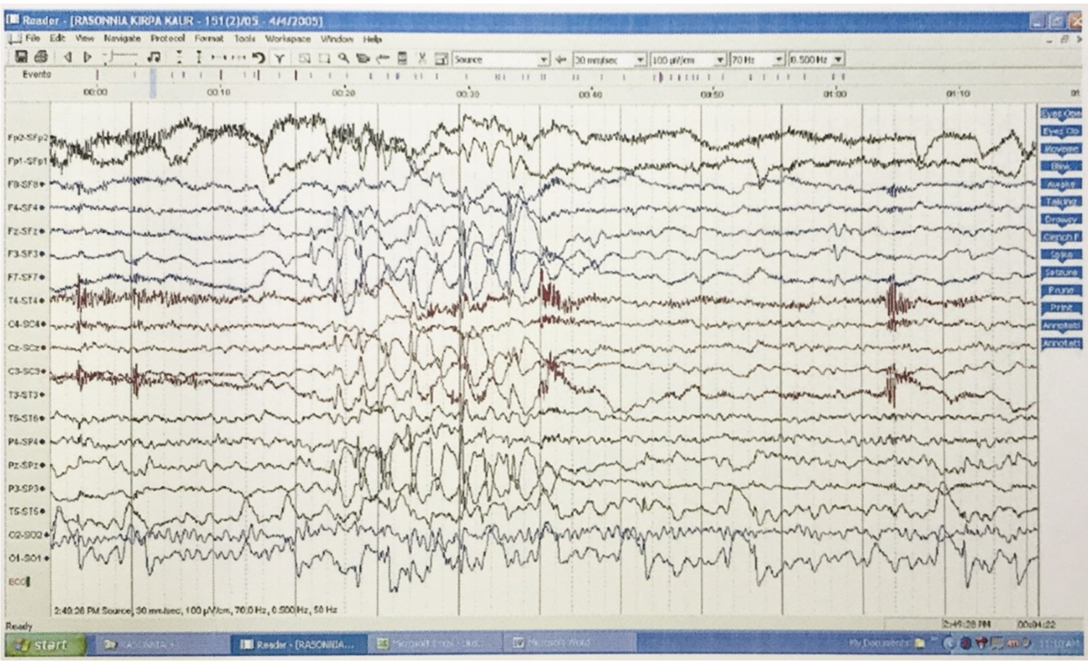

Figure 1: EEG signal recorded from Patient A at time $t = 1$ [12].

Table 1: Average Potential Difference (APD) at the sensor on $MC_{t=1}$

Sensor	X	Y	APD
F_{pz}	7.68	0	0
F_{p1}	7.3041	2.3733	52.02898438
F_{p2}	7.3041	−2.3733	6.779648438
F_3	3.3691	3.3691	19.26382813
F_4	3.3691	−3.3691	9.716523438
C_3	0	3.1812	49.30257813
C_4	0	−3.1812	16.01148438
P_3	−3.3691	3.3691	37.73242188
P_4	−3.3691	−3.3691	6.303164063
O_1	−7.3041	2.3733	3.56859375
O_2	−7.3041	−2.3733	12.700625
F_7	4.5142	6.2133	15.66375

Continued on next page

Table 1 – *Continued from the previous page*

Sensor	X	Y	APD
F_8	4.5142	−6.2133	2.464921875
T_3	0	7.68	15.07421875
T_4	0	−7.68	15.63382813
T_5	−4.5142	6.2133	4.565429687
T_6	−4.5142	−6.2133	5.765625
F_z	3.1812	0	12.84117188
C_z	0	0	8.29734375
P_z	−3.1812	0	4.4128125
O_z	−7.68	0	0

Consequently, Binjadhan [12] developed a MATLAB programming to rearrange the data in Table 1 and formed a square matrix, $A(1)$ as follows:

$$A(1) = \begin{pmatrix} 12.7006 & 3.56859 & 0 & 4.56543 & 5.76563 \\ 37.7324 & 6.30316 & 4.41281 & 49.3026 & 16.0115 \\ 15.0742 & 15.6338 & 8.29734 & 12.8412 & 19.2638 \\ 9.71652 & 15.6638 & 2.46492 & 52.0290 & 6.77965 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Therefore, for every recorded EEG data at any time t, a square matrix can be written correspondingly and form a set of square matrices denoted as $MC_n(\mathbb{R})$.

