

## Optimal backstepping control of quadrotor UAV using gravitational search optimization algorithm

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### ABSTRACT

Quadrotor unmanned aerial vehicle (UAV) has superior characteristics such as ability to take off and land vertically, to hover in a stable air condition and to perform fast maneuvers. However, developing a high-performance quadrotor UAV controller is a difficult problem as quadrotor is an unstable and underactuated nonlinear system. The effort in this article focuses on designing and optimizing an autonomous quadrotor UAV controller. First, the aerial vehicle's dynamic model is presented. Then it is suggested an optimal backstepping controller (OBC). Traditionally, backstepping controller (BC) parameters are often selected arbitrarily. The gravitational search algorithm (GSA) is used here to determine the BC parameter optimum values. In the algorithm, the control parameters are calculated using an integral absolute error to minimize the fitness function. As the control law is based on the theorem of Lyapunov, the asymptotic stability of the scheme can be ensured. Finally, several simulation studies are conducted to show the efficacy of the suggested OBC.

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## 1. INTRODUCTION

Unmanned aerial vehicles are commonly used in a variety of applications, including indoor or outdoor surveillance, remote inspection and hostile environment tracking. Quadrotor is emerging as a common platform among UAVs owing to its greater capacity for payload and greater maneuverability for single-rotor. In addition, quadrotor has a number of benefits in regards to structure, cost and motion control compared to traditional helicopters. Quadrotors have therefore received increased attention among researchers and engineers, and have also become a promising alternative for multiple unmanned military and civilian apps. However, quadrotor UAV has significant science and engineering issues due to its characteristics including underactuation, unknown nonlinearities, multivariable and high coupling. Quadrotor control is therefore becoming quite complicated and hard, primarily because of its underactuated characteristics and nonlinearities.

It is known that the quadrotor is volatile. Hence, the attitude and altitude stability problems are the primary goals for most research in this sector. Some of the methods used to control the quadrotor platform are the proportional-integral-derivative (PID) control [1-3], linear quadratic regulator (LQR) control [4-6], sliding mode control [7-9], feedback linearization control [10-12], fuzzy logic (FL)

control [13-15] and backstepping control [16-18]. In this paper, the quadrotor helicopter's stability issue is regarded. The altitude is selected as controllable DOF, along with three angles of attitude, roll-pitch-yaw. The dynamic model that describes the movements of the quadrotor helicopter and takes into account the different parameters that influence the flying structure dynamics is presented. Thereafter, a backstepping approach-based control technique is created. The backstepping control system is a non-linear control method based on the theorem of Lyapunov. Due to its recursive design and systematic methodology, the backstepping control design methods got many attention [19-23]. Unlike the feedback linearization technique with issues such as the precise model requirement, the backstepping strategy provides a choice of design instruments to accommodate nonlinearities. The benefit of backstepping compared to other control techniques is its flexibility in design because of its recurrent use of Lyapunov functions. The backstepping model concept is to pick a suitable variables state recursively to be a virtual inputs for the general system's reduced dimension subsystems and for each stable virtual controller, the Lyapunov functions are designed. The stability of the overall control system is therefore guaranteed by the designed actual control law.

Even though the backstepping technique can provide a systematic building process for controller design, it is not simple to achieve satisfactory output due to the arbitrary selection of controller parameters acquired by the backstepping technique. In order to achieve a good result, it is essential to select appropriate parameters because the incorrect choice of parameters can give a poor results and sometime cause system unstable. It is also possible to select the parameters appropriately, but it is not feasible to say that the ideal parameters are selected. In this work, GSA is used to calculate off-line the ideal parameters for the quadrotor system backstepping controller. This study mainly contributes to the backstepping control design approach utilizing GSA to maneuver a quadrotor UAV.

