

# Multivariate Change Point Estimation in Covariance Matrix Using ANN

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**Abstract.** In statistical process control, change point estimation is an essential requirement for diagnosing the source of a deviation when a process is out of control. In this study, an ANN-based method is proposed to estimate the change point in the multivariate normal process which is subjected to covariance variation. Since in a physical system parameter is correlated, correlation is kept constant to obtain realistic simulated data. Employing statistical simulation, different out of control scenarios are simulated and statistics are calculated for each scenario. This study is to predict the change point in the control chart using the simulated set and corresponding statistical sets, an ANN is adopted. The resulting model achieved a high accuracy of 90% in training and 80% testing with a reasonable level of confidence in the prediction. Also, results show that Bayesian reaches a higher accuracy than Levenberg in ANN training.

## 1. Introduction

Control charts are efficient tools in SPC for the monitoring process. [1,2,3]. Multivariate process control techniques were established by Hotelling (1947) were in his pioneering paper he suggested the use of the  $T^2$  statistic for monitoring the mean vector of multivariate processes [1]. Literature reviews show most of the research in the last 20 years has focused on developing multivariate control charts for monitoring shifts in the process mean. However, these control charts were not capable of determining the change points [2,3]. Recently, several works have been reported on the monitoring of shifts in the process mean of multivariate processes and at the same time estimating the change points using various new methods. Niaki and Khedmati [4] suggested a method for estimating the change point of a multivariate Poisson process mean vector when the type of change follows a linear trend. Then, they compared the linear trend estimator performance with a step-change point estimator.

The artificial neural network (ANN) method has also received great attention in estimating the change point by monitoring the changes in the mean vector due to its remarkable performance compared to other methods. Ahmadzadeh et al. [5] evaluated the step-change point in the mean vector of the multivariate normal process by applying ANN. An ANN-based multivariate exponentially weighted moving average (MEWMA) control chart which not only had the ability to estimates step point change in mean vector but also could identify the shifted variables was developed. Atashgar [6]

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proposed a supervised learning ANN to discover the change point with a linear trend in the mean vector of the bivariate neural network. The approach not only had the ability to estimate the change point but also could identify the deviation variables. Their proposed method includes three neural networks that can identify out of control situations, determine variables that cause change and estimate the change point with a linear trend in the mean vector of the process. Amiri et al. [7] applied an integrated supervised learning-based approach including dozens of neural networks to identify a single step change point in the mean vector of the multivariate process. The proposed method had the ability to identify out of control situation, estimate the change point of shifted variable and determine the variable/variables which contributed to shifts. However, these works generally identify change -point considering only the mean vector. While the variability of change in the multivariate process is also very important [3,7].

In this paper, our motivation is to develop a new ANN method to determine the change point (real-time) by monitoring the covariance change of a multivariable process. The first used one MEWMS<sub>AS</sub> (multivariate exponentially weighted mean squares) control chart for showing the out of control situation. In the following section, have been present the algorithm of the proposed modified ANN method with covariance change and, evaluation of its performance. Finally, the conclusion is presented.

## 2. Methodology

### 2.1 The MEWMS<sub>AS</sub> Procedure

MEWMS<sub>AS</sub> has monitored variability in the multivariate process. Let  $X_1, X_2, \dots, X_t, X_{t+1}, \dots, X_T$  are independent vectors from observations in which  $X$  is a normal distribution with  $p$  variables. The process was under control until the moment of  $\tau$  and it is a normal distribution. After the moment of  $\tau$ , a shift occurred in the process covariance matrix that causes the process distribution to alter into  $N_p \sim (\mu_0, \Sigma_1)$  in which  $\Sigma_{-1}$  is the process covariance matrix. It is assumed that the standard deviation of out of control variables remains on the new level until the discovery of out of control mode and taking corrective actions. As a result, the control chart of MEWMS<sub>AS</sub> discovered the occurred shift in the Variability the control statistic of the chart based on the MEWMS<sub>AS</sub> deviation with the approximated distribution of the sum of all elements (MEWMS<sub>AS</sub>) is  $\text{sum}[S_t]$  where its statistics are [4]:

$$Y_{ij} = \sum_0^{-1} \frac{1}{2} (X_{ij} - \mu_0) \quad (1)$$

$$S_t = (1-\lambda)S_{t-1} + \frac{\lambda}{n} \sum_{j=1}^n Y_{ij} Y'_{ij} \quad (2)$$

$$S_1 = \frac{1}{n} \sum_{j=1}^n Y_{ij} Y'_{ij} \quad (3)$$

and where its control limits are:

$$UCL_{MEWMS_{AS}} = \frac{p}{v_{AS}} \chi^2_{\frac{\alpha_{AS}}{2}, v_{AS}} \quad (4)$$

