

STRUCTURAL BEHAVIOUR OF END-PLATE CONNECTIONS ON HYBRID
BEAM TO CRUCIFORM COLUMN SECTION

TAN BOON CHEIK

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Faculty of Engineering
Universiti Teknologi Malaysia

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DEDICATION

Specially dedicated to
my beloved parents, brother, sister, and *Ker Shin*.

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ABSTRACT

Cruciform column section is an innovative column section which consists of two universal beam sections where one of the beams is cut into half along its web and welded onto the web of other beam. Distinct geometrical differences compared to conventional universal H-shaped column section are that the cruciform column has smaller width to depth ratio and lower flange thickness. Its advantages as an alternative vertical compressive member were well proven by previous researchers since 2005. This study focuses on structural behaviour of flush end plate and extended end plate connections of hybrid beam to cruciform column section. Experimental tests of four flush end-plate and four extended end-plate connections on hybrid beam to cruciform column section were conducted and the results were used to validate analytical study and finite element modelling (FEM). Analytical study was conducted based on Eurocode 3: Part 1.8 component method and existing mathematical models. It was found that Eurocode 3: Part 1.8 can be used to predict initial stiffness and moment resistance of cruciform column connections. Existing mathematical models were not able to predict moment rotation behaviour of cruciform column connections but were specifically useful for predicting connection behaviour within the limit of regression model. Hence, comprehensive finite element analysis using ANSYS 14.0 on the cruciform column connections was carried out to predict the moment rotation behaviour. Initial stiffness and moment resistance of the models were in good agreement with experimental test results with percentage difference well within 20%. In terms of failure mode, the deformation shown in FEM exhibited similar pattern with experimental tests and Eurocode 3: Part 1.8. Parametric analysis was carried out using validated FEM model. Critical zones were determined through stress distribution pattern using stress ratio and verified with linear-plastic and semi-rigid partial strength connection behaviour of cruciform column connections. From the parametric analysis, it was identified that the significant parameters for cruciform column connections were beam depth, end-plate thickness, and column flange thickness. Simple mathematical functions were developed to predict moment rotation behaviour of cruciform column connections using regression analysis and were strongly supported by statistical analysis. As compared to existing finite element models, the initial stiffness and moment resistance percentage differences were well within 15% for both single bolt row flush end-plate and extended end-plate cruciform column connections. Based on these outcomes, practising engineers will be able to predict the moment rotation behaviour of cruciform column connection conveniently and accurately using the developed mathematical function.

ABSTRAK

Tiang krusiform merupakan tiang inovatif bentuk salib yang terdiri daripada dua rasuk di mana salah satu rasuk dipotong sepanjang web dan dikimpal ke web rasuk yang lain. Perbezaan daripada segi geometri berbanding dengan tiang konvensional yang berbentuk H ialah nisbah lebar dengan kedalaman dan ketebalan bibir tiang krusiform adalah lebih kecil. Kelebihannya sebagai struktur mampatan alternatif telah dibuktikan oleh ramai penyelidik terdahulu sejak tahun 2005. Kajian ini memberi fokus kepada kelakuan struktur bagi sambungan jenis plat hujung rata dan plat hujung ditambah antara rasuk hibrid dengan tiang keratan krusiform. Ujian makmal yang terdiri daripada empat plat hujung rata dan empat plat hujung ditambah antara rasuk hibrid dengan tiang krusiform telah dijalankan dan keputusan ujian digunakan untuk pengesahan kajian analitikal dan pemodelan unsur terhingga (FEM). Kajian analitikal telah dijalankan dengan menggunakan kaedah komponen Eurocode 3: Part 1.8 dan model matematik yang sedia ada. Hasil kajian analitikal mendapati Eurocode 3: Part 1.8 boleh diguna untuk meramal kekakuan awal dan momen rintangan sambungan krusiform. Walau bagaimanapun, model matematik yang sedia ada tidak dapat meramal kelakuan putaran-momen bagi sambungan tiang krusiform, tetapi model tersebut hanya boleh diguna khusus untuk kelakuan sambungan model putaran-momen yang berada dalam julat model regresi. Oleh itu, pemodelan dan analisis unsur terhingga yang komprehensif menggunakan ANSYS 14.0 untuk sambungan krusiform telah dibuat untuk meramal kelakuan sambungan krusiform. Nilai kekakuan awal dan rintangan momen yang diperolehi daripada analisis unsur terhingga dibandingkan dengan ujian makmal dan peratusan perbedaannya didapati dalam lingkungan 20%. Daripada segi mod kegagalan, perubahan bentuk yang ditunjukkan dalam keputusan analisis unsur terhingga mempamerkan corak yang sama dengan ujian makmal dan Eurocode 3: Part 1.8. Zon kritikal ditentukan melalui taburan tegasan menggunakan nisbah tegasan dan telah disahkan dengan pemodelan plastik-lurus dan kelakuan kekakuan separa sambungan tiang krusiform. Analisis parametrik juga telah mengenalpasti parameter sambungan tiang krusiform yang penting iaitu kedalaman rasuk, ketebalan plat hujung dan ketebalan bibir tiang. Fungsi matematik yang mudah telah dibangunkan untuk meramalkan sifat sambungan tiang krusiform dengan menggunakan analisis regresi dan telah disokong oleh analisis statistik. Perbandingan dengan model unsur terhingga yang sedia ada mendapati perbezaan peratusan kekakuan awal dan rintangan momen berada dalam lingkungan 15% untuk sambungan plat hujung rata dan plat hujung ditambah. Berdasarkan hasil kajian ini, jurutera struktur boleh meramal kelakuan sambungan tiang krusiform dengan mudah dan tepat berdasarkan fungsi matematik yang dibangunkan.