## 2. QUADROTOR DYNAMIC MODEL

The quadrotor UAV comprises of four-rotors in cross-configuration, as shown in Figure 1. The dynamic equation of movement of the attitude could be deduced from the Euler equation, that could be formulated as follows [24]:

$$\begin{aligned}
 \ddot{x} &= (c_\phi s_\theta c_\psi + s_\phi s_\psi) \frac{1}{m} u_1 \\
 \ddot{y} &= (c_\phi s_\theta s_\psi - s_\phi c_\psi) \frac{1}{m} u_1 \\
 \ddot{z} &= -g + (c_\phi c_\theta) \frac{1}{m} u_1 \\
 \ddot{\phi} &= \dot{\theta} \dot{\psi} \left( \frac{I_{yy} - I_{zz}}{I_{xx}} \right) - \frac{J_r}{I_{xx}} \dot{\theta} \Omega_d + \frac{l}{I_{xx}} u_2 \\
 \ddot{\theta} &= \dot{\phi} \dot{\psi} \left( \frac{I_{zz} - I_{xx}}{I_{yy}} \right) + \frac{J_r}{I_{yy}} \dot{\phi} \Omega_d + \frac{l}{I_{yy}} u_3 \\
 \ddot{\psi} &= \dot{\theta} \dot{\phi} \left( \frac{I_{xx} - I_{yy}}{I_{zz}} \right) + \frac{1}{I_{zz}} u_4
 \end{aligned} \tag{1}$$



Figure 1. The quadrotor UAV configuration

### 3. BACKSTEPPING CONTROL OF QUADROTOR

Only the altitude and angular movement (roll-pitch-yaw) are selected as four controllable degrees-of-freedom (DOF) in this study. The dynamic model (1) can therefore be formulated as outlined below in a nonlinear dynamic equation:

$$\dot{X} = f(X) + g(X)u \quad (2)$$

where  $u$  the input and  $X$  the state vector defined below:

$$u = [u_1 \ u_2 \ u_3 \ u_4]^T \quad (3)$$

$$X = [x_1 \ x_3 \ x_5 \ x_7]^T = [z \ \phi \ \theta \ \psi]^T \quad (4)$$

From (1) and (4), the nonlinear functions  $f(X)$  and  $g(X)$  can be composed appropriately as:

$$f(X) = \begin{pmatrix} -g \\ \dot{\theta}\psi a_1 - \dot{\theta} a_2 \Omega_d \\ \dot{\phi}\psi a_3 + \dot{\phi} a_4 \Omega_d \\ \dot{\theta}\dot{\phi} a_5 \end{pmatrix} \quad g(X) = \begin{pmatrix} u_z \frac{1}{m} & 0 & 0 & 0 \\ 0 & b_1 & 0 & 0 \\ 0 & 0 & b_2 & 0 \\ 0 & 0 & 0 & b_3 \end{pmatrix} \quad (5)$$

where  $a_1 = (I_{yy} - I_{zz})/I_{xx}$ ,  $a_2 = J_r/I_{xx}$ ,  $a_3 = (I_{zz} - I_{xx})/I_{yy}$ ,  $a_4 = J_r/I_{yy}$ ,  $a_5 = (I_{xx} - I_{yy})/I_{zz}$ ,  $b_1 = l/I_{xx}$ ,  $b_2 = l/I_{yy}$ ,  $b_3 = 1/I_{zz}$ ,  $u_z = (c_\phi c_\theta)$ .

The control objective is to plan a reasonable control law for (2) so that the state direction  $X$  can track a wanted reference direction  $X_d = [x_{1d} \ x_{3d} \ x_{5d} \ x_{7d}]^T$ .

