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Distributed Adaptive Cooperative Control With Fault Compensation Mechanism for Heterogeneous Multi-Robot System

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ABSTRACT In this paper, a distributed adaptive consensus law with fault compensation for a heterogeneous multi-robot system (MRS) is proposed. The design paradigm adopted in this work involves a leader-following cooperative algorithm featuring two distinct adaptive coupling gains to compensate for multiple additive time-varying faults. Exacerbating the situation, the follower robots commissioned in the leader-following mission are non-identical due to their dynamic characteristic as normally exist in a physical setup. The capability of the proposed scheme is investigated and compared with the other two recent works in two facets; one is to gauge how the algorithm is able to mitigate faults of varying nature in the presence of heterogeneous robot(s) while maintaining the platoon formation during the leader-following task; two is the ability to cope with subsequent topology reconfiguration. The stability and the robustness of the proposed scheme against bounded time-varying faults are proven using rigorous Lyapunov analysis. The proposed control strategy exempts the use of an observer or estimator, thereby simplifies the synthesis and implementation on mobile robots. The simulation results of the proposed adaptive consensus law demonstrate the best performance as compared to the other two recent works in the presence of multiple faulty robots.

INDEX TERMS Cooperative control, multi-robot system, fault-tolerant, adaptive gain.

I. INTRODUCTION

In recent decades, there have been a plethora of research studies on the cooperative control of multi-agent systems (MASs) [1]–[4]. The application of MASs has gained interest especially in multi-robot systems (MRSs) whereby a variety of automated applications such as surveillance, search and rescue, and exploration are notable examples. Without the loss of generality, the term MRS is referred in this paper to address a practical concern involving a platoon configured MASs which is not always dynamically homogeneous as illustrated in Fig. 1. These are deployed mostly in autonomous modes with a minimum of human supervision to travel autonomously in a strategic group formation or alignment in various geographic locations and under various terrain conditions. For agility and flexibility in carrying out a remote mission, each of the so-called agent robots is equipped with different on-board instrumentation, thereby exhibiting distinctive dynamics. Such heterogeneous characteristics pose a great challenge in controlling all the robots in a network to work cooperatively [5], [6]. When deploying a cooperative MRS autonomously, the mission time of the MRS is often governed by the finite energy reserve on board. This can be remedied by careful path planning in the mission field to reduce surplus travel. Such method requires terrain description, i.e., *a priori* information, which often is unavailable.

It is imperative to preserve the integrity of the platoon formation of an MRS by ensuring that any faults occurrence can be effectively compensated. Faults in question can be

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FIGURE 1. A platoon formation of heterogeneous multi-robot systems.

either emanating from individual robot agent or it can be environmentally induced during the autonomous mission. According to Chen, a 'fault' is described as an unexpected change in the system's operation [7].

Many effective fault-tolerant control methods (FTCs) have been extensively investigated for MRSs to guarantee system stability at an acceptable level. One possible solution for achieving MRS coordination in the presence of faulty robot(s) is to locally modify the control input of the faulty robot(s) [8]. In general, FTC solutions can be divided into two main categories: passive and active. A passive FTC refers to a control design that is robust to a fault occurrence without any modification of the control system, and this method is well-suited for low-dimensional scale application [9]. An active FTC, on the other hand, allows controller configuration for fault detection, estimation, compensation, and isolation [10].

For the active FTC solutions, many effective methods based on observer and estimator have been presented in the literature [2], [4], [10]–[13]. However, the study of FTC for MRSs is relatively new [14]. For MRSs, the distributed FTC observer-based is designed for a leader-follower consensus problem with constant additive faults and multiplicative faults in [15] and [16], respectively. However, the solutions presented in that literature are generally subjected to two significant constraints as follows: (1) depending on the nature of the system dynamics in question, the observer design may require some states as inputs, and it is important to have a state measurement that is free from noise; (2) certain estimator designs may require a persistent excitation condition for convergence, which is not always achievable in practice. Recently, several published studies explored the application of neural network (NN) as estimator in the FTC. The NN has self-learning capability, which is able to estimate unknown components of the system including faults [3]. In [17], the NN is proposed to compensate faults for homogeneous MAS. Nevertheless, since NN, depending on the designer's neural nodes choice, is computationally exhaustive, event-control is employed in conjunction with NN to reduce the computational burden [3], [18].

With an increase in the number of agents, the fault compensation becomes more challenging as more data are exchanged within the system [14]. In the absence of estimation, adaptive control is also an effective tool with proven application in the FTC for both linear and nonlinear single systems [19]. In a relatively large network of agents, it is possible to design an adaptive control by adjusting the coupling gain adaptively so that the system can counteract faults to fulfills the desired objective. In [20]–[23], the robustness and convergence of an MAS are improved by selecting a sufficiently strong coupling gain. The work in [24] infers that strong coupling gain and a large number of agents imply synchronization robustness in the MAS against heterogeneity. In [25]–[29], an extensive study on distributed adaptive consensus was presented with linear homogeneous MASs with and without considering faults. In [30], a distributed adaptive consensus law was designed for a heterogeneous MAS with scalar faults, which required all followers to know the leader dynamics to compute their control inputs. In [31], a robust adaptive consensus protocol was presented with the use of a threshold update protocol (TUP), in which exchanging information with neighbors is mandatory, thus limiting the capability of the proposed law to the undirected topology.

Motivated by the abovementioned studies, this paper proposes an adaptive consensus law for a linear heterogeneous MRS with time-varying faults, where the MRS can be regarded as a nontrivial nonlinear system. Two distinctive adaptive coupling gains approach is used to compensate for the fault existence without requiring any extra *a priori* information about faults. This method exemplifies a robust approach that is pragmatic since observer or estimator design is not needed. A unidirectional communication approach is considered to ensure the practicality of the proposed solution through minimum power consumption in communication activity. Compared to the existing works in FTC methods, the proposed control method has several key contributions: (1) a new adaptive consensus control is designed for a cooperative heterogeneous MRS under the presence of multiple additive time-varying faults; (2) An adaptive consensus law is designed based on two distinct adaptive coupling gains, that rely only on the relative state information and the agent's own dynamics, both of which are practically accessible; (3) the novel adaptive consensus law is designed using a Lyapunov analysis to compensate for the effects of the fault. In addition, not only the proposed scheme is robust against fault, the scheme is also evidently robust against changes in the inter-agent communication whereby deliberate re-configuration in communication was introduced between robots to testify its prowess. This paper is organized as follows. Section II summarizes the problem formulation and provides some basic notation of the vectors used in the rest of the paper. Section III introduces the distributed adaptive consensus with a fault-tolerant mechanism. Section IV shows the results of numerical simulations, and finally, Section V presents some conclusions and speculates on future works.

II. PROBLEM FORMULATION

Cooperative control of a platoon consisting of $N+1$ heterogeneous robots moving in a straight line on a flat or rough surface along an x-axis with a constant velocity is considered in this work. The MRS leader is indexed by 0, and the N followers are indexed from 1 to N. The control objective is to ensure that all MRS agents (robots) are moving at the same velocity as the leader while keeping a constant distance between one another to avoid collisions.

The MRS is said to have achieved the desired control objectives with the lead robot having constant velocity if for any given bounded initial states

$$\begin{cases} \lim\limits_{k\to\infty} \|p_{xi}(k) - p_{x0}(k)\| = d_{xi0} \\ \lim\limits_{k\to\infty} \|v_{xi}(k) - v_{x0}(k)\| = 0 \end{cases} \tag{1}$$

where d_{xi0} is a pre-specified distance vector on the x-axis between the followers and the leader that remains constant for all i and $p_{xi}(k)$ and $v_{xi}(k)$ are the position and velocity along the x-axis, respectively, for $i = 0, 1, \ldots, N$. The leader robot moves with constant velocity under the steady-state condition (i.e., $v_{x0} = 1$).

A. GRAPH THEORY

Suppose that the information links among the follower robots within the platoon are unidirectional and there exists at least one directional link from the leader to the followers. Consider a directed graph $\mathbb{G} = (\mathcal{V}, \mathcal{E})$ with a non-empty set of nodes $\mathcal{V} = \{0, 1, \ldots, N\}$, a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and the associated adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$. An edge rooted at the i-th node and ended at the j-th node is denoted by (i, j), which means that information can flow from robot i to robot j. a_{ij} is an unweighted edge (j, i) and $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$. Robot j is called a neighbor of robot i if $(j, i) \in \mathcal{E}$. The in-degree matrix is defined as $\mathcal{D} = diag\{d_{ij}\} \in \mathbb{R}^{N\times N}$ with $d_{ij} = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix $L \in \mathbb{R}^{N\times N}$ of $\mathbb{G}$ is defined as $L = \mathcal{D} - \mathcal{A}$. If the i-th follower observes the leader then an edge $(0, i)$ between them is said to exist with the pinning gain $g_i > 0$. We denote the pinning matrix as $G = diag\{g_i\} \in \mathbb{R}^{N\times N}$, where g_i is the pinning gain with $g_i > 0$ if and only if robot i can receive information directly from the leader robot; otherwise, $g_i = 0$. It is assumed that at least one follower is connected to the leader. Denote $\mathcal{H} = L + G$, and all the eigenvalues of the matrix $\mathcal{H}$ are denoted by λ_i for $i = 0, 1, \ldots, N$ are real and positive [32], [33].