$$LCL_{MEWMS_{AS}} = \frac{p}{v_{AS}} \chi^2_{1-\frac{\alpha_{AS}}{2}, v_{AS}} \quad (5)$$

where the degree of freedom :

$$v_{AS} = \frac{n(2-\lambda)}{\lambda} \quad (6)$$

When the MEWMS<sub>AS</sub> control chart show signals that a sample data is out of control, the sample data are collected as input data into the trained networks. In this step, it is assumed that the generating data have two variables ( $P = 2$ ), a normal distribution with mean (0,0), (assumed mean = constant) unit covariance and a correlation coefficient has changed. The values of  $X = (X_1, X_2)$  are the observations, the values  $Z = (Z_1, Z_2)$  correspond to the MEWMS<sub>AS</sub> vector in (3) with  $r = 0.1$ .

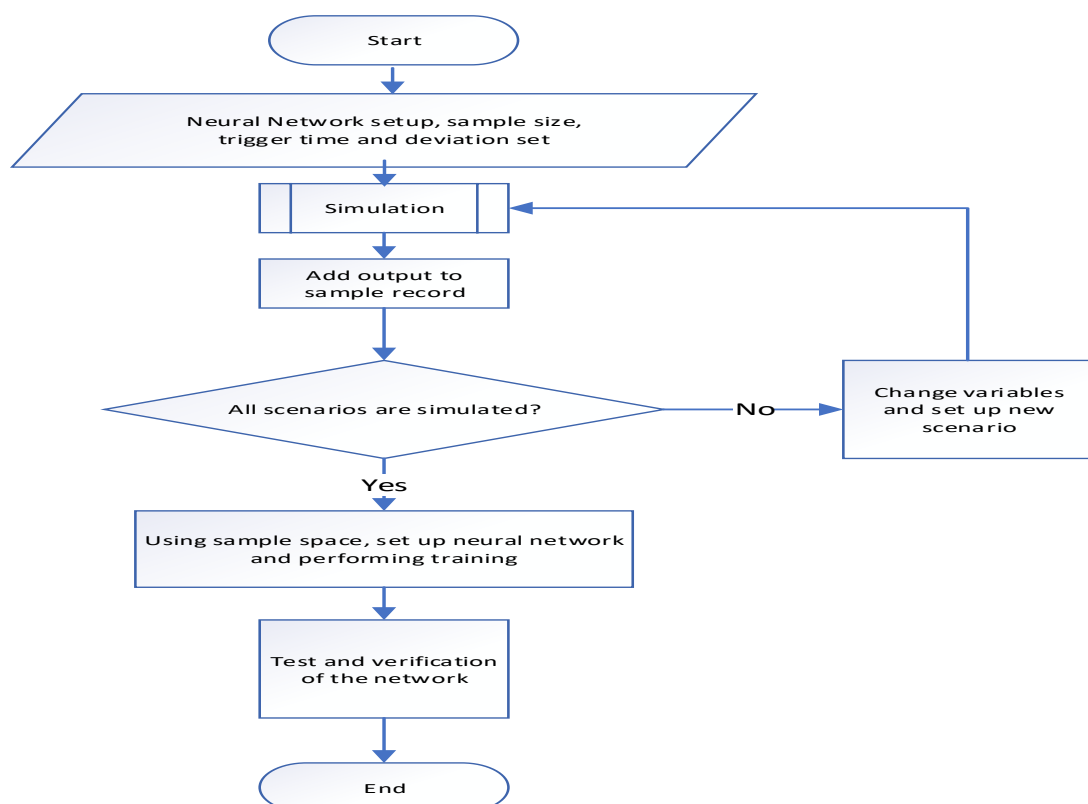
### 2.2 The proposed ANN model to estimate the change point

Assuming a normal process with  $p$  variables is under control and possesses a covariance matrix of  $\Sigma_1$  as follow:

$$\Sigma_1 = \begin{bmatrix} \delta_1^2 \sigma_1^2 & \rho \delta_1 \delta_2 \sigma_1 \sigma_2 \\ \rho \delta_1 \delta_2 \sigma_1 \sigma_2 & \delta_2^2 \sigma_2^2 \end{bmatrix} \quad (7)$$

Let each layer of the ANN model to have unique weight matrix ( $w$ ), bias vector ( $b$ ), net-input vector ( $n$ ), output vector ( $a$ ), and # of neurons ( $S$ ).

