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# A Comparison of Simulated Annealing Cooling Strategies for Redesigning a Warehouse Network Problem

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**Abstract.** Simulated annealing (SA) is considered a valuable stochastic technique for resolving difficulties associated with comprehensive multidimensional optimisation, which guarantees optimal global convergence. This study explores the use of SA to address the challenges of redesigning warehouse networks and relates the effectiveness of three (3) diverse SA cooling schedules, namely; the basic geometric, logarithmic, and linear. The broad computational findings performed and described in the study indicate that the geometric cooling schedule generates consistent, superior quality, and timely solutions compared to the other schemes.

Keywords: Simulated Annealing, Redesigning Network problems, Cooling Schedules

## 1. Introduction

Simulated annealing (SA) is a heuristic technique founded on the basis of the physical annealing procedure. Typically, SA entails the agglomeration of numerous particles in a chilled physical structure (Abramson et al., 1998). The process was established to elucidate multifaceted problems in the field of non-linear optimisation. As a result, SA has broad applications and can perform optimisation without forehand information on the structural problem, exact solution or strategy.

Conventionally, the application of SA is straightforward, since the transitions in the resolution space of the problem that needs solving are also readily derived. Moreover, the creation of a cost function, however complicated, is acquired in most cases. Transitions (or moves) result in either uphill or downhill actions in the solution space. In a typical minimisation issue, the downhill activities can be instinctively recognised, whereas the uphill variant has guaranteed prospects. Due to its ease of use, SA is a simple process for resolving important problems of practical sizes.

Previous research on cooling schedules include research analysis on mobile recommendation (Zeyang et al., 2015) and frameworks for matching dense images (Walid et al., 2017). Besides, Kai



Moriguchi et al. (2017) also examined cooling schedules by comparing two types of schemes. Consequently, the geometric and linear cooling schedules in the study identified the effectiveness of SA for optimising the thinning schedule for single forest stands. All the previous studies observed that geometric schemes provide the best results with the fastest running time (Zeyang et al., 2015; Walid et al., 2017; Kai Moriguchi et al., 2017). However, Walid et al. (2017) summarised the adaptive cooling schedule is the best for image matching. Therefore, this study recommends a flexible cooling factor model to accelerate the annealing process based on a SA schedule.

## 2. Design

### 2.1 Simulated Annealing Procedure

Before the development of the mathematical model, the following fundamental postulations were made by marginally adapting the earlier methods of Khairuddin et al., (2007). The summary of a fundamental empirical SA approach for solving combinatorial optimisation concerns is presented below:

1. Pick the empirical parameter settings to create a preliminary solution and its fee. This is described as the existing solution.
2. Acquire an adjacent solution to the existing solution through a localised exploration method.
3. Determine the cost of the adjacent solution and relate it to the present solution.
  - a. If the cost is more favourable, it is recognised as the existing solution.
  - b. If it is not cost-effective, then it is recognised as the existing solution with some likelihood. Alternatively, else, the existing solution is maintained.
4. Revise the counters and constraints and then replicate phases 2-4 pending when the suitable ending standard is achieved.

In particular, every repetition of the SA exploration technique migrates from its existing trial result to an immediate neighbour in the localised region of this result. Hence, the value of  $F(x)$  is the objective function for the existing trial result, whereas  $F(x')$  is the objective function of the current entrant in the subsequent trial result. Lastly, the term  $T_k$  measures the propensity to receive the existing candidate. This is based on the next trial solution, provided the entrant does not enhance the existing trial result. The rationale for choosing the immediate neighbour is hinged on the selection rule.

### 2.2 Move selection rule

From the direct neighbours of the actual test result, randomly choose one as the present entrant for the subsequent test result. If the aim is to maximise the objective function, the user can assent or discard the entrant as the ensuing test result according to the following criteria: If  $F(x') \geq F(x)$ , always agree to this candidate. However, when  $F(x')$  is less than  $F(x)$ , select the option with defined likelihood:  $\text{Prob}\{\text{acceptance}\} = e^{\delta}$ . Hence, the term  $\delta = (F(x') - F(x)) / T_k$ .

However, if the aim is to minimise the objective, the terms  $F(x')$  and  $F(x)$  can be inverted in the outlined equations. If the selected entrant is forbidden, recreate the procedure with a fresh arbitrarily designated direct neighbour of the actual test result. In the absence of any immediate neighbours, the algorithm must be shutdown.