B. DISTRIBUTED HETEROGENEOUS MRS MODEL

Consider a group of $N+1$ heterogeneous MRS agents, consisting of N followers and a leader that moves in a 2-D plane; the general dynamics of each robot in the platoon can be expressed as

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + f_i(k), \quad i = 0, \ldots, N \tag{2}$$

where $x_i(k) \in \mathbb{R}^n$ is the state, $u_i(k) \in \mathbb{R}^m$ is the control input, and $f_i(k) \in \mathbb{R}^n$ is the signal indicating the occurrence of faults in the i-th follower. This means that $f_i(k) \neq 0$ when node i is subject to a fault at k, and index $i \in \{0, 1, \ldots, N\}$ is the index of i-th robot in a network. We assume that the leader is fault-free. The introduced fault signal can be viewed as a system fault in the dynamics of (2), i.e., actuator faults which may be caused by physical effects or cyberattacks over the communication network [34]. Moreover, the system dynamics in (2) can be considered as those of a nonlinear system since the fault being introduced here is time-varying in nature.

To further elucidate the heterogeneity of the MRS system considered in this paper, without loss of generality, a particular basic structure of the dynamics of heterogeneous MRS agents is considered. Let position, $p_i(k) = [\, p_{xi} \; p_{yi} \,]^{\mathrm{T}} \in \mathbb{R}^2$ and velocity, $v_i(k) = [\, v_{xi} \; v_{yi} \,]^{\mathrm{T}} \in \mathbb{R}^2$ with $x_i = [\, p_i \; v_i \,]^{\mathrm{T}}$; then, the differential equation in (2) can also be written as a double integrator form whereby $\dot{p}_i = v_i$ and $m_i\dot{v}_i = -v_i + u_i$, where m_i represents the mass of the i-th robot. Let the system and input matrices (A_i, B_i) be described as

$$A_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1/m_i & 0 \\ 0 & 0 & 0 & -1/m_i \end{bmatrix}$$
$$B_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m_i & 0 \\ 0 & 1/m_i \end{bmatrix} \tag{3}$$

The heterogeneity introduced in the i-th agent is different compared to the work in [34] because each robot in the network may exhibit different masses due to the variety of mobile platforms deployed in an autonomous mission. The inertial time lag in the differential acceleration or jerk in [30] may not be suitable for the smaller rigid body dynamics of the mobile robot exemplified in this paper.

Even though the considered MRS system is specific, the main concept of this paper is applicable to other types of MAS systems or other cooperative control problems since the proposed adaptive law is only dependent on neighbors and their local information, i.e., the agents' own dynamics and relative state information.

C. FAULT MODEL

Any type of fault at any level of magnitude may immediately or gradually degrade the overall MRS performance, which leads to instability and eventually collision among the members within the platoon. Therefore, fault compensation should be investigated in designing a practical consensus law. In a case where a fault with "high" severity occurred among the MRS agent and reaches a magnitude beyond the acceptable threshold, the mission is suspended if there is no change to the current robot coordination setting. Nevertheless, to ensure that the mission can continue and complete the objective, isolation of the faulty robot within the MRS and reconfiguration of the robot's coordination setting may be required, which leads to alteration of the current communication topology.

In this paper, the considered additive fault is represented by a sudden unintended acceleration or deacceleration of a robot, which often can be due to mechanical, electronic, or software-related problems. Furthermore, the fault could transpire momentarily or continuously as represented by

Intermittent fault at time k = true if at the steady-state $v_{xi} \neq v_{x0}$ and $v_{xi} < \bar{v}_{xi}$,

Permanent fault at time k = true if at the steady-state, $v_{xi} \neq v_{x0}$ and $v_{xi} \geq \bar{v}_{xi}$

where $\bar{v}_{xi}$ is the isolation threshold. Each robot will observe its control input once per control step since the first consensus convergence is achieved during the mission. To ensure a stable, reliable, and robust MRS, an accurate measurement of the maximum allowable or tolerated fault magnitude should be quantified before the fault isolation and reconfiguration can be designed.

The main objective of the proposed consensus adaptive law is to minimize the fault strength produced by any follower robot(s) in the MRS. The magnitude of the adaptive parameters in the consensus law increases or decreases to reduce the fault magnitude at every step. In the proposed consensus law, two distinct adaptive coupling gains are employed to provide better consensus convergence for the MRS. For a continuous or permanent fault signal, the isolation threshold could be initially specified, which leads to exclusion of the faulty robot(s) and reconfiguration of the MRS coordination setting for the remaining healthy and semi-healthy robots to continue and complete the assigned mission. The semi-healthy robot is the robot with a fault magnitude below the isolation threshold value within a particular interval.

The communication graph $\mathbb{G}$ among the $N+1$ agents is assumed to satisfy the following assumptions.

Assumption 1: The graph $\mathbb{G}$ contains a directed spanning tree with the leader as the root node. The graph $\mathbb{G}$ is connected and there exists at least one path from the leader to the follower.

The stated assumption here is to highlight that a directed tree communication graph is assumed [32], [33]. A platoon of heterogeneous robots is aligned in a queue form as illustrated in Fig. 1.

Assumption 2: The pairs (A_i, B_i) are controllable and stabilizable, for $i = 0, 1, \ldots, N$.

This assumption is necessary for the state feedback control design and sufficient for the existence of a positive definite matrix, P.

Assumption 3: The desired trajectory is the Lipschitz condition and bounded, which exists a positive real constant κ such that, for all real x_1 and x_2,

$$|f(x_2) - f(x_1)| \leq \kappa\, |x_2 - x_1|$$

This assumption is required to ensure that the trajectory for all robots is continuously differentiable for (4) to function [35].

Assumption 4: The additive fault term $f_i(k)$ is bounded such that $\|f_i(k)\|_\infty < f_m$, for $i = 1, \ldots, N$ where f_m is a known positive constant.

Lemma 1 ([36]–[40]): Under Assumption 1, the matrix $L + G$ is symmetric and positive definite.

Lemma 2 ([28], [41]): If a and b are nonnegative real numbers and p and q are positive real numbers such that $1/p + 1/q = 1$, then $ab \leq a^p/p + b^q/q$, and the equality holds if and only if $a^p = b^q$.

Lemma 3 [42]: The Cauchy-Schwartz inequality states that the absolute value of the vector dot product is always less than or equal to the product of the vector norms $\left|a^{\mathrm{T}} b\right| \leq \|a\|\,\|b\|$.

III. DISTRIBUTED ADAPTIVE CONSENSUS DESIGN

The proposed control objective is to ensure that all follower robots maintain the same velocity as the leader while keeping a constant distance to avoid collision during and after the unexpected fault occurrence at any follower robot. Fig. 2 illustrates the framework of the proposed adaptive scheme.

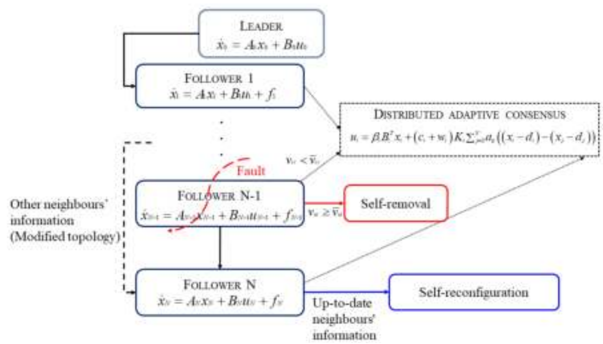


FIGURE 2. **The distributed leader-follower adaptive consensus algorithm.**

Taking the relative states of neighboring agents, the cooperative control objectives of the heterogeneous MRS in (2) and (3) are achieved when the following adaptive control law is applied to the i-th follower robot for all i.

$$\begin{aligned}
u_i &= \beta_i B_i^{\mathrm{T}} x_i + (c_i + w_i)\, K_i \sum\nolimits_{j=0}^{N} a_{ij} \left((x_i - d_i) - (x_j - d_j)\right) \\
\dot{c}_i &= -\varphi_i \,(c_i - 1)^2 \\
&\quad + \sum\nolimits_{j=1, i\neq j}^{N} \Big[a_{ij} \left((\xi_i - \xi_j)^{\mathrm{T}} + g_i \xi_i^{\mathrm{T}} \right) \\
&\quad \times \Gamma_i \left((\xi_i - \xi_j) + g_i \xi_i \right) \Big] \\
\dot{\beta}_i &= r x_i^{\mathrm{T}} B_i K_i \sum\nolimits_{j=1, i\neq j}^{N} \left[a_{ij} \left(\xi_i - \xi_j\right) + g_i \xi_i \right]
\end{aligned} \tag{4}$$

Let the consensus error $\xi_i = x_i - x_0 - d_i$ for $i = 1, \ldots, N$, where $d_i = [\, d_{xi0}\ d_{yi0}\ 0\ 0\,]^{\mathrm{T}}$ indicates the desired static formation vector, c_i and β_i denotes the adaptive coupling gains associated with $c_i(0), \beta_i(0) \geq 1$, K_i and Γ_i are the feedback gain matrices, w_i is the smooth function and r is a small positive constant that needs to be determined later. The control law at agent i is calculated using the most recently received states for the position and velocity of itself and its neighbors. Two distinct adaptive gains are employed in control input, u_i to further improve the consensus convergence and tracking. This protocol aims to ensure all robots reach consensus in position and velocity and that $p_{xi} \rightarrow p_{x0} + d_{x_{i0}}, p_{yi} \rightarrow p_{y0} + d_{y_{i0}}, v_{xi} \rightarrow v_{x0}$, and $v_{yi} \rightarrow v_{y0}$.