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LIST OF ABBREVIATIONS

BS EN1993-1-1	-	Eurocode 3: Part 1.1 (BS EN 1993-1-1: 2005)
BS EN1993-1-5	-	Eurocode 3: Part 1.5 (BS EN 1993-1-5: 2006)
BS EN1993-1-8	-	Eurocode 3: Part 1.8 (BS EN 1993-1-8: 2005)
EXP	-	Experimental results
FEM	-	Finite element modelling
LVDT	-	Linear Variable Differential Transducer
2D	-	Two-dimensional
3D	-	Three-dimension
FEP	-	Flush end-plate
EEP	-	Extended end-plate
CCUB	-	Cruciform column universal beam section

LIST OF SYMBOLS

b_b	-	width of beam
b_c	-	width of column
b_p	-	width of end-plate
d_{bolt}	-	bolt diameter
d_c	-	depth of column between fillets
d_e	-	distance from top of beam flange to edge of end-plate
d_g	-	distance between the top bolt row and bottom bolt row
f_u	-	ultimate of base material
f_{ub}	-	ultimate strength of bolt
f_y	-	yield strength of base material
$f_{y,b}$	-	yield strength of beam
$f_{y,c}$	-	yield strength of column
$f_{y,bolt}$	-	yield strength of bolt
f_{yp}	-	yield strength of end-plate
g	-	depth of beam
h_b	-	depth of beam
h_c	-	depth of column
h_i	-	distance of the i^{th} bolt row from the centre of
h_p	-	depth of end-plate
k_i	-	stiffness coefficient for its basic joint component i^{th}
k_{wc}	-	reduction factor for column web in transverse compression
n	-	shape factor
n_b	-	number of bolts in each row
p_{1-2}	-	distance from first bolt row to second bolt row
p_{2-3}	-	distance from second bolt row to third bolt row
p_f	-	distance from top of beam flange to first bolt row for flush end-
p_{fi}	-	distance from top of beam flange to first bolt row extended
r	-	root radius
t_{fb}	-	thickness of beam flange

t_c	-	thickness of column flange
t_s	-	thickness of stiffener
t_p	-	thickness of end-plate
t_{wb}	-	thickness of beam web
t_{wc}	-	thickness of column web
w	-	bolt spacing or gauge distance
z	-	lever arm
A_{bolt}	-	single bolt area
A_c	-	column cross sectional area
A_{fb}	-	beam flange area
A_s	-	tensile stress area of the bolt
A_{wb}	-	beam web area
A_{vc}	-	shear area of column
E	-	elastic modulus
E_s	-	secant modulus
$F_{c,Rd}$	-	compression resistance of joint
F_{ri}	-	resistance of bolt row i^{th} in the tension zone
I_c	-	column moment of inertia
L_b	-	bolt elongation length
M	-	applied joint moment
$M_{c,Rd}$	-	design resistance for bending
$M_{j,Ed}$	-	design moment
$M_{j,Max}$	-	maximum moment
$M_{j,Rd}$	-	moment resistance
M_o	-	reference moment
N_{Ed}	-	axial force
S_j	-	rotational stiffness
$S_{j,ini}$	-	initial stiffness
$S_{j,p}$	-	plastic stiffness
$W_{el,b}$	-	beam elastic section modulus
$W_{pl,b}$	-	beam plastic section modulus
$W_{pl,c}$	-	beam plastic section modulus
β	-	transformation parameter