$$a = f(wp + b) \quad (8)$$

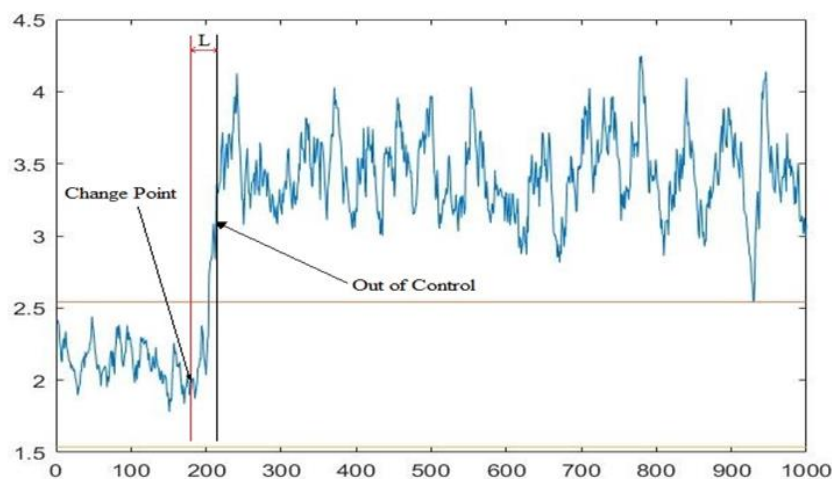


**Figure 1.** Algorithm for the simulation module.

As usual, Levenberg- Marquardt, and Bayesian have been used to train the multi-layer networks with backpropagation. Regarding the generated data, three different scenarios were considered, and each scenario involves 1000 generated data. The first scenario modifies  $\sigma_1$ , and the second scenario alters  $\sigma_2$ ; however, in the third scenario,  $\sigma_1$  and  $\sigma_2$  have been changed simultaneously.

### 3. Results

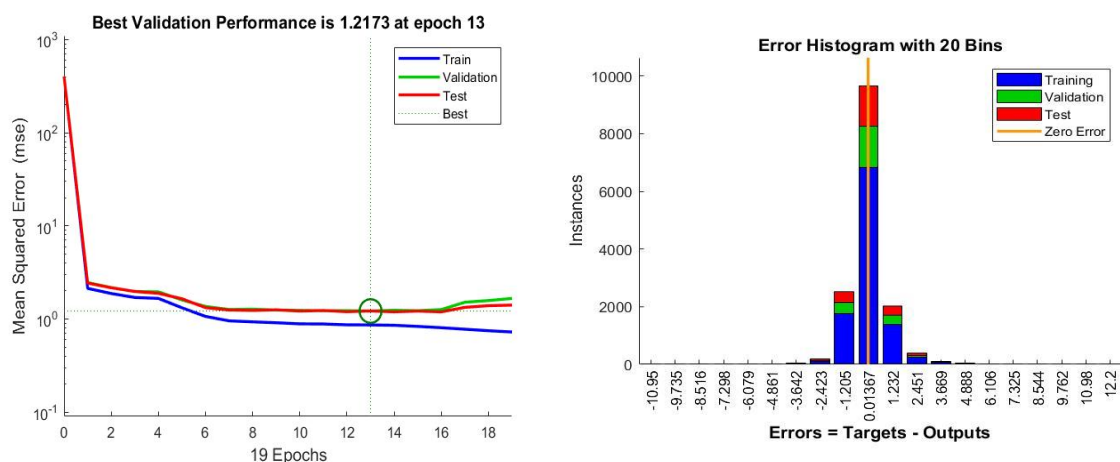
Since variance is subject to monitoring, the  $MEWMS_{AS}$  control chart has been used here as discussed earlier. Figure 2 shows how this control chart reacts to a step-change in variances of two variables under control. This graph shows the ability and fast response of the control chart to variance variations; which makes it suitable for our study. As the figure shows, L is time between out of control detection and the change point. The efficiency of estimation of L is equivalent to the accuracy of change point estimation. So, hereafter the performance of the developed ANN is discussed in terms of accuracy.



