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Statistical Estimation of ideal and realistic muon interaction on Al, Fe, and Cu absorbers

M I H M Norizan[1], **I H Hashim**[1,2,3,a)], **F Ibrahim**[1], **N N A M A Ghani**[1]

[1]Department of Physics, Faculty of Science, Universiti Teknologi Malaysia, 81300, Johor Bahru, Johor.
[2]Research Center for Nuclear Physics, Osaka University, 567-0047 Ibaraki-shi, Osaka, Japan.
[3]National Centre Particle Physics, Universiti Malaya, 50603 Kuala Lumpur, Malaysia.

[a]izyan@utm.my

Abstract. Interaction of charged muons in various absorbing materials is almost similar to other charged particles. Their behaviour has been observed by various experimental work in material science and nuclear and particle physics. A statistical model is developed in order to study the behaviour of muons after interacting with absorbing material. In this study, ideal and realistic beam structure from high intensity muon facility around the world such as at Japan Accelerator Research Complex, Research Centre Nuclear Physics, Osaka University and Paul Scherrer Institute is assumed. Three absorbing materials (aluminium, iron and copper) with thickness range of 0.00 ~ 0.12 g/cm^2 are used to observe the momentum straggling of the beams. Performance for our model was checked by comparing the value of stopping power with others work for muons with initial kinetic energy of 10-40 MeV with and without including shell effect correction term. The effect of the energy losses towards the beam structure will be discussed.

1. Introduction

Interaction of particles with matter depends on their charged and energy of the incident particle. These interactions will determine the nuclear reaction occur in the nucleus and the electron at surrounding orbits. Muon interaction is very much similar to electron interaction where positive muons tends to behave as electron and interact at 1s and 2s electronic quantum levels in muonium. While for the negative muon which has 207 times of electron mass interact at the atomic level of a muonic atom. Both μ^+ and μ^- are subject to electromagnetic and weak interaction due to Coulomb interaction, their magnetic moments and lepton flavour conservation. Low momentum muons usually stop at short range for medium-heavy nuclei absorber and their stopping probability is very high. For medium to high momentum muons, thicker absorber is required for initiating reactions.

As the muon passes through matter, it will lose its energy by atomic collisions or deflections from the incident direction due to many interactions. Energy straggling and range straggling are introduced routing in large divergences of the particles in matter [2]. The mean energy losses, the straggling associated with the energy losses is crucial for understanding the energy losses by muon [3]. The incoming particle suffer an inelastic collision with the atomic electrons of the absorbing material and elastic scattering from the nuclei. In various calculations by Bethe-Bloch formula, the muon beam structure only concerns on 1 keV energy bin. However, experimental estimation requires certain amount of beam spread to evaluate realistic muon behaviour after interaction with the materials.

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World-renowned muon facilities provide various distribution for the initial muon beam suitable for materials science, nuclear and particle physics experiment [1]. Two of them are located in Japan at the Japan Accelerator Research Complex (J-PARC) and the Research Center for Nuclear Physics, Osaka University, while another one located in Switzerland is the Paul Scherrer Institute (PSI). These facilities provide high intensity continuous or pulsed muon beams as shown in Table 1. All three facilities of our concerned shows different momentum width that affect the muon behaviour after passing through materials.

Table 1. Comparison of muons facilities around the world.

	PSI	**J-PARC**	**MuSIC**
Beam power	**1200 kW**	**1000kW**	**0.4 kW**
Beamline channel	**μE1 (Mode B)**	**D2-beamline, MLF**	**MuSIC-M1**
Muon Intensity [μ/sec]	**2 x10^4**	**1×10^4**	**0.5×10^4**
Muon Structure	**Continuous**	**Pulse**	**Continuous**
Momentum width (FWHM)	**~1%**	**~15%**	**~5%**

The present work aims to study the muon behaviour on different absorbing materials. A muon energy loss model is developed based on Bethe-Bloch formula, idealistic and realistic muon beam width are tested on various absorbing materials and thickness range. The performance of the model is verified from previous work in reference [4].