θ_{joint}	-	joint rotation
θ_{beam}	-	beam rotation
θ_{Cd}	-	rotation capacity
θ_{column}	-	column rotation
θ_{Xd}	-	permanent rotation
μ	-	is the stiffness ratio
ω	-	reduction factor
ε	-	factor depending on f_y for cross section class classification

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CHAPTER 1

INTRODUCTION

1.1 Background of Study

Bolted end-plate connection undeniably is one of the most popular choices and has been widely used as beam to column connection in steel structure. The high demand is mainly due to its high ductility and moment resisting behaviour. On the other hand, the fabrication, assembly, and erection of bolted end-plate connection are fast and simple (Davison *et al.*, 2012). Bolted end-plate connection can be categorised into flush end-plate and extended end-plate connections. The latter has an extra bolt row above beam flange as compared to flush end-plate connection to increase shear and moment resistance capacity.

Recognising the advantage as a moment resisting joint, researchers began to explore this type of connection since 1970s in terms of classification, moment rotation behaviour, and failure mode. Numerical models, analytical analysis, and experimental tests are the common methods used by researchers to study the behaviour of bolted end-plate connections. These methods are complementary and supporting each other's findings. High cost to conduct full scale experimental test is often a problem among researchers. Thus, numerical model and analytical analysis are used to predict the repeating behaviour of experimental test to reduce the cost of multiple experimental test.

The steel industry is moving forward with the aid of researchers. Improvement and optimisation on steel joint have been carried out throughout the century and never ceased. The challenges faced are to reduce the cost of construction, faster and more effective construction methods, and at the same time not neglecting the safety of end users. New members and configurations are introduced to provide an optimal design. For instance, cruciform column section was introduced for solving sophisticated

connection design at minor axis of H-shape steel column and at the same time saving in steel weight (Tahir and Shek, 2005; Tahir *et al.*, 2009).

1.2 Background

Cruciform column section is an innovative compound member introduced as an alternative vertical compressive resistance member. This section which is also known as cruciform column universal beam (CCUB) section, is a combination of two universal beam sections as shown in Figure 1.1. One of the universal beams is cut into half at the centre of minor axis and welded to the other universal beam. Fillet weld resistance must be higher than the parent material to avoid welding failure.

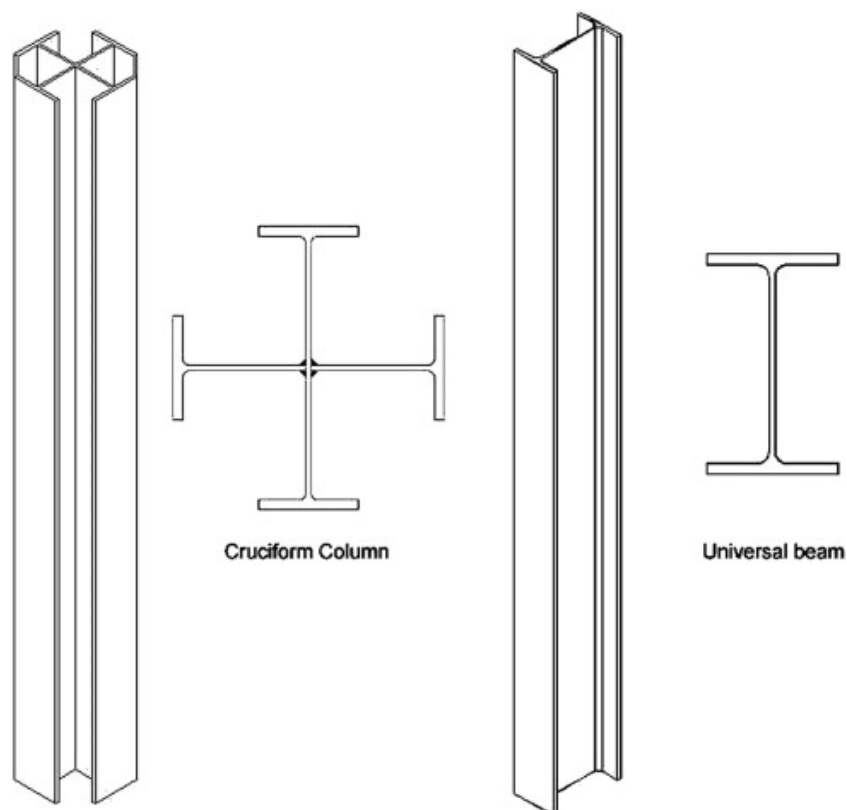


Figure 1.1 Cruciform column and universal beam sections (Tahir *et al.*, 2009)