The closed-loop dynamics of the heterogeneous MRS can be obtained by substituting (4) into (2) as follows:

$$\dot{x}_i = \left(A_i + \beta_i B_i B_i^{\mathrm{T}}\right) x_i + (c_i + w_i) B_i K_i \sum_{j=0}^{N} a_{ij} \left(\xi_i - \xi_j\right) + f_i \quad (5)$$

Based on (5), the closed-loop consensus error dynamics, $\dot{\xi}_i$ can be expressed as

$$\dot{\xi}_i = A_i\xi_i + \beta_i B_i B_i^{\mathrm{T}} x_i + (c_i + w_i) B_i K_i [\sum_{j=1}^{N} a_{ij} \left(\xi_i - \xi_j\right) + g_i (\xi_i - \xi_0)] + f_i + (A_i - A_0)x_0 - B_0 u_0 \quad (6)$$

where $\dot{d}_i = [0\ 0\ 0\ 0]^{\mathrm{T}}$ and $A_i d_i = 0$, which yields the network-based error dynamics

$$\dot{\xi} = \bar{A}\xi + (\bar{c} + \bar{w}) \bar{B}\bar{K} (L + G) \xi + \bar{\beta}\bar{B}\bar{B}^{\mathrm{T}} x + \bar{f} + \left(\bar{A} - I_N \otimes A_0\right) (1 \otimes x_0) - (I_N \otimes B_0) (1 \otimes u_0) \quad (7)$$

where $\bar{A} \triangleq diag(A_1, \ldots, A_N)$, $\bar{B} \triangleq diag(B_1, \ldots, B_N)$, $\bar{K} \triangleq diag(K_1, \ldots, K_N)$, $\bar{f} = [f_1\ f_2\ \ldots f_N\ ]^{\mathrm{T}}$, $\bar{c} \triangleq diag(c_1, \ldots, c_N)$, $\bar{w} \triangleq diag(w_1, \ldots, w_N)$, and $\bar{\beta} \triangleq diag(\beta_1, \ldots, \beta_N)$.

Remark 1: Note that the consensus law is based solely on the dynamics of the agent itself and the information of the neighboring agent. The formulation of agent consensus law, u_i is inspired by the adaptive strategy in [28] and [30]. In comparison to the adaptive law in [28] and [30], the novel adaptive law has two distinct features. First, unlike the adaptive protocol in [28], which employs a single term and an adaptive coupling gain, the adaptive protocol in (4) introduces two terms and two distinct adaptive coupling gains, c_i and β_i. As a consequence, the errors in the synchronization and control input are effectively suppressed, thus improving the convergence. Second, contrary to the adaptive protocol in [30], which is dependent on the leader's dynamics and uses a combination of constant and adaptive gains in two separate terms to further attenuate the heterogeneity of the agents, the proposed adaptive strategy (4) introduces a law that is independent of the leader's dynamics and has two distinct adaptive gains in two separate terms, allowing the MRS to be robust against time-varying and "high" severity faults while also improving the execution characteristics of the distributed MRS.

The following theorem presents a result on the design of the robust adaptive consensus law.

Theorem 1 *For a graph satisfying Assumption 1, the N robots in (2) and (3) for $i = 1, \ldots, N$ reach consensus under a leader-follower based protocol (4) with two distinct adaptive gains, c_i and β_i, and gain matrices, $K_i = -B_i^{\mathrm{T}} P_i$ and $\Gamma_i = P_i B_i B_i^{\mathrm{T}} P_i$, $w_i = \xi_i^{\mathrm{T}} P_i \xi_i$ where $P_i > 0$ is a symmetric matrix, which is a solution to the algebraic Riccati equation (ARE),*

$$\bar{A}\bar{P} + \bar{P}\bar{A}^{\mathrm{T}} - 2\bar{B}\bar{B}^{\mathrm{T}} < -\bar{Q} \quad (8)$$

where $\bar{P} \triangleq diag(P_1, \ldots, P_N)$, $\bar{Q} \triangleq diag(Q_1, \ldots, Q_N)$, and Q_i is a symmetric positive definite matrix and the coupling gains,c_i and β_i converges to finite values for $k \to \infty$.

Proof: Consider the following Lyapunov function

$$V = \sum_{i=1}^{N} \xi_i^{\mathrm{T}} P_i \xi_i + (c_i - \gamma_{i1})^2 + (\beta_i - \gamma_{i2})^2 \quad (9)$$

where γ_{i1} and γ_{i2} are positive scalars to be determined later. From (9), since $P_i > 0$, it can be seen that V is positive definite with respect to ξ_i, c_i, and β_i for $i = 1, 2, \ldots, N$. The time derivative of V along the trajectory of (6) can be obtained as

$$\begin{aligned}\dot{V} &= 2\sum_{i=1}^{N} \xi_i^{\mathrm{T}} P_i [A_i\xi_i + \beta_i B_i B_i^{\mathrm{T}} x_i \\ &\quad + (c_i + w_i) B_i K_i \sum_{j=1, i\neq j}^{N} [a_{ij} \left(\xi_i - \xi_j\right) + g_i\xi_i] + f_i \\ &\quad + (A_i - A_0)x_0 - B_0 u_0] \\ &\quad + 2\sum_{i=1}^{N} (c_i - \gamma_{i1}) \dot{c}_i + 2\sum_{i=1}^{N} (\beta_i - \gamma_{i2}) \dot{\beta}_i \end{aligned} \quad (10)$$

By substituting $K_i = -B_i^{\mathrm{T}} P_i$, with the adaptive law, $\dot{c}_i$ and $\dot{\beta}_i$ defined in (4), it is obvious to arrive at the following:

$$\begin{aligned}\dot{V} &= 2\sum_{i=1}^{N} [\xi_i^{\mathrm{T}} P_i A_i \xi_i + \beta_i \xi_i^{\mathrm{T}} P_i B_i B_i^{\mathrm{T}} x_i \\ &\quad - (c_i + w_i) \xi_i^{\mathrm{T}} P_i B_i B_i^{\mathrm{T}} P_i [\sum_{j=1, i\neq j}^{N} a_{ij} \left(\xi_i - \xi_j\right) + g_i\xi_i] \\ &\quad + \xi_i^{\mathrm{T}} P_i f_i + \xi_i^{\mathrm{T}} P_i (A_i - A_0) x_0 - \xi_i^{\mathrm{T}} P_i B_0 u_0] \\ &\quad + 2\sum_{j=1, i\neq j}^{N} \sum_{i=1}^{N} [-\varphi_i (c_i - \gamma_{i1}) (c_i - 1)^2 \\ &\quad + (c_i - \gamma_{i1}) [a_{ij} \left(\left(\xi_i - \xi_j\right)^{\mathrm{T}} + g_i \xi_i^{\mathrm{T}}\right) \Gamma_i \left(\left(\xi_i - \xi_j\right) + g_i\xi_i\right)]] \\ &\quad + 2\sum_{i=1}^{N} (\beta_i - \gamma_{i2}) [-r x_i^{\mathrm{T}} B_i B_i^{\mathrm{T}} P_i \\ &\quad \times \sum_{j=1, i\neq j}^{N} [a_{ij} \left(\xi_i - \xi_j\right) + g_i\xi_i]] \end{aligned} \quad (11)$$

Define $\Gamma_i = P_i B_i B_i^{\mathrm{T}} P_i$; then, it follows from (11) that

$$\begin{aligned}\dot{V} &= \sum_{j=1, i\neq j}^{N} \sum_{i=1}^{N} [2\xi_i^{\mathrm{T}} P_i A_i \xi_i + 2\beta_i \xi_i^{\mathrm{T}} P_i B_i B_i^{\mathrm{T}} x_i \\ &\quad - 2 (c_i + w_i) \xi_i^{\mathrm{T}} \Gamma_i [a_{ij} \left(\xi_i - \xi_j\right) + g_i\xi_i] \\ &\quad + 2\xi_i^{\mathrm{T}} P_i f_i + 2\xi_i^{\mathrm{T}} P_i (A_i - A_0) x_0 - 2\xi_i^{\mathrm{T}} P_i B_0 u_0 \\ &\quad - 2\varphi_i (c_i - \gamma_{i1}) (c_i - 1)^2 \\ &\quad + 2 (c_i - \gamma_{i1}) [a_{ij} \left(\left(\xi_i - \xi_j\right)^{\mathrm{T}} + g_i \xi_i^{\mathrm{T}}\right) \\ &\quad \Gamma_i \left(\left(\xi_i - \xi_j\right) + g_i\xi_i\right)] - 2r (\beta_i - \gamma_{i2}) x_i^{\mathrm{T}} B_i B_i^{\mathrm{T}} \\ &\quad \times P_i [a_{ij} \left(\xi_i - \xi_j\right) + g_i\xi_i]] \end{aligned} \quad (12)$$

Let $\bar{\Gamma} = \bar{P}\bar{B}\bar{B}^{\mathrm{T}}\bar{P}$ where $\bar{\Gamma} \triangleq diag(\Gamma_1, \ldots, \Gamma_N)$ for $i = 1, \ldots, N$. Then, (12) can be rewritten into a compact form as follows

$$\begin{aligned}\dot{V} &= 2\xi^{\mathrm{T}} \bar{P}\bar{A}\xi + 2\xi^{\mathrm{T}} \bar{\beta}\bar{P}\bar{B}\bar{B}^{T} x \\ &\quad - 2\xi^{T} (\bar{c} + \bar{w}) \bar{\Gamma} (L + G)\xi \\ &\quad + \sum_{i=1}^{N} [2\xi_i^{\mathrm{T}} P_i f_i + 2\xi_i^{\mathrm{T}} P_i (A_i - A_0) x_0 - 2\xi_i^{\mathrm{T}} P_i B_0 u_0 \\ &\quad - 2\varphi_i (c_i - \gamma_{i1}) (c_i - 1)^2] \\ &\quad + 2\xi^{\mathrm{T}} (\bar{c} - \gamma_1) \bar{\Gamma} (L + G)^2 \xi \\ &\quad - 2r x^{\mathrm{T}} \left(\bar{\beta} - \gamma_2\right) \bar{B}\bar{B}^{\mathrm{T}} \bar{P} (L + G) \xi \end{aligned} \quad (13)$$

where $\bar{c} = diag(c_1, c_i, \ldots, c_N) \in \mathbb{R}^{N\times N}$, $\bar{w} = diag(w_1, w_i, \ldots, w_N) \in \mathbb{R}^{N\times N}$, $\gamma_1 \triangleq diag(\gamma_{11}, \ldots, \gamma_{N1}) \in \mathbb{R}^{N\times N}$, $\gamma_2 \triangleq diag(\gamma_{12}, \ldots, \gamma_{N2}) \in \mathbb{R}^{N\times N}$, $\bar{\beta} = diag(\beta_1, \beta_i, \ldots, \beta_N) \in \mathbb{R}^{N\times N}$, and r are the positive scalars.