Therefore, if the considered existing entrant is superior to the actual test result, it must be approved as the resulting test result. However, if inferior, the likelihood of approval is contingent the measure of how bad it is (or the magnitude of  $T$ ). Conversely, the rule of move selection typically approves a phase that is just somewhat downhill but never a sharp downward phase. Beginning with a comparatively high  $T$  value (as typically observed during Simulated Annealing) dramatically enhances the likelihood of approval. Typically, this facilitates continuous examination in a nearly arbitrary pattern. Similarly, slowly reducing the  $T$  value during the search (as typically observed during Simulated Annealing) increasingly lowers the likelihood of approval, which highlights that climbing is mostly upward. Therefore, over time, the selection of  $T$  values affects the measure of uncertainty in the procedure that permits the downward phase.

The typical technique of deploying the move selection rule to ascertain the likelihood of approval of a specific downward phase is through the random assessment of numbers from 0 to 1. These random numbers are assumed to be random interpretations of an identical distribution from 0 to 1. Numerous techniques have been proposed for creating such random numbers. Assuming a random number is less than  $\text{Prob}\{\text{acceptance}\}$ , a downward phase will be approved, or else reject.

The justification for adopting the specific equation of  $\text{Prob}\{\text{acceptance}\}$  outlined by the move selection rule during Simulated Annealing is due to its resemblance to the process of physical annealing. Initially, the procedure entails the extreme temperature melting of metal or glass, and gradually cooling pending when the substance attains a stable state of low energy with the anticipated physical characteristics. Typically, the atomic energy level in the substance, at any specified temperature  $T$  during the procedure, fluctuates although this tends to decline. The mathematical model of the manner in which the energy levels vary postulates that random variations occur but that only selective enhancements are recognised. Specifically, the likelihood of an increase is acceptable once the temperature  $T$  is similar to the form  $\text{Prob}\{\text{acceptance}\}$  based on the rule of move selection for Simulated Annealing.

Similar to the process of physical annealing, an essential consideration during the design of an algorithm to resolve optimisation problems during Simulated Annealing is to pick and use a suitable temperature schedule. Due to the physical annealing example, the term  $T$  in the Simulated Annealing algorithm is defined as temperature. The schedule in question must define an original and somewhat high  $T$  value along with other gradually decreased values. Also, the number of iterations (moves) must be defined for each value of  $T$ . The choice of the outlined variables to resolve the problem under deliberation is a critical dynamic when considering the algorithmic effectiveness. However, initial testing could be accepted to guide the choice of parameters used in the algorithm.

### 2.2.1 Pseudo code design of the SA algorithm for problem minimization

Pick an opening solution  $x$ ;

Pick an opening temperature  $T_k > 0$ ;

Choose temperature alteration counter  $k = 0$ ;

Repeat

Set repetition counter =  $n$  (number of iterations to be performed at each temperature)

Repeat

Choose a solution  $x'$  in  $N(x)$ , a neighbour of  $x$ ;

Calculate  $\delta = F(x') - F(x)$ ;

If  $\delta < 0$  then  $x = x'$

else if  $\text{random}(0,1) < \exp(-\delta/T_k)$  then  $x = x'$ ;

If  $F(x') < F(x)$  then set  $\hat{x} = x$  and  $\hat{T} = T_k$

Else keep  $\hat{x}$

$n = n+1$ ;

until  $n = N(k)$ ;

$k = k+1$ ;

$T_k = \text{cooling function}(T_k, k), T_k \leq T_{k-1}$

Up until the stopping criterion is satisfied.

### 2.3 Annealing Schedule

The cooling schedule is the heart of SA. Hence, the optimisation is primarily performed in the middle stages of the cooling schedule. First of all, a reasonable initial temperature is set. Next, the cooling rate ( $\alpha$ ) between 0 and 1 is set. It is essential to state that the fast schedule for cooling is comparable to the greedy algorithm. However, significant computation will be required if the prolonged cooling schedule is selected. Therefore, it is essential to pick the suitable cooling rate to ensure that the global optima can be reached using minimum computation. The temperature of the SA algorithm is steadily reduced to fulfil the conditions:

$$T_i > 0, \forall i$$

and

$$\lim_{h \rightarrow \infty} T_i = 0$$

Typically, there is a concession between the cooling schedule rate and quality of the solutions acquired. If the temperature is gently reduced, superior solutions are achieved but with higher computation time. The temperature  $T$  can be updated in different ways El-Ghazali Talbi, (2009):

- Linear – Based on the minor linear program, the temperature  $T$  is defined by the relation follows:

$$T = T - \beta$$

where  $\beta$  is a stated value of the constant.