In a steel frame system, beam to column joint is often simplified as ideally pinned or fully rigid. These two extreme approaches do not represent the actual frame behaviour. Figure 1.2 shows pin, semi-rigid, and rigid joint frame. The pinned joint is

assumed to be simply supported. The beam will bear the full moment of the applied loads, whereas the columns are required to resist axial load and minimal moment from the beam. These assumptions result in a heavy and deep beam. In rigid joint frame, beam will transfer large part of the moment to column through connection. Hence larger columns are required to sustain the end moments of beam. A more complicated fabrication of connection must be provided. These two approaches could cause unrealistic and incorrect prediction in actual frame system which lead to wastage of materials. In fact, the actual frame system exhibits a behaviour between the two mentioned approaches, which is the semi-rigid joint (Cabrero and Bayo, 2005; Díaz *et al.*, 2011b; Girão Coelho, 2013; Saggaff *et al.*, 2007; Weynand *et al.*, 1998).

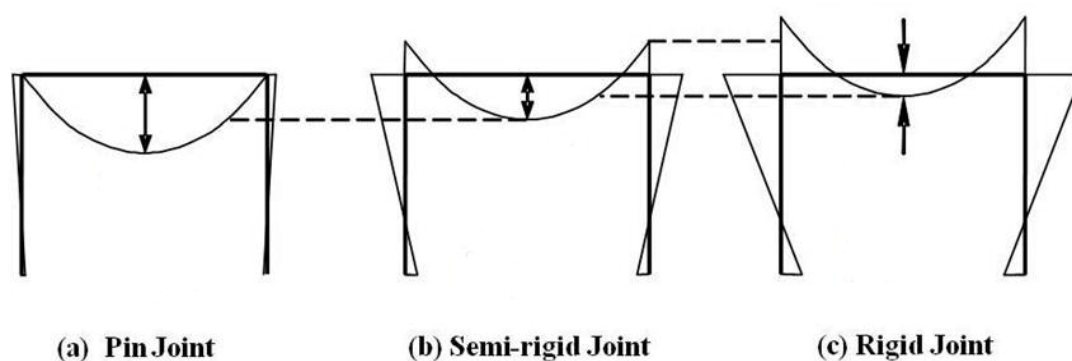


Figure 1.2 Pin, semi-rigid, and rigid frame (Tahir *et al.*, 2005)

1.3 Problem Statement

Previous studies had concluded that the performances of the cruciform column section in terms of axial resistance capacity and wind moment frame resistance capacity are the most cost efficient as compared to existing H-shape column section (Shek *et al.*, 2015; Tahir and Shek, 2005; Tahir *et al.*, 2009). The advantages of cruciform column section will be further discussed in Section 2.2. The moment rotation behaviour and failure mode of beam to cruciform column connection are still in the grey area. Prediction with a factored constant has been used to represent flexibility of cruciform column connection. For example, a factored increase was applied on calculated sway deflection to take into consideration of semi-rigid

connection (Shek *et al.*, 2015). Hence, identifying moment rotation behaviour of cruciform column connection helps to fill gap of research.

The proposed cruciform column section uses I-shaped universal beams instead of conventional H-shaped column. Due to geometrical differences, mainly width to depth ratio and flange thickness, hypothetically existing analytical methods are unable to predict the moment rotation behaviour accurately in terms of initial stiffness and moment resistance (Tan *et al.*, 2015). Numerous prediction models are developed to predict moment rotation behaviour of a joint but the models are often explicitly for the proposed connection. It is difficult to incorporate all geometrical and mechanical properties for all connections (Mohamadi-Shoore and Mofid, 2011).

Developing mathematical model to predict beam to cruciform column bolted end-plate connection requires accurate and large number of results. Due to cost and time constraint, it is impossible to conduct all configurations using experimental tests. Hence, comprehensive three-dimensional finite element models for the proposed beam to cruciform column bolted end-plate connections are required.