$$MC_n(\mathbb{R}) = \left\{ [\beta_{ij}(z)_t]_{n\times n} \mid \ i,j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R} \right\},$$

where, $\beta_{ij}(z)_t$ is a potential difference reading of EEG signals from a particular ij sensor at time t. In addition, Binjadhnan and Ahmad [13] transformed the set $MC_n(\mathbb{R})$ into a set of upper triangular matrices $MC_n''(\mathbb{R})$ using QR real Schur triangularization, written as follows:

$$MC_n''(\mathbb{R}) = \left\{ [\beta_{ij}(z)_t]_{n\times n} \,|\beta_{ij}(z)_t = 0, \forall 1 \le j < i \le n, i,j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R} \right\}.$$

Furthermore, the set $MC_n''(\mathbb{R})$ forms a semigroup under matrix multiplication [13]. Later, the semigroup $MC_n''(\mathbb{R})$ is decomposed via Krohn-Rhodes decomposition technique and yields its elementary components [12].

3. Elementary EEG signals as analogy of prime numbers

Prime numbers are considered to be the building blocks for any positive integers (refer to the Fundamental Theorem of Arithmetic [14]). Similarly, any invertible square matrix of EEG signals can be represented as the product of its elementary components, or precisely written as Theorem 1.

Theorem 1. [12] *Any invertible square matrix of EEG signal readings at time t can be written as a product of elementary EEG signals in one and only one way.*

Furthermore, Binjadhnan [12] introduced divisibility among square matrices of EEG signals as Definition 1.

Definition 1. [12] If $A(t)$ and $B(t)$ are EEG signals, we say that $A(t)$ divides $B(t)$, written as $A(t) \mid B(t)$, if there exists an EEG signals square matrix $M(t)$ such that $B(t) = A(t)M(t)$, where $B(t)$ is an invertible EEG signal.

Later, Ahmad Fuad and Ahmad [15] proved that any elementary EEG signals can be written as a summation of its simpler parts via Jordan-Chevalley decomposition technique.

Theorem 2. [15] *Let $D(t)$ be a diagonal matrix of EEG signals at time t. Suppose $D(t)$ is decomposed by using the Jordan-Chevalley decomposition, which produces the summation of its semisimple ($D(t)_S$) and nilpotent ($D(t)_N$) parts. Then $D(t)_S = D(t)$ and $D(t)_N = \mathbf{0}$.*

Theorem 3. [15] *Let $U(t)$ be a unipotent matrix of EEG signals at time t. Decomposing $U(t)$ using Jordan-Chevalley decomposition will produce the summation of its semisimple $U(t)_S$ and nilpotent $U(t)_N$ parts. Then, $U(t) = U(t)_S + U(t)_N$ and $U(t)_S = I$, where I is the identity matrix.*

Further observation revealed that the 'decomposition' similar to that of Jordan-Chevalley decomposition of elementary EEG signals (Theorem 2 and 3) could be found in prime numbers as well. In other words, some primes can be represented as a sum of two primes, and the representation is unique which is discussed in section 4.

4. Pseudo–Goldbach Theorem

Theorem 4. *Let $p > 2$ be a prime such that $p = a + b$ with $a, b \in \mathbb{Z}^+$. Then, either one of the following is true:*

(i) Either a or b is prime;
(ii) Both a and b are primes;
(iii) Both a and b are not primes.

Proof Let $p > 2$ be a prime such that $p = a + b$ with $a, b \in \mathbb{Z}^+$. Therefore, p can be listed, as shown in Table 2.

Table 2: List of possible p as a sum of integers a and b where $a, b \geq 0$

p	$=$	a	$+$	b
	$=$	1		p
	$=$	2		$p-1$
	$=$	3		$p-2$
p	$=$	$\vdots$	$\vdots$	$\vdots$
	$=$	$p-2$		3
	$=$	$p-1$		2
	$=$	p		1

The list is written by arranging a by ascending order, and b by descending order, where $a, b \in \{1, 2, \ldots, p\}$. From the list, it can be seen that the column a may contain primes or not, similar to column b, as shown in Figure 2.

Figure 2: The possibility of a and b being a prime or not.