2. Muon energy losses model

A useful model to predict the muon energy losses is by implementing the ideal and real cases for the incident beams. By using a C++ program, the statistical simulations generate the muon momentum distribution and thus shows impact of beam straggling after material absorber. Incident muon beams will be assumed to irradiate the absorbing material. Muon will suffer energy losses and energy straggling is introduced in the absorbing material. The energy losses of muons are known by calculating the stopping power given by the Bethe-Bloch formula in Eq. 1. In this study, muon beams are assumed to irradiate three different absorbing materials; Al, Fe and Cu. Due to the energy losses, S. Kumar et al. [3] states that the range of muon momentum becomes wider as the thickness of the absorbing material increase. In order to observe the momentum straggling, this model include calculations for the absorbing materials with the thickness of 0.04 g/cm^2, 0.08 g/cm^2 and 0.12 g/cm^2.

The mean energy loss per unit path length is given by quantity called stopping power or simply -dE/dx given by Bethe-Bloch formula [5]

$$-\frac{dE}{dx} = 2\pi N_a r_e{}^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[ln\left(\frac{2m_e\gamma^2 v^2 W_{max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right] \quad (1)$$

with $2\pi N_a r_e m_e c^2 = 0.1535\ cm^2/g$, r_e is defined as an electron radius which is 2.817×10^{-13} cm and the m_e is the rest mass of an electron. The constant Avogadro's number, N_a is $6.022 \times 10^{23}\ mol^{-1}$ and z is a charge of incident particle in unit of e. The Lorentz factor, γ is defined as $1/(\sqrt{1-\beta^2})$, where $\beta$is the ratio of velocity of the incident particle, v to the speed of light, c. The maximum energy transfer in a single collision is known as W_{max}. Here the parameter involving the absorbing material is the density of the material, ρ, the atomic number of the absorbing material, Z the atomic weight of the absorbing material, A, the mean excitation potential, I, and the density correction, δ.

Shell corrections, C accounts for the effect which arises when the velocity of the incident particle is comparable or smaller than the orbital velocity of the bound electron. At such energies, the assumption that the electron is stationary with respect to the incident particle is no longer valid and the Bethe-Bloch formula breaks down. The correction is generally small in number and give there an empirical formula for this correction given by [6]:

$$\begin{aligned} \boldsymbol{C(I,\eta)} &= [(\mathbf{0.422377}\boldsymbol{\eta}^{-2} + \mathbf{0.0304043}\boldsymbol{\eta}^{-4} - \mathbf{0.00038106}\boldsymbol{\eta}^{-6}) \times \mathbf{10}^{-6}\boldsymbol{I}^{2}] \\ &+[(\mathbf{3.850190}\boldsymbol{\eta}^{-2} - \mathbf{0.1667989}\boldsymbol{\eta}^{-4} + \mathbf{0.00157955}\boldsymbol{\eta}^{-6}) \times \mathbf{10}^{-9}\boldsymbol{I}^{3}] \end{aligned} \tag{2}$$

where $\eta = \beta\gamma$ and I is the mean excitation potential in eV. Hence, in order to understand and illustrate the interactions of muons, development of the energy losses model is required to give a picture of the experiments. The energy losses of the incident beam include the effect of the material surrounding the experimental area which include elements that make up the set-up such as aluminium, iron and copper.

Comparison with calculation from Groom *et al.* [4] have been made and summarized in Table 2. In general, the agreement is regarded as adequate, but is worse for higher energy muons. In the second run, the simple shell corrections as given by Eq. 2 has been used, and under this condition, the value of stopping power somewhat overcorrects.

Table 2. Comparison of stopping power for muons (in MeV g^{-1} cm^2) including and excluding the shell correction, with those from Groom *et al.* [4].

Energy (MeV)	10 MeV	20 MeV	30 MeV	40 MeV
Aluminium (Z=13)				
This calculation				
Without shell correction	6.40	3.98	3.13	2.70
With shell corrections	6.26	3.83	2.98	2.55
Groom et al. [4]	6.19	3.80	2.96	2.53
Iron(Z=26)				
This calculation				
Without shell correction	5.71	3.58	2.83	2.45
With shell corrections	5.56	3.44	2.68	2.30
Groom et al. [4]	5.49	3.40	2.65	2.27
Copper(Z=29)				
This calculation				
Without shell correction	5.49	3.45	2.73	2.36
With shell corrections	5.35	3.31	2.59	2.22
Groom et al. [4]	5.28	3.28	2.56	2.20

Based on the comparison of different calculation with and without shell correction term for our statistical energy losses model, we conclude that the calculations for this model must include shell correction terms in order to best re-enact the real-life conditions. Compared to Groom et al. [4] calculations, the muon stopping power at 10 MeV are higher by 1.1%, 1.2% and 1.3% through Al, Fe and Cu respectively. Without the shell correction, the stopping powers for this case are higher by 3.4%, 4.0% and 3.9% respectively. Through lower Z material, the agreement is more precise with Groom et al. [4]. In all cases, the agreement improves with increasing energy.