1.4 Objectives of Study

The aim of this research is to study the structural behaviour of hybrid beam to cruciform column bolted flush end-plate and extended end-plate connections. Moment rotation comparisons are conducted in terms of initial stiffness, moment resistance, and failure mode between experimental test, analytical analysis, and numerical analysis. Lastly, mathematical models are developed to predict moment resistance using regression analysis and supported by statistical test. These aims consist of the following objectives:

1. To conduct full scale experiment tests on hybrid beam to cruciform column bolted end-plate connections for validation on finite element analysis.

2. To assess the accuracy of existing design code and mathematical models predicting moment rotation behaviour of cruciform column connections using analytical analysis.
3. To simulate moment rotation behaviour of cruciform column connections using three-dimensional finite element models and validate with experimental test results.
4. To develop mathematical models for predicting the initial stiffness and moment resistance of cruciform column connections.

1.5 Scope of Study

The scope of this research involves full scale experimental test, analytical study, numerical and parametric analysis, and regression analysis supported by statistical test on beam to cruciform column bolted flush end-plate and extended end-plate connections. Figure 1.3 shows the flowchart of research.

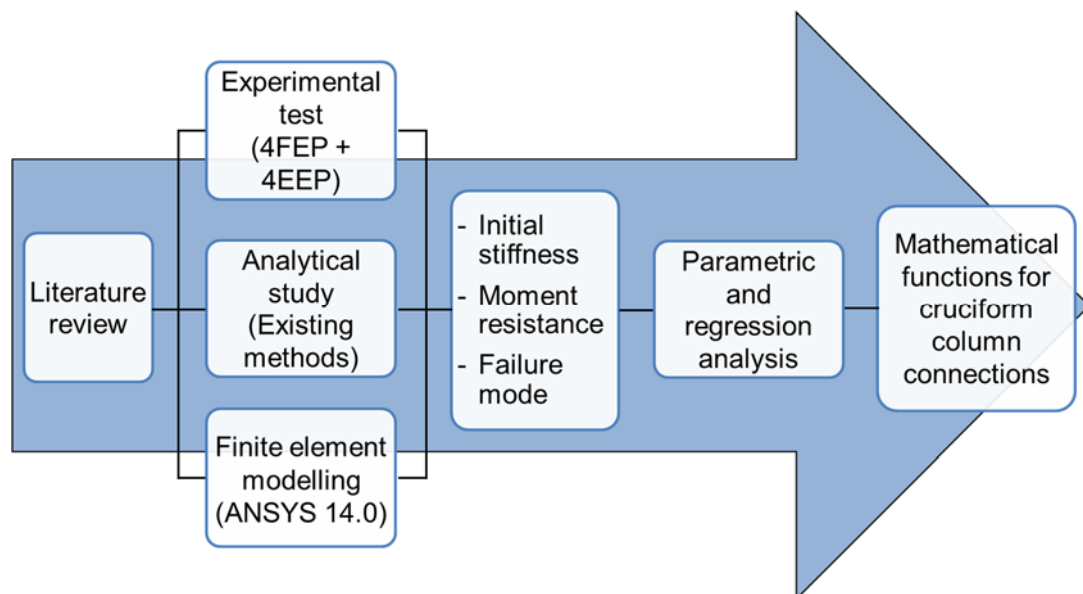


Figure 1.3 Flowchart of research

Full scale experimental tests are carried out to identify the actual behaviour of cruciform column connections. There are a total of 8 experiment test specimens which consist of 4 flush end-plate and 4 extended end-plate connections with varied beam depth, beam width, beam flange thickness, beam web thickness, end-plate thickness, end-plate width, and bolt diameter. The experimental test results are used to validate analytical study and numerical analysis.

Analytical study is carried out to predict initial stiffness and moment resistance of cruciform column connections. It is based on component method using Eurocode 3: Part 1.8 (BSI, 2005b; SCI and BCSA, 1995; SCI and BCSA, 2013), and mathematical models by Frye and Morris (Frye and Morris, 1975), Kukreti *et al.* (Kukreti *et al.*, 1990; Kukreti *et al.*, 1989), Krishnamurthy *et al.* (Krishnamurthy *et al.*, 1979), Bahaari and Sherbourne (Bahaari and Sherbourne, 1997), and Mohamadi and Mofid (Mohamadi-Shoore and Mofid, 2011). These methods are used and compared to experimental results to identify the accuracy and reliability on cruciform column connections.

