

H_{∞} mixed sensitivity optimization for high speed tilting trains

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ABSTRACT

The industrial norm of tilting high speed trains, nowadays, is that of Precedence tilt (also known as Preview tilt). Precedence tilt, although successful as a concept, tends to be complex (mainly due to the signal interconnections between vehicles and the advanced signal processing required for monitoring). Research studies of early prior to that of precedence tilt schemes, i.e. the so-called Nulling-type schemes, utilized local-per-vehicle signals to provide tilt action (this was essentially a typical disturbance rejection-scheme) but suffered from inherent delays in the control). Nulling tilt may still be seen as an important research aim due to the simple nature and most importantly due to the more straightforward fault detection compared to precedence schemes. The work in this paper presents a substantial extension conventional to robust H_{∞} mixed sensitivity nulling tilt control in literature. A particular aspect is the use of optimization is used in the design of the robust controller accompanied by rigorous investigation of the conflicting deterministic/stochastic local tilt trade-off.

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1. INTRODUCTION

Tilting trains is a worldwide accepted technology concept in high speed railway transportation. It has been successfully established as a part of modern railway vehicle technology with many high-speed train services worldwide fitted with tilt [1-3] and an increasing interest for regional express trains as well as recently attempt to apply in metro systems [4]. The tilting concept is quite straightforward whereby usually a tilting mechanism (that is inverted pendulum-like platform) is employed to (mostly actively) lean the vehicle body inwards on track corners hence reducing the lateral acceleration level experienced by passengers. The particular benefit from tilting trains use is reduction in journey times due to increase of speed on track corners. From a more local (South-East Asia) area viewpoint, use of tilting service could be beneficial to the Malaysia High Speed Railway project since it presents a potentially cost effective solution to consider and hence potentially avoid building an extensive (and expensive) new rail-track infrastructure.

In most cases of high speed tilting trains, active control is used to perform the tilting action and active tilting train systems technology has been greatly improved by the major contributions of control engineering [1-2]. Initial control design attempts on tilting trains employed the so-called "nulling" tilt control approach [5], i.e. feedback control from a single lateral accelerometer mounted on the body of the vehicle (regarding required tilt) passenger vehicle. This early attempt proved to be challenging to perform sufficiently fast response on the curve transitions without causing a degradation of ride quality on (straight) track misalignment as well as system stability.

As mentioned already nowadays tilting trains employ a command-driven system in which a signal from an accelerometer on a non-tilting part of the precedent vehicle (and sometimes from a database) commands the required tilting angle, with a straightforward tilt angle feedback controller locally ensuring that each vehicle tilts to the indicated tilt angle [6, 7]. This solution is commonly known as tilt with precedence i.e. utilise preview-tilt information from the previous vehicle with a sufficient level of filtering to be applied to remove the effect of track irregularities on the tilt command signal. Essentially tilt-with precedence attempts to improve the performance issues of “Nulling”-type. The preview-tilt approach is the currently accepted in industrial practice in tilting trains systems, but it can be a complex overall scheme; amongst other things it must reconfigure when the train changes direction, still difficult to provide a satisfactory performance for the leading vehicle of the train. It must be noted that GPS systems are used in some cases to provide the “when-to-tilt” (preview) command via track database information, although issues of signal quality communication, delays, and constraints due to tunnels may affect operation and add further complexity [1, 8, 9].

Previous studies of control applications in railway exist [10-12]. In recent publication specifically, many studies has been done before [13-15] and more recent studies in [16-18]. Although there is no depth investigation via optimization relative to tilt control interest-cost function H_∞ mixed sensitivity controller design. It is proven to be difficult to achieve the trade off between deterministic and stochastic in tilt control performance via manually designed controller [7].

This paper presented, exactly this, i.e. the impact of optimizing H_∞ mixed sensitivity controller in achieving improved results on the tilt control trade off between deterministic and stochastic performance via optimizing weight constraints. The controller performance is rigorously assessed using both frequency-domain and time-domain (simulation) analysis.

2. MODELLING

The simple tilting train setup use here with anti roll bar as tilt across secondary tilting mechanism. The end view vehicle is presented in Figure 1. The details mathematical expression can be referred to [16]. As expected in dynamic behavior railway system, actuator dynamics parameters systems are selected to provide damping of 50% and 3.5hz bandwidth. Linearised version of non linear behavior system is used here as it gives good approximation for analysis and designing robust controller. The overall roll angle from the horizontal (track elevation + expected tilt) shall not exceed 14 degrees.