Invoking Lemma 1, $(L+G) > 0$ and taking the upper bound of the solution to ARE in (8) with careful selection of γ_1 such that $0 < \left\|(\bar{c}+\bar{w})(L+G) + (\bar{c}-\gamma_1)(L+G)^2\right\| < 1$, with $\bar{w} = diag(w_1, w_i, \ldots, w_N) \in \mathbb{R}^{N\times N}$, in particular,

$$\bar{A}\bar{P} + \bar{P}\bar{A}^{\mathrm{T}} - 2\left((\bar{c}+\bar{w})(L+G) + (\bar{c}-\gamma_1)(L+G)^2\right)\bar{B}\bar{B}^{\mathrm{T}} \leq \bar{A}\bar{P} + \bar{P}\bar{A}^{\mathrm{T}} - 2\bar{B}\bar{B}^{\mathrm{T}} < -\bar{Q} < 0 \quad (14)$$

holds, then,

$$\begin{aligned}\dot{V} \leq\ & -\xi^{\mathrm{T}}\bar{Q}\xi + 2\xi^{\mathrm{T}}\bar{\beta}\bar{P}\bar{B}\bar{B}^{\mathrm{T}}x \\ & + \sum\nolimits_{i=1}^{N}[2\xi_i^{\mathrm{T}}P_i f_i + 2\xi_i^{\mathrm{T}}P_i(A_i - A_0)x_0 - 2\xi_i^{\mathrm{T}}P_i B_0 u_0 \\ & -2\varphi_i(c_i - \gamma_{i1})(c_i-1)^2] \\ & -2rx^{\mathrm{T}}\left(\bar{\beta}-\gamma_2\right)\bar{B}\bar{B}^{\mathrm{T}}\bar{P}(L+G)\xi \end{aligned} \quad (15)$$

Taking the triangular inequality as in Lemma 3, yields the following upper bound for (15),

$$\begin{aligned}\dot{V} \leq\ & -\xi^{\mathrm{T}}\bar{Q}\xi + 2\xi^{\mathrm{T}}\bar{\beta}\bar{P}\bar{B}\bar{B}^{\mathrm{T}}x + 2\xi^{\mathrm{T}}\bar{P}(\bar{A} - I_N\otimes A_0)(1\otimes x_0) \\ & -2\xi^{\mathrm{T}}\bar{P}(I_N\otimes B_0)(1\otimes u_0) \\ & -2\mathrm{tr}\left(\varphi(\bar{c}-\gamma_1)(\bar{c}-I_N)^{\mathrm{T}}(\bar{c}-I_N)\right) \\ & +2\xi^{\mathrm{T}}\bar{P}\bar{f} - 2rx^{\mathrm{T}}\left(\bar{\beta}-\gamma_2\right)\bar{B}\bar{B}^{\mathrm{T}}\bar{P}(L+G)\xi \end{aligned} \quad (16)$$

where $tr(\bullet)$ is the trace operation of a matrix and $\varphi \triangleq diag(\varphi_1, \ldots, \varphi_N) \in \mathbb{R}^{N\times N}$. Then, proceeding from (16),

$$\begin{aligned}\dot{V} \leq\ & -\xi^{\mathrm{T}}\bar{Q}\xi - 2\left\|\varphi(\bar{c}-\gamma_1)(\bar{c}-I_N)^{\mathrm{T}}(\bar{c}-I_N)\right\|_F \\ & +2\|\xi\|\left\|\bar{P}\bar{A} - I_N\otimes A_0\right\|\|1\otimes x_0\| \\ & -2\|\xi\|\left\|\bar{P}I_N\otimes B_0\right\|\|1\otimes u_0\| + 2\|\xi\|\left\|\bar{P}\bar{f}\right\| \\ & +2\|\xi\|\left[\bar{\beta}\bar{P}\bar{B}\bar{B}^{\mathrm{T}} - r\left(\bar{\beta}-\gamma_2\right)\bar{B}\bar{B}^{\mathrm{T}}\bar{P}(L+G)\right]\|x\| \end{aligned} \quad (17)$$

where $\|\bullet\|_F$ indicates the Frobenius norm. Further applying the triangular inequality to (17) yields,

$$\begin{aligned}\dot{V} \leq\ & -\xi^{T}\bar{Q}\xi - 2\|\varphi\|_F\|\bar{c}-\gamma_1\|_F\|\bar{c}-I_N\|_F^2 \\ & +2\|\xi\|\left\|\bar{P}\bar{A} - I_N\otimes A_0\right\|\|1\otimes x_0\| \\ & -2\|\xi\|\left\|\bar{P}I_N\otimes B_0\right\|\|1\otimes u_0\| + 2\|\xi\|\left\|\bar{P}\bar{f}\right\| \\ & +2\|\xi\|\left[\bar{\beta}\bar{P}\bar{B}\bar{B}^{\mathrm{T}}\left(I_N - r(L+G)(I_N - \gamma_2\bar{\beta}^{-1})\right)\right]\|x\| \end{aligned} \quad (18)$$

Completing the square of the term in (18) further concludes the upper bounds

$$\begin{aligned}\dot{V} \leq\ & -\lambda_{\min}\left(\bar{Q}\right)\|\xi\|^2 - 2\lambda_{\min}(\varphi)\|\bar{c}-\gamma_1\|_F\|\bar{c}-I_N\|_F^2 + \breve{F} \\ & +2\|\xi\|^2 + \left\|\bar{P}\bar{A} - I_N\otimes A_0\right\|^2\|1\otimes x_0\|^2 \\ & +\left\|\bar{P}I_N\otimes B_0\right\|^2\|1\otimes u_0\|^2 \\ & +2\|\xi\|\left[\bar{\beta}\bar{P}\bar{B}\bar{B}^{\mathrm{T}}\left(I_N - r(L+G)(I_N - \gamma_2\bar{\beta}^{-1})\right)\right]\|x\| \end{aligned} \quad (19)$$

where $\lambda_{\min} = (\bullet)$ represents the minimum singular values of the matrix in question and $\breve{F} = 2\|\xi\|\left\|\bar{P}\bar{f}\right\|$ represents the faults occurring within the network. Let $\Pi_0 = \left\|\bar{P}\bar{A} - I_N\otimes A_0\right\|^2\|1\otimes x_0\|^2 + \left\|\bar{P}I_N\otimes B_0\right\|^2\|1\otimes u_0\|^2$

From (17), r is chosen such that the following equation holds $r(L+G) \geq (I_N\otimes 1)$. Eventually,

$$\begin{aligned}\dot{V} \leq\ & -\lambda_{\min}\left(\bar{Q}\right)\|\xi\|^2 + 2\|\xi\|^2 - 2\lambda_{\min}(\varphi)\|\bar{c} \\ & -\gamma_1\|_F\|\bar{c}-I_N\|_F^2 - R + \breve{F} + \Pi_0 \end{aligned} \quad (20)$$

where $R = 2\|\xi\|\left[\bar{\beta}\bar{P}\bar{B}\bar{B}^{\mathrm{T}}\left(r(L+G)(I_N - \gamma_2\bar{\beta}^{-1}) - I_N\right)\right]\|x\|$.

To guarantee the consensus convergence, it is important to have $\lambda_{\min}(\bar{Q}) > 2$(Please see Remark 2). By choosing γ_2 such that $R \geq \Pi_0 + \breve{F}$, we can obtain that $\dot{V} \leq 0$, and thus V is bounded. From (20), according to LaSalle's Invariance principle [35], it follows that the consensus error ξ asymptotically converges to zero within the compact set of which the size of the invariance depends on the fault magnitude, the boundedness of the leader trajectory x_0 and the leader's control input. The adaptive gains, c_i and β_i are ultimately bounded. Thus, the proof is completed. ■

Remark 2: The simulation parameters were selected by design based on the aforementioned Theorem 1. Suppose $V_1 \subset V$, i.e., $V_1 = \sum_{i=1}^{N}\xi_i^{\mathrm{T}}P_i\xi_i$ and the condition $\lambda_{\min}(\bar{Q}) > 2$ holds; then, the term in (20): $-\lambda_{\min}\left(\bar{Q}\right)\|\xi\|^2$ should be bounded as $\dot{V}_1 \leq -\eta V_1$, where $\eta = \lambda_{\min}(\bar{Q})/\lambda_{\max}(\bar{P})$ denotes the convergence rate of the consensus. We should choose r appropriately so that $R \geq \Pi_0 + \breve{F}$ holds. In addition, the adaptive coupling gain condition, i.e., $\bar{c} > \gamma_1$ should be adhered to guarantee the overall performance of the MRS during the adversity of time-varying faults while driving the consensus convergence error close to zero.