- Geometric – Based on the geometric program, the temperature is revised based on the relation:

$$T = \alpha T$$

Where  $\alpha \in [0,1]$ . This is considered the most widespread function for cooling. Based on previous findings in the literature, the value  $\alpha$  is typically within the range 0.5 to 0.99.

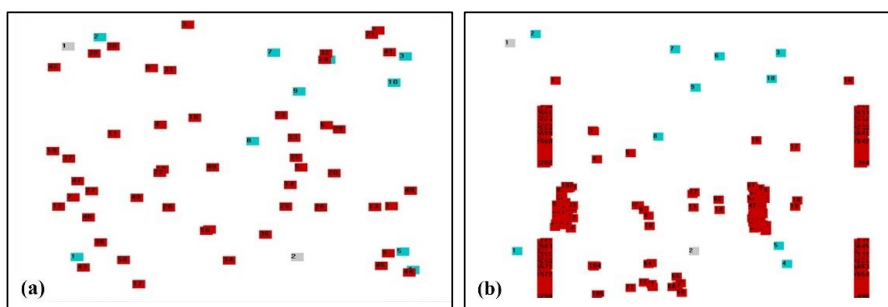
- Logarithmic – This is based on the relation:

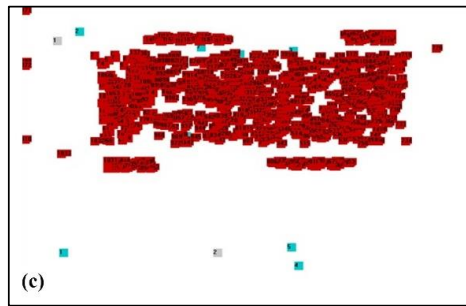
$$T_i = \frac{T_0}{\log(i+10)}$$

This program is considered too sluggish for practical applications, despite its capacity to deliver convergence proof to any global optimum.

### 3. Results and Analysis

Figure 1 presents the problem characteristics of three data sets. As observed, a varied type of distribution exists within the sets (Figure 1a-c). Data set 1 in Figure 1(a) exhibited a well-distributed network compared to the Data sets 2 and 3 in Figures 1(b) and (c), respectively. Data set 2 presented a cluster pattern of distribution type. However, Data Set 3 depicted a cramped distribution of the data set (Figure 1(c)). All of the data sets confined a similar coordinate; the warehouse and plant points.





**Figure 1.** Distribution of transshipment problem for (a) Data Set 1; (b) Data Set 2 and (c) Data Set 3

Table 1 summarises the characteristics of all data sets. It can be concluded that the number of customers for Data Set 1, 2, and 3 are 50, 654, and 1060, respectively.

**Table 1.** Characteristics of the Problem

| Data   | Number of Plants | Number of Warehouses | Number of Customers |
|--------|------------------|----------------------|---------------------|
| Data 1 | 2                | 10                   | 50                  |
| Data 2 | 2                | 10                   | 654                 |
| Data 3 | 2                | 10                   | 1060                |

Table 2 compares the result of three schemes across thirty runs with the maximum iteration set at 19000. The average achievement for each scheme regardless of the time taken has been conveyed. As expected, the logarithmic scheme found the best solutions with the smallest standard deviation based on average cost estimation. However, as discussed in the previous sub-section, the logarithmic approach took a long time to reach the stopping criteria.

Based on the considerations of this study, the geometric scheme offered the best performance within reasonable running time and cost estimation. This is in good agreement with Peprah et al. (2017) whose previous study observed that geometric schemes produce faster cooling rates, which are suitable for the annealing process.