Finite element method modelling software, ANSYS 14.0 is used for three-dimensional modelling on the proposed cruciform column connections. Finite element models consisting of 4 flush end-plate and 4 extended end-plate beam to cruciform column connections are validated with experimental test results. They are also used numerically to determine initial stiffness, moment resistance, failure mode, and stress distribution.

Validated finite element models are further expanded to conduct parametric analysis on beam depth, beam width, beam web thickness, beam flange thickness, column web thickness, column flange thickness, end-plate thickness, bolt diameter, and bolt gauge distance. The results are used to identify the influencing factor of each parameters towards the behaviour of the proposed cruciform column connections. At the same time, a range of upper and lower limit for each parameters are determined to perform multiple regression analysis.

Development of mathematical functions representing moment rotation curve of cruciform column connections are carried out using regression analysis on initial stiffness, plastic stiffness, reference moment, and shape function. Regression analysis is carried out based on the identified influencing factors and ranges. By using 11 key parameters with a total of 25 variations, a total of 6912 combinations of joints are created. 50 combinations are randomly selected for regression analysis. Statistical significance F-tests are used to determine the significance and reliability of the regression model.

1.6 Significance of Study

Research works on cruciform column section have been carried out since year 2005 and the advantages of cruciform column section are well proven (Shek *et al.*, 2015; Tahir and Shek, 2005; Tahir *et al.*, 2009). The main aims are to introduce and implement cruciform column section in steel industry. This study is important to fill the gap in cruciform column connections behaviour.

Comprehensive three-dimensional modelling for cruciform column flush end-plate and extended end-plate connections are made. Detailed study on the behaviour of cruciform column connection can be done without depending on experimental test. On the other hand, new mathematical models are introduced to predict the moment rotation behaviour of cruciform column connections. These mathematical models are supported by statistical test to provide a reliable prediction on moment rotation behaviour of cruciform column connections. Economical moment rotation behaviour of semi-rigid frame design is adopted.

The developed mathematical models can be applied to computer software. Beam section, column section, end-plate thickness, connection configuration, and material strength can be determined in accordance to the required shear and moment resistance. This makes introducing the proposed new cruciform column section easier to be approached and accepted by steel industry practitioner.

By completing this study, huge database specifically for cruciform column connection is created. More than 200 models of cruciform column connections with various geometrical configurations and material properties are generated and analysed numerically. The moment rotation curve of each models contains important results such as initial stiffness and moment resistance. These models and results can be used as reference for future studies.

1.7 Thesis Outline

This thesis consist of 6 chapters. The first chapter, Chapter 1 gives an overview of this thesis on structural behaviour of end-plate connections on hybrid beam to cruciform column section. A general introduction followed by background study on cruciform column section are discussed. Problem statements, objectives, scope, and significance of this study are highlighted.

Chapter 2 is the literature review of this study. A detailed insight of previous study on cruciform column section and semi-rigid connection are presented. Advantages of cruciform column section are summarised. On the other hand, methods of predicting moment rotation behaviour of a joint are also discussed based on Eurocode 3: Part 1.8 and existing mathematical models proposed by previous researchers. The proposed methods used are evaluated and adopted to develop mathematical model predicting the proposed cruciform column connections.

Chapter 3 begins with the methods and procedures used to conduct the full scale experimental programme of end-plate connections on hybrid beam to cruciform column section. The results obtained from experimental test are briefly discussed and are used to validate analytical and finite element models. Then, analytical study on cruciform column connections using component method based on Eurocode 3: Part 1.8 and existing mathematical models are conducted. Accuracy and suitability of each methods are evaluated.

Chapter 4 focuses on finite element modelling. Modelling procedures, techniques, and assumptions are explained. Finite element modelling is carried out and validated with experiment test results. Stress distribution of each test specimens are explored and discussed based on stress ratio of each individual components of a joint. The most critical component for cruciform column connections is identified. In

Chapter 5, the validated model is used and expanded for parametric analysis to identify the influencing factor of all components (parameters) for cruciform column connections. The range of each respective investigated parameters are identified based on practical and common sizes. These results are used to conduct regression analysis to develop mathematical models predicting the behaviour of cruciform column flush end-plate and extended end-plate connections. Statistical analysis is performed to check the significance and validity of the developed mathematical function.

Lastly, the findings of this study are concluded and summarised in Chapter 6. In addition, recommendation of works for future study are proposed.

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