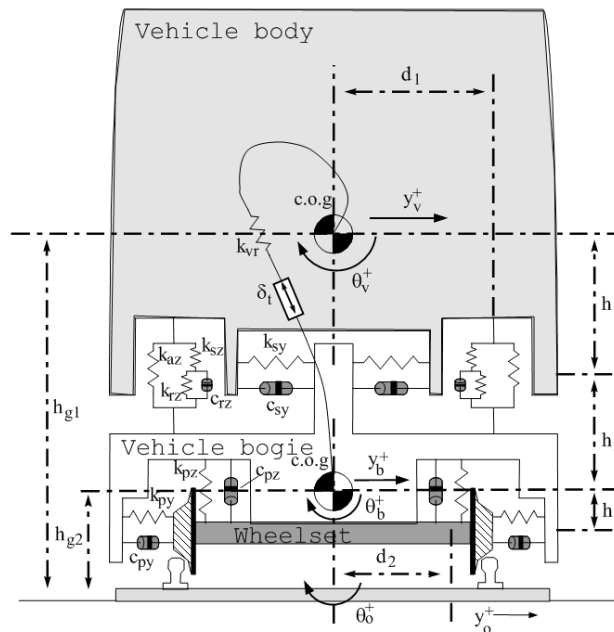


Figure 1. End view of anti roll bar (ARB) tilting vehicle structure [7]

The nominal transfer function from mathematical model in [16] is given by (1)

$$G(s) = \frac{Y_{e.cd}}{\partial_a} = \frac{2753(s+2618)(s+4073)(s-29.36)(s-6.02)(s^2+7.65s+24.44)(s^2+4.825s+15870)}{(s+23.2)(s^2+5.11s+8802)(s^2+22s+4836)(s^2+29.15s+4888)} \quad (1)$$

With the state vector, control input and exogenous input vectors $w(t)$ are dropped for simplicity.

$$x = [y_v \ \theta_v \ y_b \ \theta_b \ \dot{y}_v \ \dot{\theta}_v \ \dot{y}_b \ \dot{\theta}_b \ \theta_r \ \delta_t \ \dot{\delta}_t \ y_w \ (y_w)]^T, u = [\delta_{ti}]$$

$$w = [R^{-1}\dot{R}^{-1}\theta_o \ \dot{\theta}_o \ \ddot{\theta}_o \ y_o \ \dot{y}_o]^T \quad (2)$$

The details definition of values and parameters in (2) was presented in [16]. This represent 60% tilt compensation of effective cant deficiency and ideal control input tilt angle $\Delta_{(t-i)}$. From nominal transfer function in (1), it is noticeable that the existence of non minimum phase(NMP) zeros located at 29.36 and 6.02. It is well known that the existence of NMP zeros on the right hand plane will constraint the bandwidth. It is difficult to design good and robust controller due to this NMP zeros location that is close to origin. There are two exogenous inputs used as assessments in this model, (i) deterministic track input and (ii) straight track misalignment in lateral direction also known as stochastic track input [19]. The stochastic track input velocity spectrum is represented by,

$$S_r(f_t) = \frac{(2\pi)^2 \Omega_1 v^2}{f_t} (m/s)^2 Hz^{-1} \quad (3)$$

Where v is the vehicle speed (58 m/s – 30% higher than non-tilt) and f_t is the temporal frequency in Hz. For simulation purpose, the value of Ω_1 is chosen according to typical medium-quality rail track which is $0.33 \times 10^{-8} m$. For deterministic track inputs a rail track corner with maximum cant angle θ_o^{max} of 6 degree, maximum curve radius $R_{max} = 1km$, transition length = 145m at each end and track length of 1.2km are used in simulation process. The weight lateral acceleration of passenger by w_z Sperling index is assessed for ride quality purpose [20].

2.1. H_∞ mixed sensitivity controller design

The H_∞ mixed sensitivity controller design is useful in order to achieve robust control performance. With a robust design, the stability of the system in nominal plant can be achieved. In tilt control especially, manual design of H_∞ mixed sensitivity was investigated in [21]. However, it is proven that by using manual design, the tilt control performances trade off are difficult to achieved. Here, the SISO advanced (integer-order) robust approach is extended by using optimization in tuning the related weighting functions for the H_∞ mixed sensitivity design. The feedback structure of the proposed controller designed is shown in Figure 2.