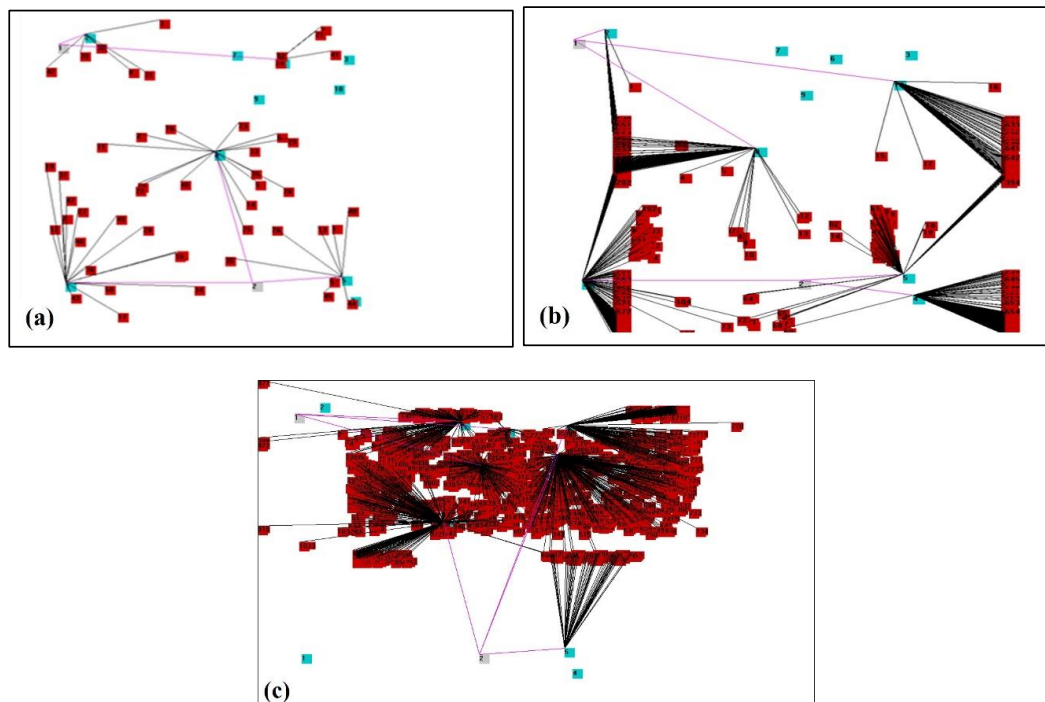
**Table 2.** Comparison of 3 schemes, time and cost varying

| Data                 | Data 1    |          |             | Data 2    |           |             | Data 3    |           |             |
|----------------------|-----------|----------|-------------|-----------|-----------|-------------|-----------|-----------|-------------|
|                      | geometric | linear   | logarithmic | geometric | linear    | logarithmic | geometric | linear    | logarithmic |
| Average cost         | 11601.73  | 11884.68 | 11397.60    | 172290.33 | 172315.27 | 172063      | 212783.20 | 216850.70 | 212756      |
| Standard deviation   | 216.77    | 242.77   | 0.0         | 435.27    | 444.34    | 0.0         | 13.83     | 1425.14   | 0.0         |
| Best solution        | 11397.6   | 11397.6  | 11397.6     | 172063    | 172063    | 172063      | 212756    | 212756    | 212756      |
| Average running time | 6.14      | 4.38     | 73.92       | 9.93      | 4.82      | 455.60      | 13.82     | 4.97      | 645.07      |

Based on the previous results, geometric schemes were selected for further improvement. Each temperature was run with four iterations before the decrease in temperature. It can be seen in Table 3 that the result supported the idea with improvements.

**Table 3.** Enhanced geometric for all three data sets

|                      | Data 1   | Data 2    | Data 3    |
|----------------------|----------|-----------|-----------|
| Average cost         | 11522.08 | 172069.50 | 212788.07 |
| Standard deviation   | 228.57   | 19.83     | 93.69     |
| Best solution        | 11397.60 | 172063    | 212756    |
| Average running time | 9.94     | 37.67     | 54.38     |



**Figure 2.** The best solution output of transshipment problem for (a) Data Set 1; (b) Data Set 2 and (c) Data Set 3

#### 4. Conclusion

This study recommended flexible models of cooling schemes for simulated annealing. The objective is to enhance the process of annealing and comparative examine its effectiveness against other geometric, linear and logarithmic cooling schemes in the literature. The findings revealed that the geometric scheme generates the fastest cooling rates compared to the other two schemes, which, although exhibited slower cooling rates are still applicable to the process of annealing.

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