The development of the backstepping control is defined:

*Step 1:* Error is defined:

$$e_1 = x_{1d} - x_1 \quad (6)$$

where  $x_{1d}$  is the required path indicated by the reference. The tracking error derivative can then be described as:

$$\dot{e}_1 = \dot{x}_{1d} - \dot{x}_1 \quad (7)$$

The first feature of Lyapunov is selected as:

$$V_1(e_1) = \frac{1}{2} e_1^2 \quad (8)$$

The derivative of  $V_1$  is:

$$\dot{V}_1(e_1) = e_1 \dot{e}_1 = e_1 (\dot{x}_{1d} - \dot{x}_1) \quad (9)$$

$\dot{x}_1$  could be seen as a virtual control. The required virtual control value recognized as a stabilizing feature can be set as:

$$\alpha_1 = \dot{x}_{1d} + k_1 e_1 \quad (10)$$

where  $k_1$  is a +ve gain and ought to be decided by the GSA.

By substituting its required value for virtual control, (9) then turns into:

$$\dot{V}_1(e_1) = -k_1 e_1^2 \leq 0 \quad (11)$$

*Step 2:* The deviation from the required value of the virtual control can be described as:

$$e_2 = \dot{x}_1 - \alpha_1 = \dot{x}_1 - \dot{x}_{1d} - k_1 e_1 \quad (12)$$

The derivative of  $e_2$  is expressed as:

$$\begin{aligned} \dot{e}_2 &= \ddot{x}_1 - \dot{\alpha}_1 \\ &= f(x_1) + g(x_1)u_1 - \ddot{x}_{1d} - k_1 \dot{e}_1 \end{aligned} \quad (13)$$

The second Lyapunov function is selected as:

$$V_2(e_1, e_2) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \quad (14)$$

The derivative of (14) is:

$$\begin{aligned} \dot{V}_2(e_1, e_2) &= e_1\dot{e}_1 + e_2\dot{e}_2 \\ &= e_1(\dot{x}_{1d} - \dot{x}_1) + e_2(\ddot{x}_1 - \ddot{\alpha}_1) \\ &= e_1(-e_2 - k_1e_1) + e_2(f(x_1) + g(x_1)u_1 - \ddot{x}_{1d} - k_1\dot{e}_1) \\ &= -k_1e_1^2 + e_2(-e_1 + f(x_1) + g(x_1)u_1 - \ddot{x}_{1d} - k_1\dot{e}_1) \end{aligned} \quad (15)$$

Step 3: For  $\dot{V}_2(e_1, e_2) \leq 0$ , the  $u_1$  is chosen:

$$u_1 = \frac{1}{g(x_1)}(e_1 + k_1\dot{e}_1 + \ddot{x}_{1d} - f(x_1) - k_2e_2) \quad (16)$$

where  $k_2$  is a +ve gain and ought to be also decided by the GSA. Replacing (16) in (15), then:

$$\dot{V}_2(e_1, e_2) = -k_1e_1^2 - k_2e_2^2 = -E^TKE \leq 0 \quad (17)$$

where  $E = [e_1 \ e_2]^T$  and  $K = \text{diag}(k_1, k_2)$ . Since  $\dot{V}_2(e_1, e_2) \leq 0$ ,  $\dot{V}_2(e_1, e_2)$  is negative semi-definite. Hence, the system will be stabilized by (16).

#### 4. OPTIMAL BACKSTEPPING CONTROL

A controller (16) was intended to stabilize the subsystem in the past section. The coefficients  $k_1, k_2$  are parameters of control and must be positive in order to meet stability criteria. These parameters are chosen by trial and error in the standard backstepping technique. To overcome this limitation, the GSA is used to choose the optimal control parameters value. To select the best parameters of backstepping controller, the GSA is used off-line and controller performance differs with adjusted parameters. The quadrotor system consists of four subsystems, as mentioned above. Thus, there are eight control parameters to be chosen at the same time. An integral absolute error (IAE) is used in this work to assess the controller's performance. The index IAE is defined as below [25]:

$$IAE = \int_0^t |e(t)| dt \quad (18)$$

Since the system is consists of four subsystems, a vector IAE for the whole scheme is therefore considered as  $IAE_T = [IAE_A \ IAE_R \ IAE_P \ IAE_Y]$ , where the subscripts are signified for altitude, roll, pitch and yaw subsystem, respectively. GSA aims at minimizing the fitness function J, stated as:

$$J = IAE_T \cdot W \quad (19)$$

Where  $W = [W_1 \ W_2 \ W_3 \ W_4]^T$  is weighting-vector utilized to establish the importance of the several parameters and "W" changes in the range of 0-1. Same weights for the four objectives to be achieved are considered in our case to indicate the error indexes minimizations are correspondingly essential. The time-domain simulation is performed for the simulation period,  $t$  for the fitness function calculation. This fitness function needs to be minimized in order to expand the system response in terms of the settling time, overshoots and steady-state errors.