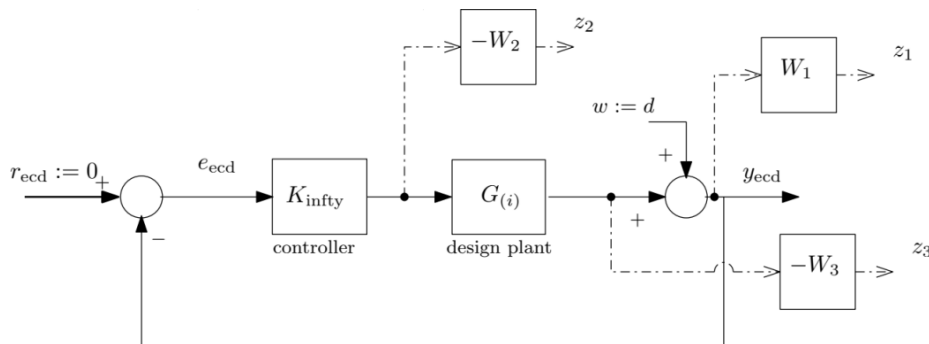


Figure 2. H_∞ mixed sensitivity feedback structure

W_1 , W_2 and W_3 is frequency weight for sensitivity $S(jw)$, control sensitivity $(KS(jw))$ and complimentary sensitivity $(T(jw))$. The overall minimization is summarized in (4), where ∞ (means infinity norm)

$$\min_{K \in S} \begin{Bmatrix} W_1(1+GK)^{-1} \\ W_2K(1+GK)^{-1} \\ W_3GK(1+GK)^{-1} \end{Bmatrix} \infty \tag{4}$$

Here, we designed W_3 as multiplicative uncertainty bound covers NMP zeros in (1) based on the previous paper by [19]. This bound is illustrated in Figure 3. W_1 and W_2 is obtained via optimization. We presented two cases of optimization, P_1 (all the weight sensitivity are included in the optimization process) and P_2 where W_3 is excluded.

$$W_3(jw) = \frac{-j70.76w}{(176.7 - w^2) + j35.38w} \tag{5}$$

$$W_1 = \frac{(s/(M^{1/2})) + W_{B*})^2}{(s + W_{B*}(A^{1/2}))^2} \tag{6}$$

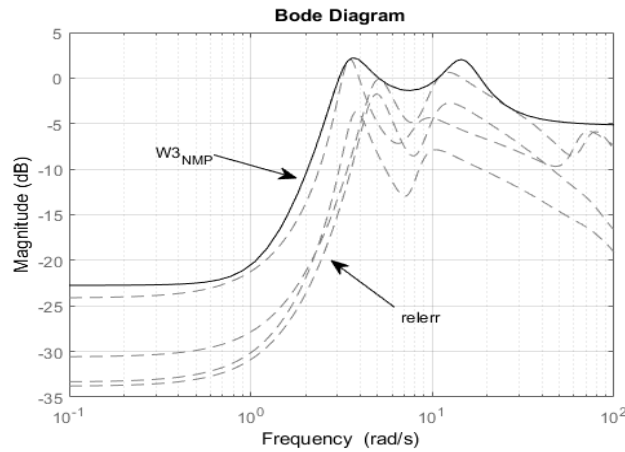


Figure 3. Multiplicative uncertainty bound W3

The second order transfer function formula for W_1 in (6) was introduced by [22, 23] where W_{B*} is maximum frequency bandwidth, M is the maximum peak of Sensitivity ($S(jw)$) and A is maximum steady state error. The initial value of upper and lower bound of W_1 was obtained from previous PID design by [24, 25]. The presented framework is designed to have closed-loop stability, good tracking or disturbance rejection performance and robust stability. However, robust performance analysis assessment will not be included in this paper. The optimization process is implemented via *fmincon()* in MATLAB. The minimization problem for H_∞ mixed sensitivity is given by,

$$\begin{aligned} & \text{Minimize } f(x) = P_{CT \text{ standing}} \\ & \text{Subject to } < \text{constraint} = \text{rdq} \leq 7.5\% > \end{aligned} \tag{7}$$

Where P_{CT} standing is the percentage of standing passengers feeling uncomfortable on curve transition and r.d.q is ride degradation quality compared to non tilting trains based on Europe standard [20]. The results of tuned weight sensitivity via optimization is presented in Table 1.

Table 1. Weight sensitivity parameters for P1 and P2

Minimization ID	W_1	W_2	W_3
P_1	$\frac{s^2 + 1.317s + 0.4337}{1.255s^2 + 0.06102s + 0.0007416}$	0.9355	as in eq. (5)
P_2	$\frac{s^2 + 3.06s + 2.341}{2s^2 + 0.1375s + 0.00236}$	0.7043	NA

3. RESULTS AND ANALYSIS

The nominal tilt performance and performance margins for both P_1 and P_2 with full order controller is presented in Table 2 and Table 3. The results for both cases shows very satisfactory and improved tilt performance compared to the one presented in [24, 25]. By not including complimentary sensitivity in optimization problem, P_2 gives better P_{CT} performance for both standing and seating (less is better in this case). Also for both cases, less than 7.5% ride quality degradation is obtained. Both trade-off between deterministic and stochastic that is difficult to achieved via manual design can be achieved via H_∞ mixed sensitivity optimization.