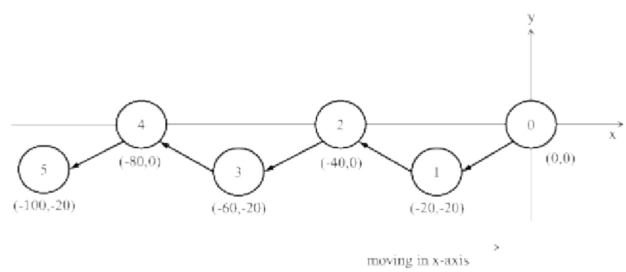


FIGURE 3. Leader-follower directed graph topology.

IV. SIMULATION RESULTS

Consider a heterogeneous MRS that moves along the x-axis of a two-dimensional coordinate frame and is connected by a directed communication topology, as shown in Fig. 3.

Let $x_i = [p_{xi}\ p_{yi}\ v_{xi}\ v_{yi}]^{\mathrm{T}} \in \mathbb{R}^4$, where p_{xi} and v_{xi} indicate the position and velocity of the i-th robot along the x-axis, respectively, while p_{yi} and v_{yi} indicate the position and velocity of the i-th robot along the y-axis, respectively. Let $u_i = [u_{xi}\ u_{yi}]^{\mathrm{T}} \in \mathbb{R}^2$ be the control input and $f_i = [0\ 0\ f_{xi}\ f_{yi}]^{\mathrm{T}} \in \mathbb{R}^4$ be the additive fault signal associated with robots 1, 2, and 4. Six different types of intermittent time-varying faults are defined as follows

$$\begin{aligned} f_{x1} &= a, \quad 30 \leq kT \leq 50 \\ f_{y1} &= -a, \quad 30 \leq kT \leq 50 \\ f_{x2} &= 0.1t - 2b, \quad 30 \leq kT \leq 50 \\ f_{y2} &= -0.1t + 2b, \quad 30 \leq kT \leq 50 \\ f_{x4} &= d + \omega, \quad 40 \leq kT \leq 80 \\ f_{y4} &= -d + \omega, \quad 40 \leq kT \leq 80 \end{aligned}$$

where ω is the white noise, k is the time index and T is the sampling period, which is equal to 0.001 s. The fault parameters are set to $a = 2, b = 1.5$, and $d = 1.5$, which yields the specific fault signal depicted in Fig. 4.

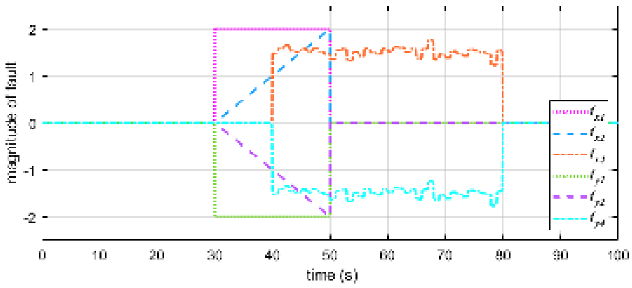


FIGURE 4. Fault signals f_{xi} and f_{yi}.

The fault signal is simulated either as a rectangular signal or a soft bias signal (slope) at a different instance. The fault magnitude in Fig. 4 is categorized as "low" severity. There is no fault for robot 3 and robot 5.

The parameters m_i are arbitrarily selected and tabulated below in Table 1.

TABLE 1. Parameters m_i.

Index	0	1	2	3	4, 5
m_i	0.1	0.2	10	2	0.5

Hence, the dynamics of the i-th mobile robot is characterized by the following matrices

$$A_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & -10 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & -0.1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & -0.5 \end{bmatrix},$$

$$A_4 = A_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 10 & 0 \\ 0 & 10 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 5 & 0 \\ 0 & 5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad B_4 = B_5 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2 & 0 \\ 0 & 2 \end{bmatrix},$$

It is assumed that the robots are communicating with one another according to the information graph shown in Fig. 3. For the proposed adaptive law, the simulation parameters are designed as $r = 5, c_i(0) = 2$, and $\beta_i(0) = 1$ for $i = 1, \ldots, N$. By solving the LMI in (8), the feasible solution matrix P_i is obtained as

$$P_1 = \begin{bmatrix} 0.0406 & 0 & 0.0338 & 0 \\ 0 & 0.0373 & 0 & 0.0677 \\ 0.0338 & 0 & 0.4082 & 0 \\ 0 & 0.0135 & 0 & 0.0731 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.5521 & 0 & 1.3611 & 0 \\ 0 & 0.8458 & 0 & 0.5756 \\ 1.3611 & 0 & 11.5188 & 0 \\ 0 & 5.7562 & 0 & 18.4969 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0.6408 & 0 & 0.6366 & 0 \\ 0 & 0.6451 & 0 & 0.4759 \\ 0.6366 & 0 & 2.3685 & 0 \\ 0 & 0.9519 & 0 & 3.1305 \end{bmatrix}$$

$$P_4 = P_5 = \begin{bmatrix} 0.1939 & 0 & 0.1719 & 0 \\ 0 & 0.1961 & 0 & 0.2282 \\ 0.1719 & 0 & 0.9251 & 0 \\ 0 & 0.1141 & 0 & 0.4414 \end{bmatrix}$$

Then, the feedback gain matrices, K_i and Γ_i are given by

$$K_1 = \begin{bmatrix} -0.1690 & 0 & -2.0410 & 0 \\ 0 & -0.0677 & 0 & -0.3657 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -0.1361 & 0 & -1.1519 & 0 \\ 0 & -0.5756 & 0 & -1.8497 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -0.3183 & 0 & -1.1842 & 0 \\ 0 & -0.4759 & 0 & -1.5653 \end{bmatrix}$$

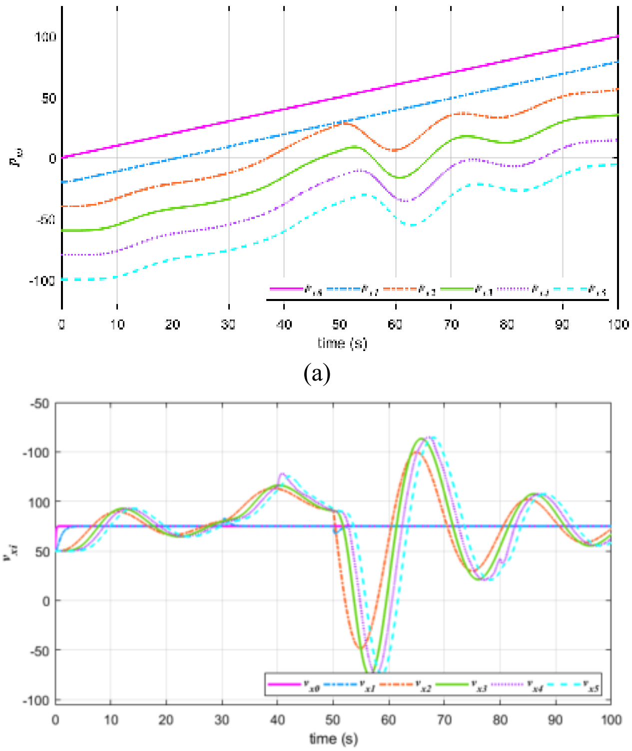
$$K_4 = K_5 = \begin{bmatrix} -0.3437 & 0 & -1.8501 & 0 \\ 0 & -0.2282 & 0 & -0.8829 \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} 0.0286 & 0 & 0.3449 & 0 \\ 0 & 0.0229 & 0 & 0.1238 \\ 0.3449 & 0 & 4.1657 & 0 \\ 0 & 0.0248 & 0 & 0.1337 \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} 0.0185 & 0 & 0.1568 & 0 \\ 0 & 0.0331 & 0 & 0.1065 \\ 0.1568 & 0 & 1.3268 & 0 \\ 0 & 1.0647 & 0 & 3.4213 \end{bmatrix}$$

$$\Gamma_3 = \begin{bmatrix} 0.1013 & 0 & 0.3769 & 0 \\ 0 & 0.1132 & 0 & 0.3725 \\ 0.3769 & 0 & 1.4024 & 0 \\ 0 & 0.7450 & 0 & 2.4501 \end{bmatrix}$$

$$\Gamma_4 = \Gamma_5 = \begin{bmatrix} 0.1182 & 0 & 0.6360 & 0 \\ 0 & 0.1042 & 0 & 0.4030 \\ 0.6360 & 0 & 3.4231 & 0 \\ 0 & 0.2015 & 0 & 0.7794 \end{bmatrix}$$



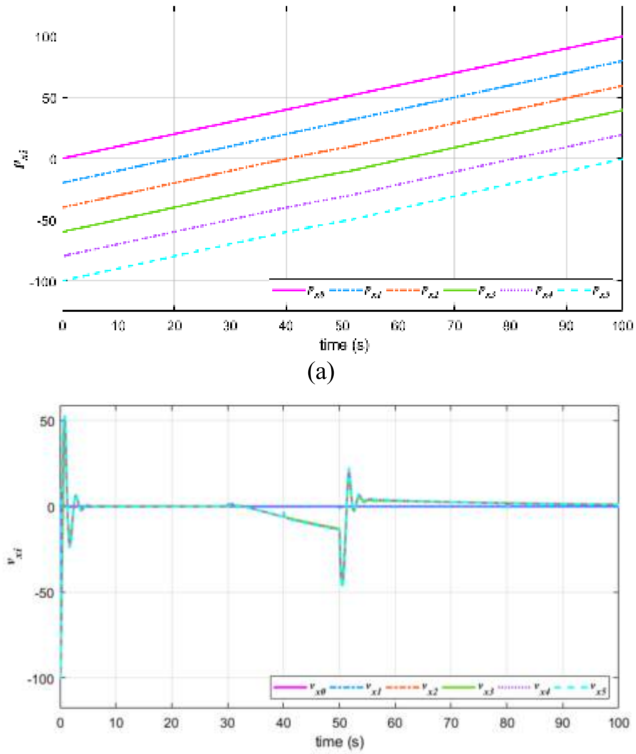
(a)

(b)

FIGURE 5. **(a) Position p_{xi} and (b) velocity v_{xi} without adaptive consensus law.**

The leader moves along the x-axis with a constant velocity. To show the effectiveness of the proposed adaptive law, the results are compared with [28] and [30]. Referring to Fig. 5, for the first 30 s of the simulation, all robots move synchronously together to achieve cruising velocity $v_{x0} = 1m/s$ with a constant distance vector of 20 m.