By rule of product, there are two ways of getting a (prime or not prime) and two ways of getting b (prime or not prime), which produce four combinations, namely

(i) a or b are primes;
(ii) a and b are not primes;
(iii) a is a prime, and b is not a prime;
(iv) a is not a prime, and b is a prime.

In short, it can be summarized that

(i) a or b is prime;
(ii) a and b are primes;
(iii) a and b are not primes.

Moreover, Theorem 5 states the uniqueness of any prime p when it is presented as a summation of two primes, p_1 and p_2.

Theorem 5. *Let p, p_1 and p_2 be primes such that $p = p_1 + p_2$. The presentation is unique except for the order.*

Proof Let p, p_1, p_2 be primes such that $p = p_1 + p_2$. Therefore, p can be listed as below:

p	=	**a**	+	**b**
	=	1		p
	=	2		$p-1$
	=	3		$p-2$
p	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	=	$p-2$		3
	=	$p-1$		2
	=	p		1

From the list, it is evident that we have

$$p = p_1 + p_2 = p_2 + p_1.$$

Hence, the presentation is unique, except for the order.

Now, the main result of this paper is presented as Theorem 6. It is slightly different from Goldbach conjecture in the sense that Theorem 6 states that for any prime p that can be represented as a sum of two primes, one of them must be even (2). More importantly, the summation is unique, as much as the Jordan–Chevalley decomposition of elementary EEG signals is unique. In addition, the smallest prime number, 2, is considered as the semisimple matrix with respect to Theorem 2 and Theorem 3.

Theorem 6 (Pseudo-Goldbach Theorem). *If $p = p_1 + p_2$ and $p \neq p_1 \neq p_2$ and all of them are primes, then p_1 and p_2 must be even (2) and odd.*

Proof Let p, p_1 and p_2 be primes such that $p = p_1 + p_2, p \neq p_1 \neq p_2$ and $p > 2$. There are three cases to be considered:

CASE I (both even):
Let $p_1 = 2n$ and $p_2 = 2m$, where $m, n \in \mathbb{Z}^+$. Therefore, $p_1 + p_2 = 2n + 2m = 2(n + m)$ is even. The only even prime is 2 $\implies n + m = 1$ which is not possible.

CASE II (both odd):
Let $p_1 = 2n + 1$ and $p_2 = 2m + 1$, where $m, n \in \mathbb{Z}^+$. Therefore

$$\begin{aligned} p_1 + p_2 &= (2n + 1) + (2m + 1) \\ &= 2(n + m + 1) - \text{which is an even prime} \end{aligned}$$

The only even prime is 2 $\implies n + m = 0$ which is not possible.

CASE III (one of them is even):
Let $p_1 = 2n$ and $p_2 = 2m + 1$, where $m, n \in \mathbb{Z}^+$. Therefore

$$\begin{aligned} p_1 + p_2 &= 2n + (2m + 1) \\ &= 2(n + m) + 1 > 2. \end{aligned}$$

$\therefore$ Hence, p_1 and p_2 must be even and odd.

To illustrate Theorem 6, every possible outcome for p written in the form of $p = a + b$ such that $3 \leq p \leq 100$ with $a,\ b \in \mathbb{Z}^+$ is determined using MATLAB programming (refer to Appendix). Their uniqueness is apparent and highlighted in Table 3. If several numbers can be represented as a sum of two primes, then

(i) one of them must be 2;

(ii) and if it does, then the presentation is unique, as highlighted in Table 3.

Table 3: List of possible $p = a + b$, where $3 < p < 100$, $a, b \in \mathbb{Z}^+$

5	$= 1 + 4$	7	$= 1 + 6$	13	$= 1 + 12$	19	$= 1 + 18$
	$= \boxed{2 + 3}$		$= \boxed{2 + 5}$		$= \boxed{2 + 11}$		$= \boxed{2 + 17}$
			$= 3 + 4$		$= 3 + 10$		$= 3 + 16$
					$= 4 + 9$		$= 4 + 15$
					$= 5 + 8$		$= 5 + 14$
					$= 6 + 7$		$= 6 + 13$
							$= 7 + 12$
							$= 8 + 11$
							$= 9 + 10$

$$
\begin{aligned}
31 &= 1+30 \\ &= \boxed{2+29} \\ &= 3+28 \\ &= 4+27 \\ &= 5+26 \\ &= 6+25 \\ &= 7+24 \\ &= 8+23 \\ &= 9+22 \\ &= 10+21 \\ &= 11+20 \\ &= 12+19 \\ &= 13+18 \\ &= 14+17 \\ &= 15+16
\end{aligned}
$$