Table 2. Nominal tilt performance for P_1 and P_2

H_∞ mixed sensitivity optimization		P_1	P_2
P_{CT}	Standing (% of passeng.)	48.703	46.41
related	Seated (% of passeng.)	13.029	11.912
Ride quality	Tilting train	3.059	3.052
(passenger comfort)	Degradation. (%)	7.413	7.166

Table 3. Performance margin

Cases	P_1	P_2
Gain margin (dB)	8.306	6.243
Phase margin (deg)	52.682	44.513
Bandwith (rad/s)	1.25	1.23
$\ S(j\omega)\ _\infty$	1.625	1.925

Stable performance margins can be seen on Figure 4. Both cases show satisfactory open loop response with satisfies gain and phase margin. In term of lateral acceleration, P_2 case perform faster than P_1 . This can be see in Figure 5.

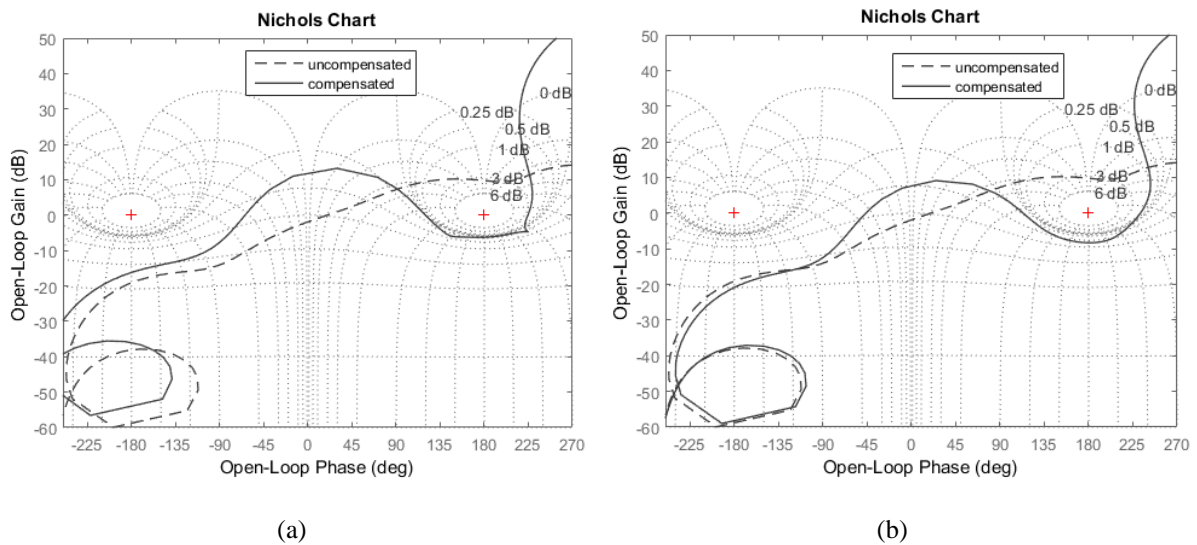


Figure 4. Open loop frequency response for P_1 (a) and P_2 (b)

Although in nominal performance with the introduction of complimentary sensitivity W_1 less superior compared to P_2 case, it is expected to gives better robust performance . More conservative controller designs tend to provide better robust performance but can be far from the desired aim of improving Nulling-type tilt control performance.

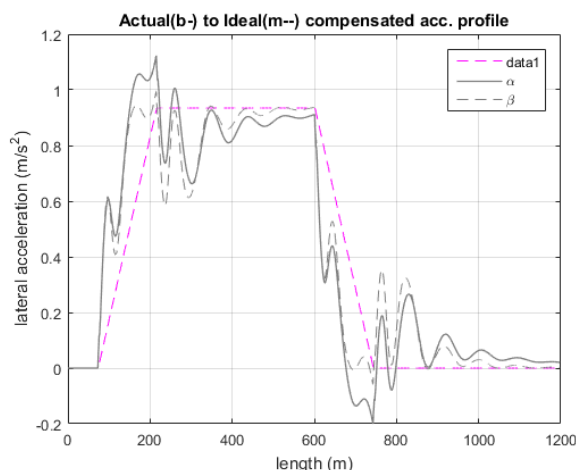


Figure 5. Lateral acceleration for P_1 (alpha) and P_2 (beta)

4. CONCLUSION

This paper has presented an optimized viewpoint of H_∞ mixed sensitivity design for tilt control (emphasizing the single input single output (SISO) problem aspect in this application). The impact of the proposed controller in nominal performance was showcased via extensive simulation results. With the presented benefits of proposed controller with respect to tilting train control performance, the (optimized) H_∞ -mixed sensitivity controller design approach can be applied to other active suspension vehicle problems. Future points of interest relating to the extension of this work are controller reduction (while maintaining robust performance) considering a gain-scheduled framework.

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