However, since the robots were unable to achieve convergence within 30 s, fault occurrence further deteriorates both position and velocity signals, causing large oscillations. Following (20) in Theorem 1, without adaptive gains c_i and β_i, any increase in fault magnitude (F term) would cause $\dot{V} > 0$, thus, V is no longer guaranteed to be bounded value. During the fault occurrence, the changes in trajectories are influenced not only by the fault signals but also by each particular robot's dynamics. In comparison to robot 1, robot 2 produced position and velocity signals that were unsmooth as the rate of acceleration slowed. Robot 4, however, followed the robot 2 trajectories since the output of robot 2 indirectly connected to robot 4 via robot 3. In the absence of the proposed adaptive law, the resulting trajectories of the agents are similar to those reported in [34], as shown in Fig. 5.



(a)

(b)

FIGURE 6. **Consensus performance for Hu's law [30]: (a)p_{xi} trajectories; (b) v_{xi} trajectories.**

The results being presented in Figs. 6 to 8 show the effectiveness of the adaptive consensus law in the presence of the time-varying and additive faults.

The velocity curve shown in Fig. 7(b) indicates that the adaptive law proposed in [28] has a slower convergence performance than that of the proposed adaptive law in Fig. 8(b). The proposed adaptive law has good tracking properties; however, consensus results for [30] outweighed the proposed law as depicted in Fig. 6(b).

In comparison to [28], only one adaptive coupling gain is used in the control input. In [30], the control input contains two coupling gains, but only one of them is designed to be adaptable. Therefore, this paper introduces two distinct adaptive coupling gains in the consensus law to produce relatively rapid velocity convergence while ensuring robust stability of the mobile robot system, as shown in Fig. 8.

According to Theorem 1, adaptive gain β further attenuate the heterogeneity as long as $R \geq \Pi_0 + \breve{F}$ is hold. This is the key advantage of the proposed adaptive compared to [28] and [30]. In addition, the r parameter is also part of the R, which

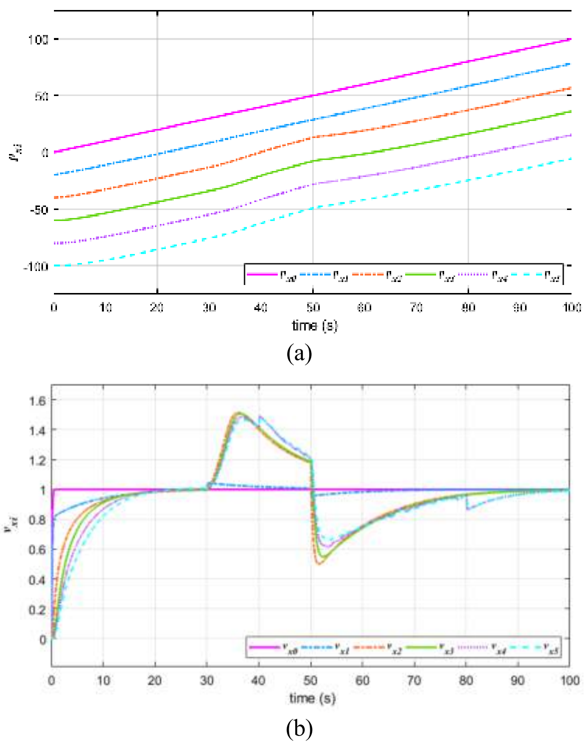


FIGURE 7. Consensus performance for Lv's law [28]: (a) p_{xi} trajectories; (b) v_{xi} trajectories.

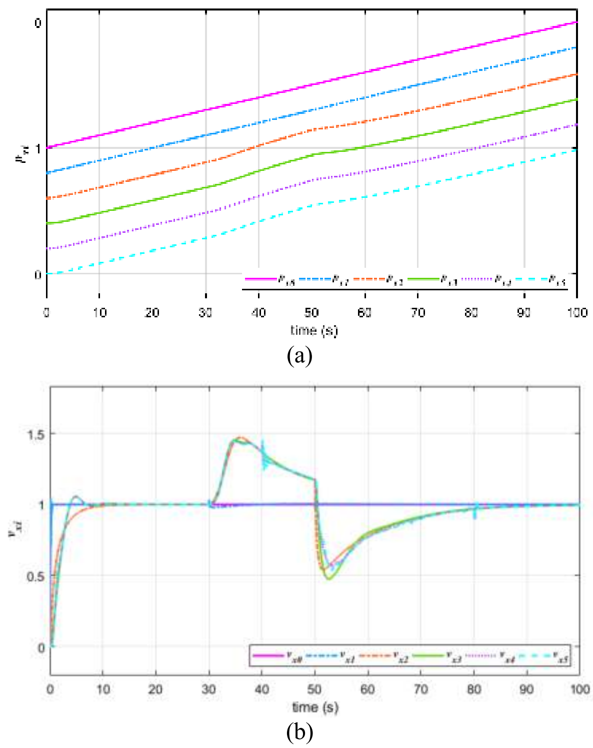


FIGURE 8. Consensus performance for the proposed law: (a)p_{xi} trajectories; (b) v_{xi} trajectories.

simply acts as a tuneable parameter to increase or decrease the rate of the agents' response according to the user preference.

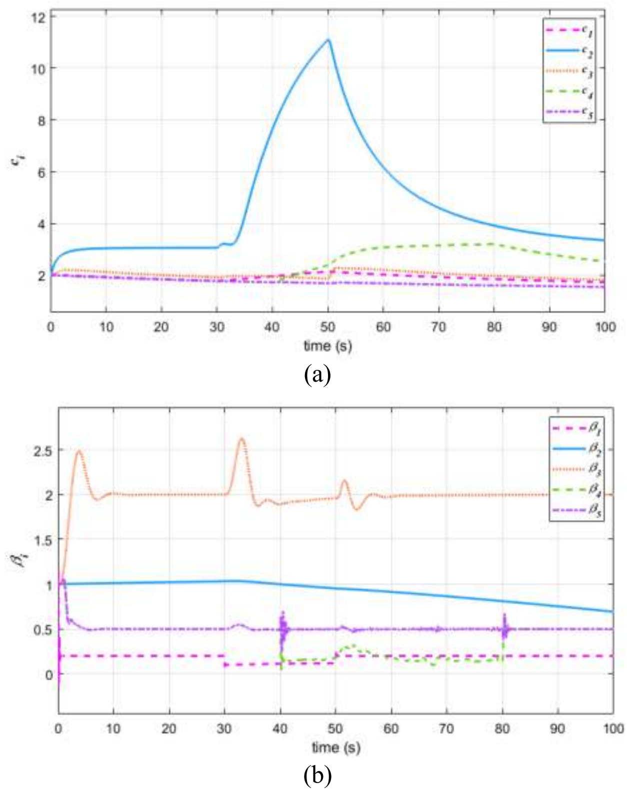


FIGURE 9. (a) adaptive gain c_i; (b) adaptive gain β_i.

For position tracking, all simulated adaptive algorithms are capable of minimizing fault strength, avoiding collisions, and allowing faulty robots to quickly revert to the desired position after the fault is removed. In Fig. 9, the coupling gains, c_i and β_i for the proposed law converged to a new finite value to counteract the occurrence of the fault.

To demonstrate the robustness of the algorithm, a further comparison is made between [30] and the proposed law, with the magnitude of f_{x2} and f_{y2} increased tenfold to signify "high" severity and with the remaining faults unchanged. The performance of the laws is illustrated in Figs. 10 and 11.

Figs. 10 and 11 show that with a larger magnitude of f_{x2} and f_{y2}, [30] produces a longer convergence time, more than twice that of the proposed adaptive law.

In addition, the proposed adaptive law produces the same convergence time as in the previous results in Fig. 8(b) demonstrating the robustness of the proposed consensus law. Furthermore, as shown by the relative velocity difference in Figs. 8(b) and 11(b), all robots are strongly connected during both normal and fault conditions.

Remark 3: [30] and the proposed consensus law both have two coupling gains in two separate control input terms. Unlike [30], which used a combination of constant and time-varying gains, the adaptive design of the proposed consensus law employs two distinct adaptive coupling gains to enhance convergence.

The trajectories of the control input u_{xi} for the three adaptive laws are depicted in Fig. 12.