#### 5. SIMULATION RESULTS

The performance of the suggested strategy is assessed in this section. The parameters values of the quadrotor used in the simulations are given as [26];  $m=0.5$  kg,  $l=0.2$  m,  $g=9.81$  m/s<sup>2</sup>,  $I_{xx}=I_{yy}=4.85 \times 10^{-3}$  kgm<sup>2</sup>,  $I_{zz}=8.81 \times 10^{-3}$  kgm<sup>2</sup>,  $b=2.92 \times 10^{-6}$  Ns<sup>2</sup>,  $d=1.12 \times 10^{-7}$  Nms<sup>2</sup>. Then, GSA is utilized to tune the backstepping control parameters. The following values are allocated in this research for optimization of controller parameters:

- a. The number of agents (masses)=15;
- b. The search space dimension=8 ( i.e.,  $k_{i=1...8}$ );
- c. The parameters searching ranges are restricted to [0, 15];
- d. The number of maximum iteration=20;
- e. Optimization process is repeated for 20 times;
- f. The simulation time,  $t$  is equal to 10s;

The best optimized controller value is selected among the obtained finest value. Figure 2 shows the range of the fitness function and the iterations number. In the meantime, the evolution of the control parameters is shown in Figure 3 over the iteration number. The GSA technique can prompt convergence and achieve desired fitness value through about 20 iterations.