In this model, the calculations are done on the low-energy muons at common momentum as used in the three muon facilities; J-PARC, PSI and MuSIC (30 MeV/c). Two cases of muon beams will be considered which is realistic and ideal muon beams. The distribution function of the real beam case is in the form of a Gaussian function. Meanwhile, ideal beam means no tail effect in the incident beams. These are standard that can be achieved at their highest efficiency and performance in these advanced muon facilities. Different from the practical muon beams, the true value has 100% probability of lying within the limits p-σ and p+σ.

The intensity *I* of the muon beams correspond to the integral of the spectra. In a real experiment, the range can be determined by passing a beam of particles at the desired energy through different thicknesses of the material under test and measuring the ratio of transmitted to incident particles [6]. In this model, the transmission ratio is obtained by taking the number of counts of the resulting muon beams normalized to the number of counts of the incident muon beams. From the number-distance curve, the mean range is the range where roughly half of the particle is absorbed. For the purpose of finding a well-defined number for mean range of the muons, the incident muon beams are assumed to irradiate the three-absorbing materials with the thickness starting from 0.02 g/cm^2 with increments of 0.02 g/cm^2 until the transmission ratio becomes zero.

3. Results and discussion

3.1 Muon energy losses in Al, Fe, and Cu of varying thickness

The energy losses spectra through varying thickness of absorbing materials in the range of 0.00 – 0.12 g/cm^2 were plotted. After the irradiation, for real and ideal beam cases the mean incident momentum is 30 MeV/c from J-PARC muons facilities through Al, Fe and Cu are presented in Figure 1 and 2 as a function of the projectile momentum respectively. From Figure 1, the momentum distribution of the incident muons through increasing thickness in the range of 0.00 – 0.12 $g/cm^2$is broadened widely from 3.77 to 10.93, 10.46, and 9.75 MeV/c for Al, Fe and Cu respectively. Then, for each absorbing material with thickness 0.12 g/cm^2, the incident muons loses their momentum by at most 16.66, 14.86, and 13.29 MeV/c respectively.

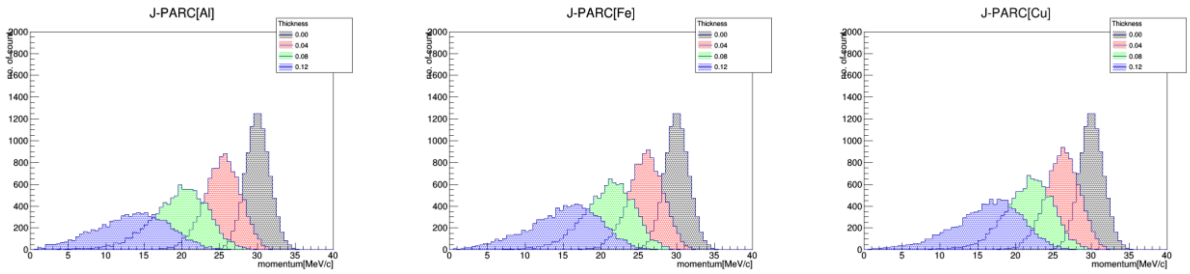

Figure 1. Momentum distribution for real cases of muon from J-PARC at various thickness after irradiation through different absorbing materials: (a) Al (b) Fe and (c) Cu.

From Figure 2, in Al, almost all muons did not lose all their energy except through the thickness of 0.12 g/cm^2 of aluminium where only 0.08% of muons stopped in the target. Meanwhile for Fe and Cu, both show non-stopped muons in the calculated beam at a given thickness. The momentum distribution of the incident muons through increasing thickness in the range of 0.00 – 0.12 g/cm^2 is broadened widely from 3.1 to 17.4, 11.5, and 9.9 MeV/c for Al, Fe and Cu respectively.