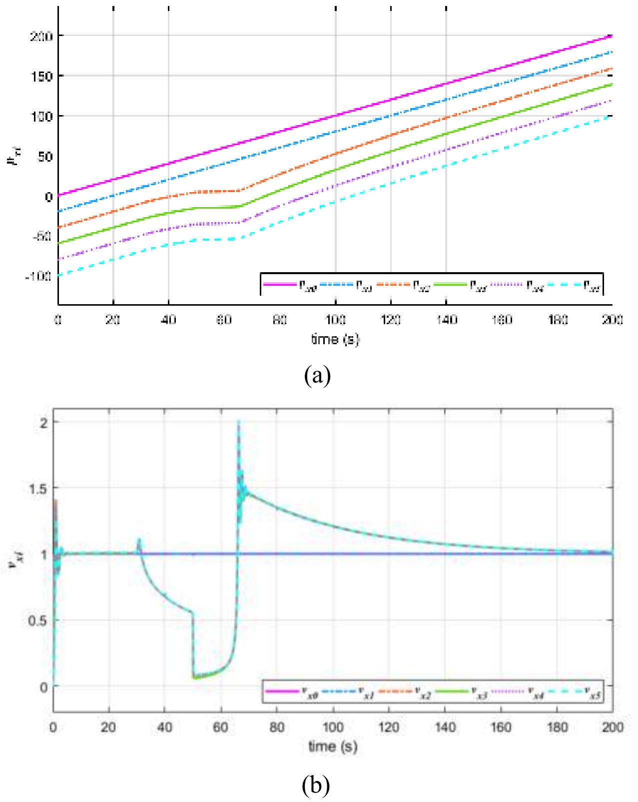


FIGURE 10. **Consensus performance for Hu's law with "high" severity faults: (a) p_{xi} trajectories; (b) v_{xi} trajectories.**

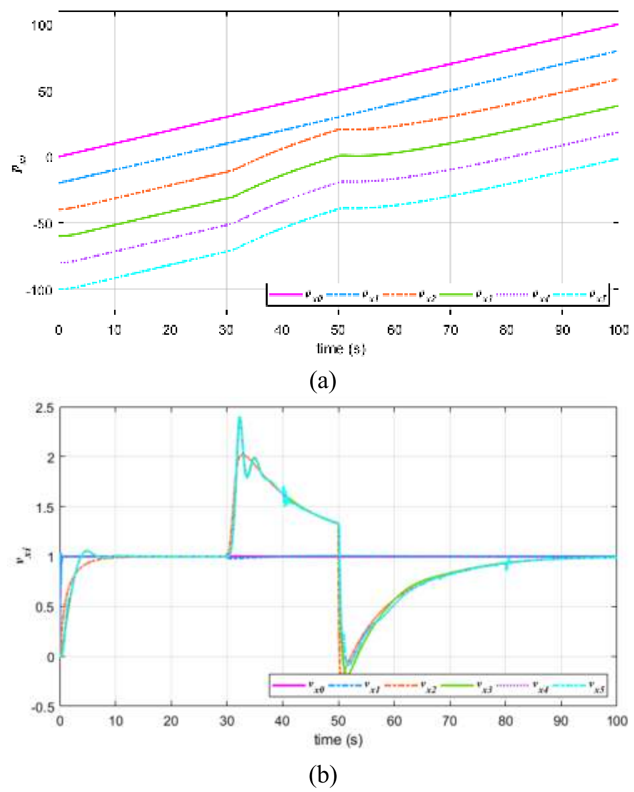


FIGURE 11. **Consensus performance for the proposed law with "high" severity faults: (a) p_{xi} trajectories; (b) v_{xi} trajectories.**

According to Fig. 12(a) and (c), it is observed that both Hu's law and the proposed law exhibit very high overshoot in the beginning instances. Referring to the inset images in Fig. 12(a-c), Hu's law produced high control effort at 50 s during the fault occurrence period, in contrast to the smooth and non-fluctuating control input u_{x2} obtained using Lv's law and the proposed law. When compared to Lv's law, the proposed law has a slightly shorter convergence time at the start of the mission and at $t > 70s$. With the application of two distinct adaptive gains, the proposed consensus law shows the capability to effectively suppress the MRS heterogeneity under both transient and steady-state conditions. Despite the slightly aggressive value of the control input u_{xi} as shown in Fig. 12(c), the control effort produced is acceptable and satisfactory.

Remark 4: The results in Fig. 12 highlight that the introduced novel approach of two distinct adaptive gains did not incur exhaustive control efforts, while awarding a high degree of robustness against the time-varying faults.

As Remark 4 implies, low control effort, as evidently illustrated by Fig. 12, translates to light controller computation, which is amenable to a remote practical application when energy resources are scarce.

Moreover, to analytically compare the transient controllers' efforts for the three adaptive laws, ISE and IAE are utilized for the velocity error, $|v_{xi}(k) - v_{x0}(k)|$ as the controller performance indices.

TABLE 2. **Adaptive law performance indices.**

Adaptive law method	Robot	ISE	IAE	ISE $10\times f_2$	IAE $10\times f_2$
Proposed	Robot 1	0.008551	0.2347	0.00855	0.2347
	Robot 2	3.932	12.82	16.5	25.78
	Robot 3	4.595	13.15	17.71	26.09
	Robot 4	4.328	13.01	16.93	25.95
	Robot 5	4.343	13.01	17.01	25.96
Hu's [30]	Robot 1	0.01577	0.1586	0.01577	0.1586
	Robot 2	0.4548	3.358	18.12	31.38
	Robot 3	0.5542	3.519	18.45	31.66
	Robot 4	0.5853	3.627	18.11	31.44
	Robot 5	0.6177	3.707	18.22	31.56
Lv's [28]	Robot 1	0.181	2.405		
	Robot 2	5.175	15.55		
	Robot 3	5.868	16.33		
	Robot 4	6.46	17.86		
	Robot 5	6.888	18.59		

According to the performance indices in Table 2, the proposed adaptive law outperforms Hu's law [30] for relatively large fault magnitudes but outperforms Lv's law [28] for small fault magnitudes. The presented results, validating the effectiveness of the proposed adaptive law, demonstrate that it is more applicable than the existing adaptive laws.

In this paper, the general additive type of faults is explicitly considered. The fault is assumed to be intermittent, and the maximum magnitude that can be tolerated is bounded by f_m.

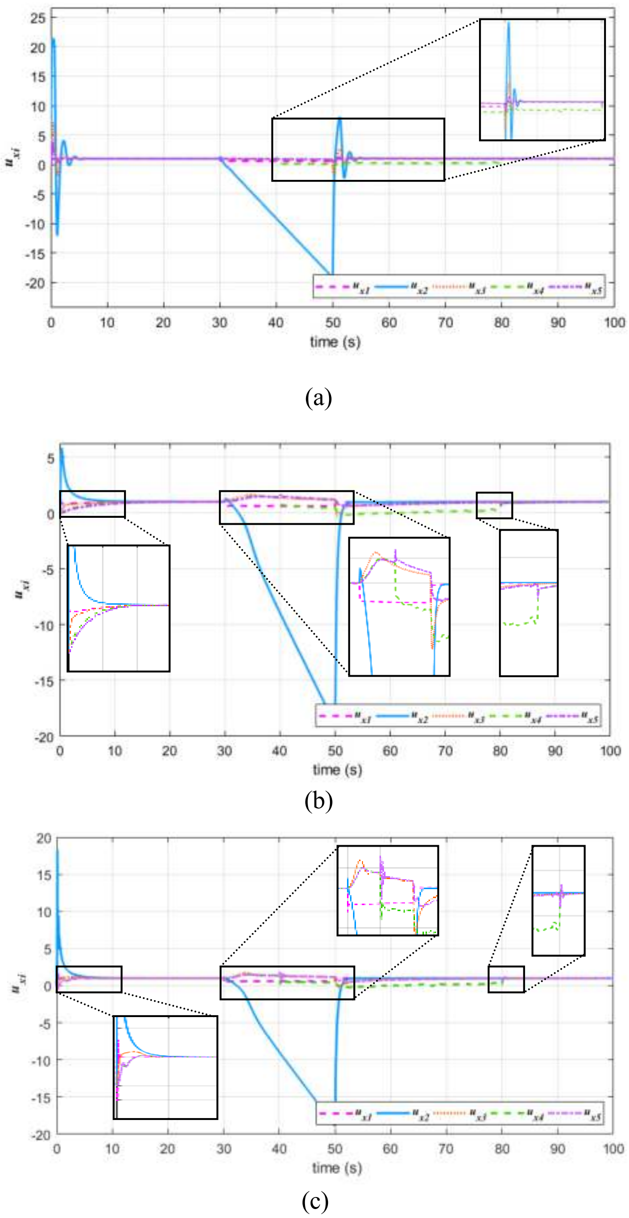


FIGURE 12. Control effort: (a) Hu's law [30]; (b) Lv's law [28]; (c) the proposed law.

A fault magnitude that exceeds f_m represents a condition where the entire MRS may become unstable, and the positioning of the robots can result in a collision. According to Fig. 11(b), as the magnitude of the fault increases, so does the magnitude of the follower robot's velocity. In practice, the limits on the actuator operation range should not be exceeded to prevent mechanical failure of the robots and to maintain optimal MRS operation. Hence, to complete the mission despite faulty teammates with f_m or permanent faults, an active fault tolerance strategy can be designed to remove the faulty robot(s) from the team and allow the remaining healthy and semi-healthy (fault occurrence below the maximum limit) robots to automatically reconfigure themselves.

The exclusion of faulty robots from the team could be executed by employing fault isolation thresholds. In this case, all robots must observe their control inputs and isolate themselves if they exceed a certain threshold by withdrawing from the mission and cutting off communication so that the remaining healthy and semi-healthy robots can automatically adjust their adaptive coupling parameters in their consensus laws to account for changes in the communication topology. Since all the robots rely solely on the relative state difference with their neighbors to compute the consensus law, this isolation process can be achieved.