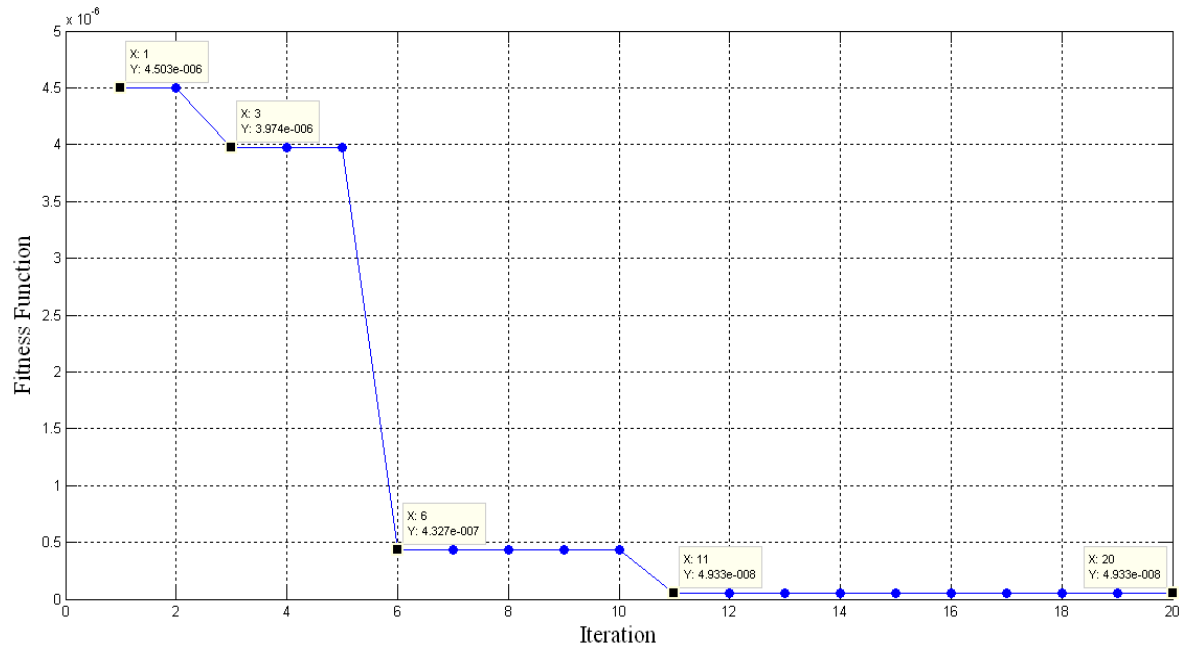


Figure 2. Fitness function with iterations number

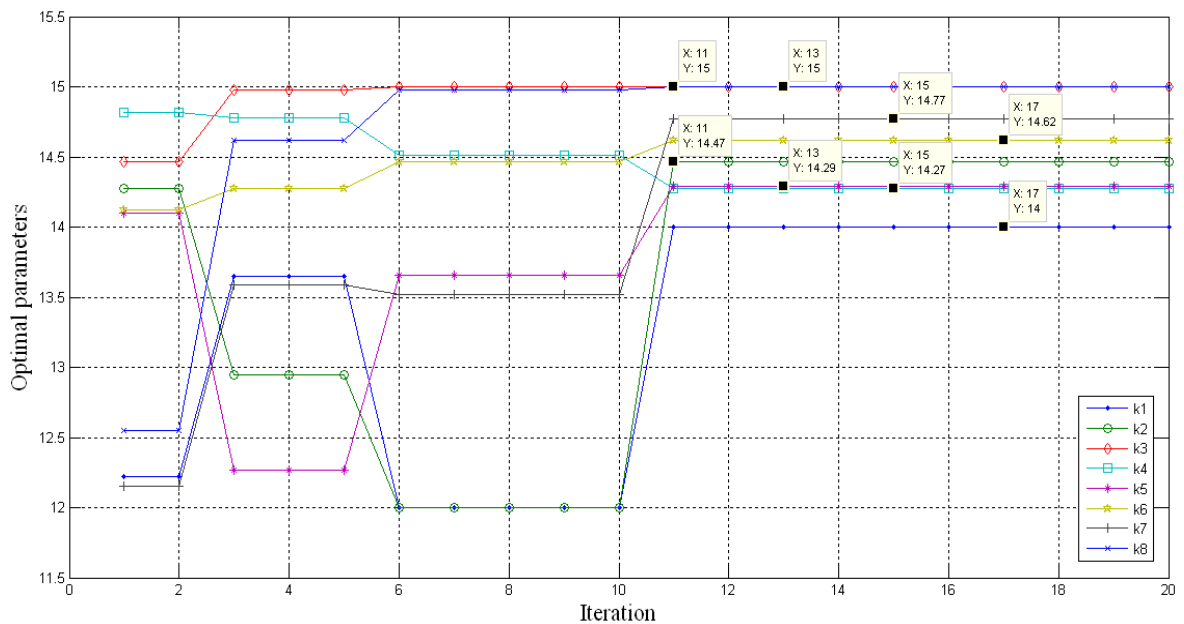


Figure 3. Control parameters versus iterations number

To investigate the efficacy of the suggested optimal backstepping controller, two simulation studies were conducted on the quadrotor. The first simulation is conducted for stabilizing problem and the second is performed to investigate attitude tracking problem.

### 5.1. Simulation experiment 1: stabilizing problem

The altitude/attitude control experiment is implemented in this simulation. The quadrotor is required to achieve and hold at a desired altitude/attitude. The desired altitude/attitude are set at  $x_{id} = [z_d, \phi_d, \theta_d, \psi_d] = [15, 0, 0, 0]^T$ . The initial states are set to  $z = 0, \phi = 0.2, \theta = 0.2$  and  $\psi = 0.2$ . By using the proposed OBC, it can be seen that the quadrotor can fly toward the desired altitude and manage to maintain at the desired altitude/attitude as shown in Figure 4. It can be also observed in Figure 4, the attitude converges quickly to stabilize the quadrotor.