**Figure 2.** Performance of  $MEWMS_{AS}$  with  $\Delta\sigma = 0.5$ .

#### 3.1 Developing neural network model

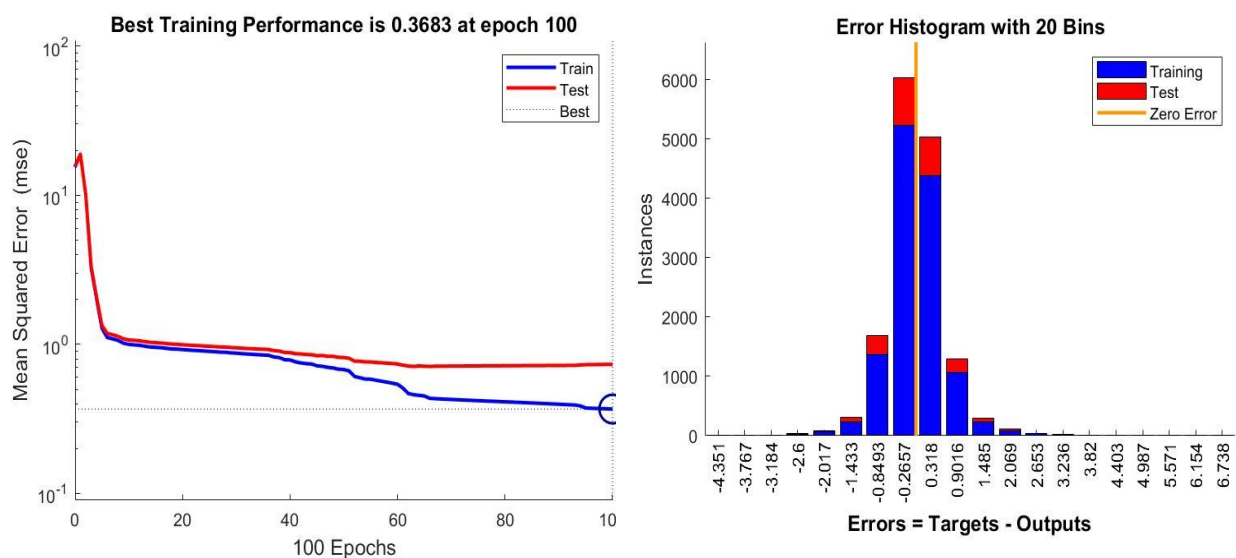
To develop the ANN model, two network configurations and models are tested. Figure 3 shows the performance and error distribution for the Levenberg-Marquardt model with 5 hidden layers. The accuracy achieved is 85% training, 75% validation and 75% testing.



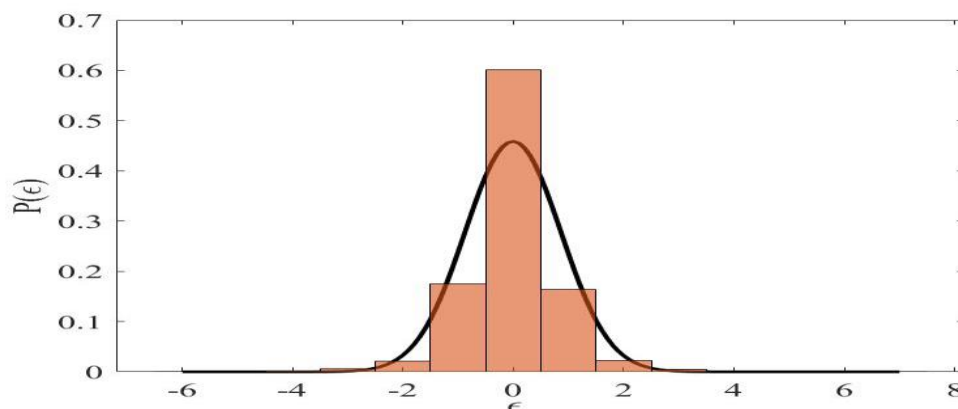
**Figure 3.** Performance (left) and error (right) diagrams for ANN with 5 hidden layers and Levenberg model.

Finally, the best accuracy is obtained using the Bayesian training model with 7 hidden layers. The best accuracy is 90% for training and 82% for testing. The performance of the model is presented in Figure 4. After obtaining the model, the goodness and confidence level of the model should be evaluated. To investigate, 1000 new cases are simulated. Detection time is estimated by the model and error of estimation is evaluated by comparing it with real-time obtained from the simulation. Then a normal distribution probability distribution function is fitted to the error data. The analysis shows that

data are normally distributed with a PFD with ( $\mu = -0.00635$ ,  $\sigma = 0.87$ ) as shown in Figure 5. Then probability for different error values is calculated and presented in Table 1.



**Figure 4.** Performance (left) and error (right) diagrams for ANN with 7 hidden layers and Bayesian model.



**Figure 5.** The error of ANN with 7 hidden layers.

**Table 1.** Probability of errors

Error	Probability
$ \epsilon  < 0.5$	0.4300
$ \epsilon  < 1$	0.7500
$ \epsilon  < 2$	0.9785
$ \epsilon  < 3$	0.9990

According to the above table, the presented model can estimate the change point with less than 2 sampling time unit accuracy with a confidence level of 95%.

#### 4. Conclusion

A neural network-based model is developed here to estimate the change point in the covariance matrix of a process. Though the model is developed for a two-variable system, the method can be generalized for multivariate systems as well. Different neural network size is used, and it can be concluded that:

- a) The Bayesian model is more suitable than Levenberg.
- b) 7 hidden layer helps the best results. Accuracy of 90% for training and 82% for testing is achieved.
- c) The developed model can estimate the change point accurately. the confidence level of 95% is achieved for the absolute error values less than 2.

Based on the obtained result one may conclude that ANN can accurately and efficiently be used for change point estimation, despite the problem stochastic nature.

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