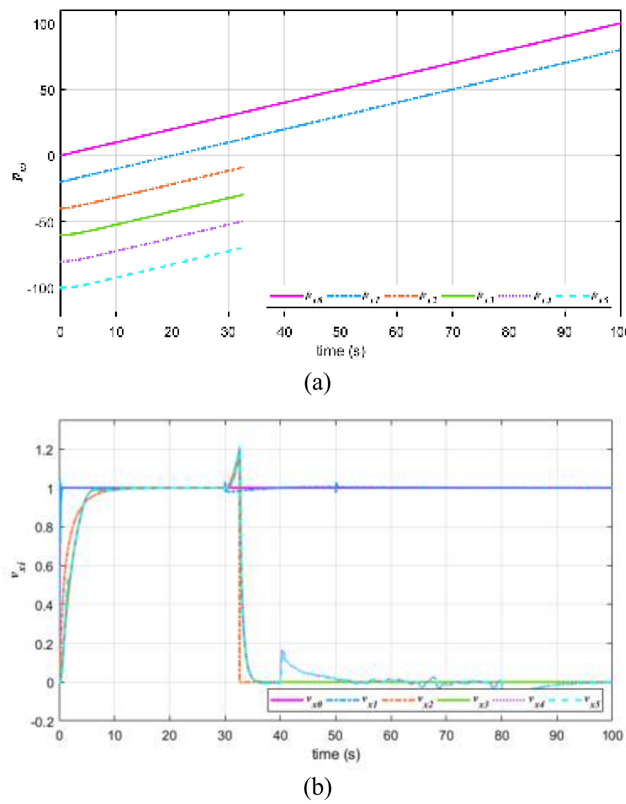


FIGURE 13. Isolation of robot 2: (a) p_{xi} trajectories; (b)v_{xi} trajectories.

There is, however, a clear limitation of the automatic isolation sequence using the current unidirectional topology. For instance, in Fig. 13, due to the presence of a fault above the threshold, robot 2 automatically initiates self-removal from the MRS and stops moving, while the remaining robots reorganize themselves to continue participating in the MRS to complete the assigned mission.

However, because all robots are unidirectionally connected to a single robot, the ejection of robot 2 from the MRS causes the immediate neighbor of robot 2 to adjust the adaptive parameters based on the position and velocity of robot 2. A cascading effect on the remaining robots that are indirectly connected to robot 2 leads to a failed mission. Therefore, alternatively, each robot can be connected to at least two neighbors to reduce the possibility of total failure.

Remark 5: To ensure that the coordination of the MRS remains stable during isolation, a new restriction is implemented where all the followers must be connected to two or more neighbors to maintain global synchronization. However, the optimal topology should be investigated since more neighbors does not always guarantee better consensus convergence. For further discussion, please refer to [39].

For comparison, the topology in Fig. 3 is modified by adding a communication link between robots 1 and 3, as illustrated in Fig. 14 by the red dashed line.

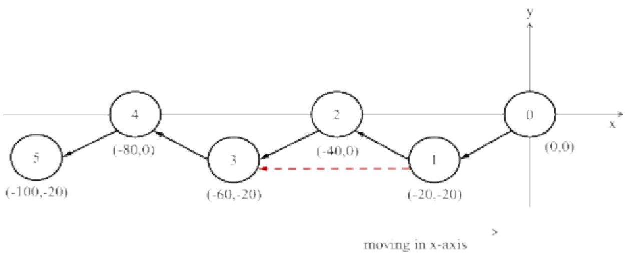


FIGURE 14. Modified topology.

With the additional communication link, the isolated faulty robot 2 does not affect the remaining healthy robots in the MRS, as shown in Fig. 15. The immediate neighbors of robot 2 automatically recalculate their adaptive consensus law to cope with the changes in their relative state information with the remaining neighbors. In addition, fast convergence can be achieved immediately after the faulty robot is removed. Both adaptive gains converge to finite values. It is noted that since both robots 1 and 4 are relatively lightweight compared to robot 2, the "low" severity fault subjected to these robots, as depicted in Fig. 4, has a minimal effect on the agents' position and velocity.

It is worth mentioning that the MRS with a permanent fault required more information exchange to effectively isolate the fault. However, because the amount of information exchanged in the network is proportional to the number of communication links, communication demand can be minimized by limiting the number of neighbors with whom each agent is permitted to communicate and determine the optimal network topological design for a high probability of permanent fault. The results obtained are congruent with the analysis in Theorem 1, whereby as long as the condition of $R \geq \Pi_0 + \check{F}$ and Lemma 1 are fulfilled, multiple time-varying faults in the MRS can be accommodated by using the proposed adaptive law. Contrary to work in [31], the proposed law can be applied to the case of directed and undirected network topology.

The proposed adaptive law performance indices for each robot before and after isolation are tabulated in Table 3 .

The results presented in Table 3 suggest that the proposed adaptive law with modified topology is much more acceptable for efficient and robust fault-tolerant control, mainly for multiple time-varying faults. The simulation proved that MRS reconfiguration can be done adaptively without the use of a sophisticated control algorithm.

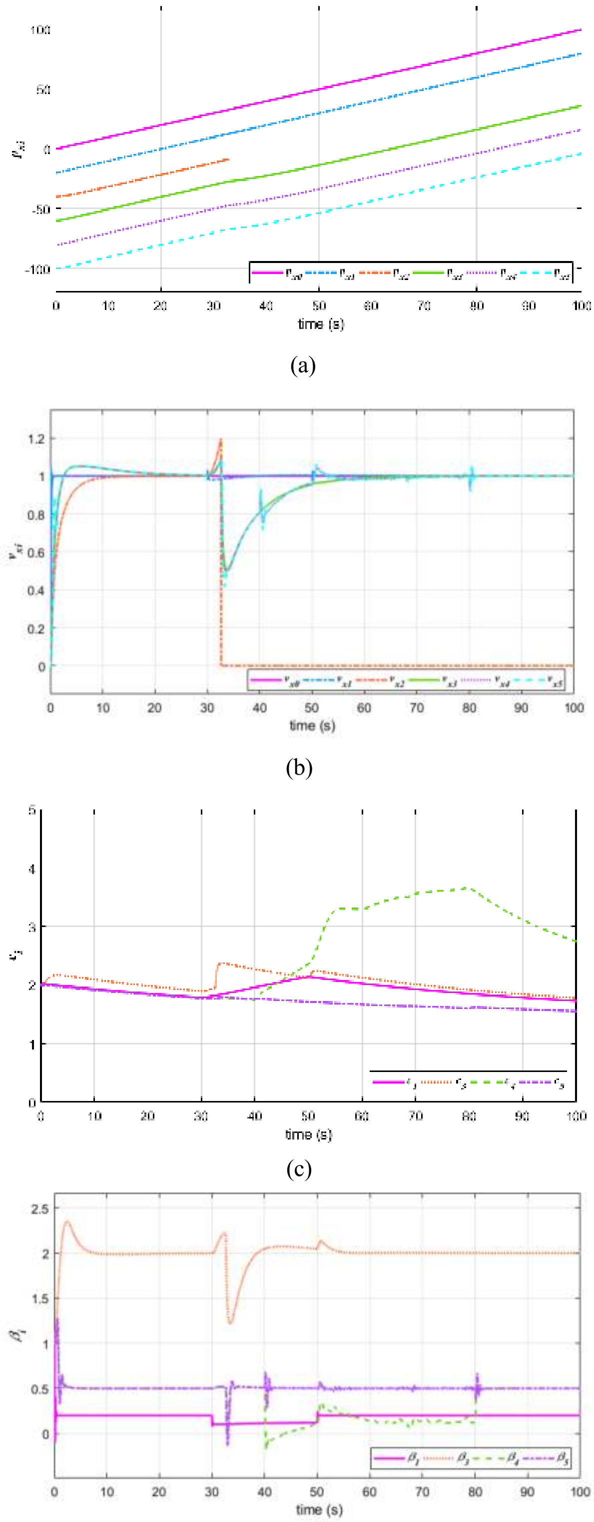


FIGURE 15. Modified topology: (a) p_{xi} trajectories; (b) v_{xi} trajectories; (c) adaptive gain c_i; (d) adaptive gain β_i.

TABLE 3. Proposed adaptive law performance indices before and after isolation.

Proposed law	Robot	ISE	IAE
Before isolation	Robot 1	0.008551	0.2347
	Robot 2	3.932	12.82
	Robot 3	4.595	13.15
	Robot 4	4.328	13.01
	Robot 5	4.343	13.01
Isolation with Topology in Fig. 3	Robot 1	0.008551	0.2347
	Robot 2	n/a	n/a
	Robot 3	68.04	68.76
	Robot 4	68.16	68.79
	Agent 5	68.24	68.83
Isolation with modified topology in Fig. 14	Robot 1	0.008551	0.2347
	Robot 2	n/a	n/a
	Robot 3	1.345	5.142
	Robot 4	1.423	5.293
	Robot 5	1.508	5.379

V. CONCLUSION

In this paper, a distributed leader-follower adaptive consensus law for a linear heterogeneous MRS is proposed. The proposed consensus law employs two distinct adaptive gains to improve tracking and convergence performance to ensure a safe separation between the robots in the presence of multiple additive time-varying fault occurrences. The proposed strategy allows for maintaining a limited communication burden; i.e., the unidirectional information exchanged among neighbors for relative state computations. Simulation results of the MRS verified the effectiveness of the proposed adaptive law. Future research may be devoted to an extension of the current work to a nonlinear MRS, switching topology, and communication delay.

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