$$
\begin{aligned}
43 &= 1+42 \\ &= \boxed{2+41} \\ &= 3+40 \\ &= 4+39 \\ &= 5+38 \\ &= 6+37 \\ &= 7+36 \\ &= 8+35 \\ &= 9+34 \\ &= 10+33 \\ &= 11+32 \\ &= 12+31 \\ &= 13+30 \\ &= 14+29 \\ &= 15+28 \\ &= 16+27 \\ &= 17+26 \\ &= 18+25 \\ &= 19+24 \\ &= 20+23 \\ &= 21+22
\end{aligned}
$$

$$
\begin{aligned}
61 &= 1+60 \\ &= \boxed{2+59} \\ &= 3+58 \\ &= 4+57 \\ &= 5+56 \\ &= 6+55 \\ &= 7+54 \\ &= 8+53 \\ &= 9+52 \\ &= 10+51 \\ &= 11+50 \\ &= 12+49 \\ &= 13+48 \\ &= 14+47 \\ &= 15+46 \\ &= 16+45 \\ &= 17+44 \\ &= 18+43 \\ &= 19+42 \\ &= 20+41 \\ &= 21+40 \\ &= 22+39 \\ &= 23+38 \\ &= 24+37 \\ &= 25+36 \\ &= 26+35 \\ &= 27+34 \\ &= 28+33 \\ &= 29+32 \\ &= 30+31
\end{aligned}
$$

$$
\begin{aligned}
73 &= 1+72 \\ &= \boxed{2+71} \\ &= 3+70 \\ &= 4+69 \\ &= 5+68 \\ &= 6+67 \\ &= 7+66 \\ &= 8+65 \\ &= 9+64 \\ &= 10+63 \\ &= 11+62 \\ &= 12+61 \\ &= 13+60 \\ &= 14+59 \\ &= 15+58 \\ &= 16+57 \\ &= 17+56 \\ &= 18+55 \\ &= 19+54 \\ &= 20+53 \\ &= 21+52 \\ &= 22+51 \\ &= 23+50 \\ &= 24+49 \\ &= 25+48 \\ &= 26+47 \\ &= 27+46 \\ &= 28+45 \\ &= 29+44 \\ &= 30+43 \\ &= 31+42 \\ &= 32+41 \\ &= 33+40 \\ &= 34+39 \\ &= 35+38 \\ &= 36+37
\end{aligned}
$$

The results point to the resemblance to the elementary EEG signals and are summarized in Table 4.

Table 4: Compatibility of EEG signals to positive integers.

	EEG signals	**Positive integers**
Divisibility	Definition 1	**Definition 2.** [14] If a and b are integers, we say that a divides b if there is an integer c such that $b = ac$.
Unique factorization theorem.	Theorem 1.	Fundamental Theorem of Arithmetic [14].
Building blocks	Diagonal EEG signals group and the unipotent EEG signals group.	Prime numbers.
Highlights	Jordan-Chevalley decomposition of EEG signals (Theorem 2 and Theorem 3).	Pseduo-Goldbach Theorem (Theorem 6).

5. Conclusion

It is shown that the elementary EEG signals can be viewed as prime numbers, and vice versa. Theorem 6 is the manifestation of the Jordan-Chevalley decomposition of elementary EEG signals observed in prime numbers. The results provide a supporting evidence that the EEG signals during seizures behave similar to that of prime numbers, indicating that it contains a pattern. This pattern can be studied to facilitate the epileptic seizure's prediction.

Acknowledgments

This research is supported by Fundamental Research Grant Scheme FRGS PY/2020/05155 awarded by Ministry of Education, Malaysia. The first author gratefully acknowledge financial support from Yayasan Sultan Iskandar.

Appendix

```
N=100;
A=primes(N);
A=A(1,2:end);
B=cell(length(A),1);
j=0;

for i=1:length(A)
    l=floor(A(i)/2);
    ii=1:l;
    C=zeros(length(ii),2);
    C(:,1)=ii';
    a=A(i);
    a=repmat(a,length(ii),1);
    C(:,2)=a-C(:,1);
    B{i}=C;
end
```

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