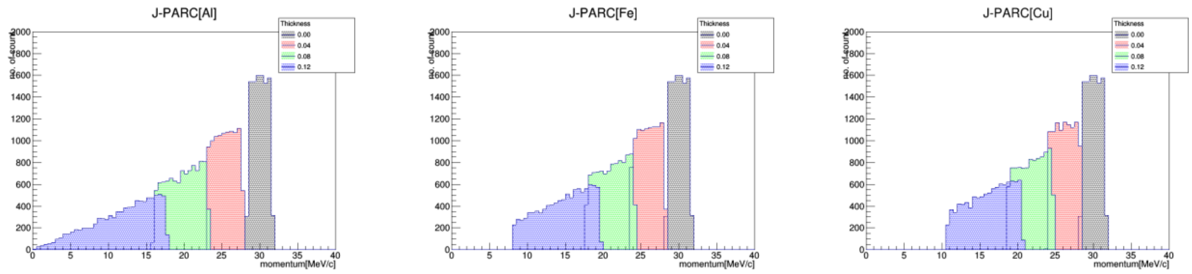

Figure 2. Momentum distribution for ideal cases muon from J-PARC at various thickness after irradiation through different absorbing materials: (a) Al (b) Fe and (c) Cu.

Figure 3 plots the width of the beams against the thicknesses of the absorbing materials to visualise the broadening pattern of muons through these materials. The width broadening of the beams shows a similar pattern as reported by Newhauser *et al.* [7] as they observed the beam width broadening of proton in water.

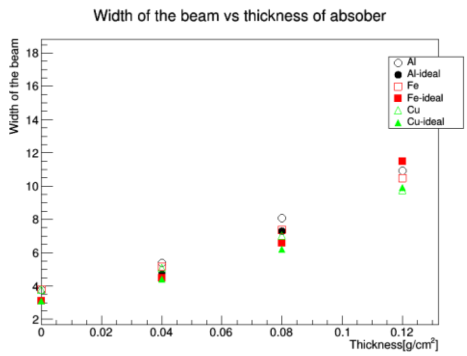

Figure 3. Broadening of the beam width in different material due to energy losses.

The analysis on the range of muons is done by calculating the muon transmission ratio. The result of our calculation for the J-PARC, MuSIC and PSI real and ideal muon beam cases are shown in Figure 4 (a) and (b) respectively. It is observed that the value of mean range constant despite different initial momentum distribution, is found to be 0.053, 0.021 and 0.019 cm through Al, Fe and Cu respectively. Besides, the cut-off in ideal beam cases observed to be sharper, which means that they show less pronounced straggling than in the real beam cases.

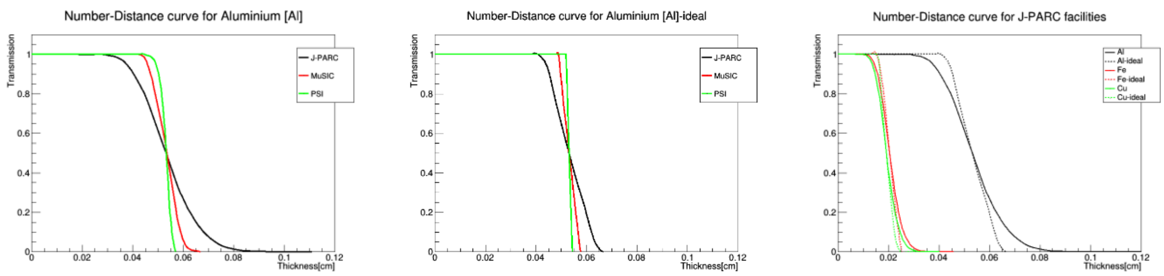

Figure 4. Number-distance curve; (a) for a real case in Al, (b) for an ideal case in Al, and (c) for J-PARC facility only in Al, Fe and Cu.

Figure 4(c) show that muons through aluminium has the highest mean range compared to iron and copper. This can be explained by the density of the material which affects the range of muons in material.

The stopping power increases with increasing density [8]. Hence, the range is inversely proportional to the density of the absorbing material.

4. Conclusions

This study successfully developed a statistical model to understand the muon behaviour when interacting with matter. After checking the performance of the statistical model, we conclude that shell corrections must be included into the stopping power formula in order to get a better result. The effect of the increasing path length on the width of beam distribution for muons passing through different absorbing materials at various thicknesses was clearly demonstrated where the width gets wider an absorbing material. The density of the absorbing material affects the mean range of the muon beam as the range is inversely proportional to the density. Finally, the ideal beam demonstrates smaller straggling effect as compared to the real beam cases.

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