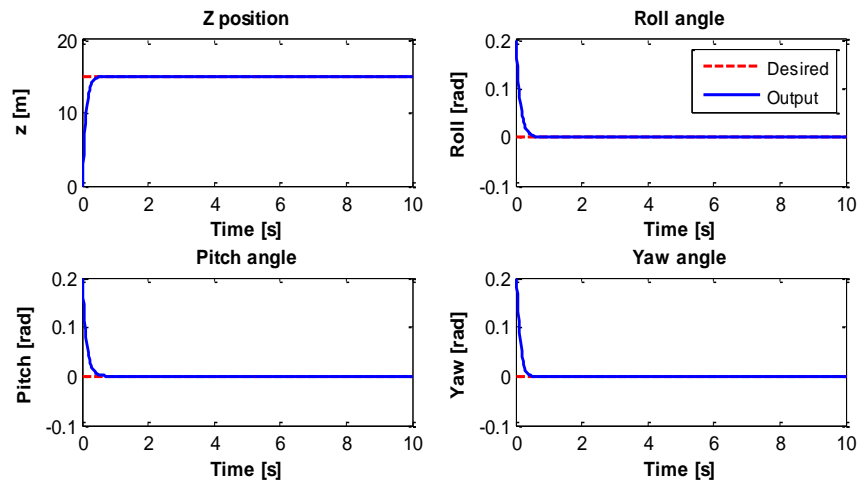


Figure 4. Altitude/attitude response

### 5.2. Simulation experiment 2: attitude tracking problem

The attitude tracking control experiment is conducted in this simulation. The quadrotor is required to track a desired reference signal. The sinusoidal signals are utilized as a trajectory to the attitude angles. It can be observed in Figure 5, the quadrotor manages to follow the trajectories efficiently. The findings also indicate that, the OBC manage to give a satisfactory tracking performance since the tracking error is small.

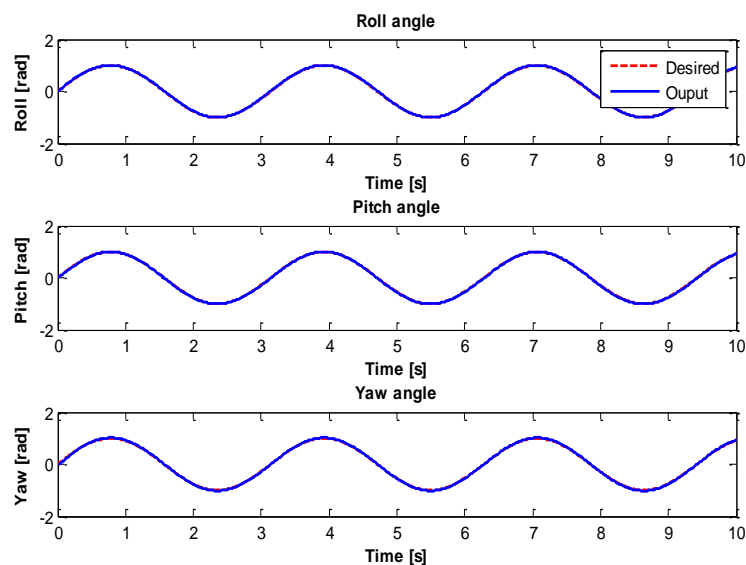


Figure 5. Attitude tracking response

## 6. CONCLUSION

In this paper, the application of an optimal backstepping controller for maneuvering a quadrotor UAV is successfully demonstrated. First, the quadrotor's mathematical model is introduced. The optimal backstepping controller is then developed in which the controller can select the parameters automatically via GSA. To ensure the system's stability, the backstepping control design is based on the Lyapunov function. Finally, the OBC is applied for a quadrotor UAV stabilization and trajectory tracking missions. The results indicate that the suggested control scheme can achieve a good transient and tracking response.

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