JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 10, Number 4, Winter 1998

NUMERICAL CONFORMAL MAPPING FOR EXTERIOR REGIONS VIA THE KERZMAN-STEIN KERNEL

A.H.M. MURID, M.Z. NASHED AND M.R.M. RAZALI

Dedicated to Philip M. Anselone, in appreciation of a long friendship and lasting contributions

ABSTRACT. A simple numerical method is described for computing a conformal map from a domain exterior to a smooth Jordan curve in the complex plane onto the exterior of the unit disk. The numerical method is based on a boundary integral equation similar to the Kerzman-Stein integral equation for interior mappings. Typical examples show that numerical results of high accuracy can be obtained provided that the boundaries are smooth.

1. Introduction. Let Γ be a Jordan curve, Ω^+ and Ω^- be the interior and exterior of Γ, respectively. It is well known, see, e.g., [**7**, p. 374], that there exists a unique function $\tilde{R}$, called the *exterior mapping function* of Ω, which maps Ω^- onto the exterior of the unit disk such that $\tilde{R}(\infty) := \lim_{z\to\infty} \tilde{R}(z) = \infty$ and the Laurent series of $\tilde{R}$ at ∞ has the form

$$(1.1) \qquad \tilde{R}(z) = \gamma^{-1} z + a_0 + a_1 z^{-1} + \cdots, \qquad \gamma > 0,\ z \in \Omega^-.$$

The positive constant γ is called the *capacity* of Γ, or the *transfinite diameter* of Ω^+.

Since Γ is a Jordan curve, $\tilde{R}$ can be extended to a homeomorphism of the closure $\Omega^- \cup \Gamma$ to the closure $|w| \geq 1$. Thus as $z(t)$ traverses along Γ, the image point $\tilde{R}(z(t))$ describes the unit circle. Hence

$$(1.2) \qquad \tilde{R}(z(t)) = e^{i\theta(t)}, \qquad 0 \leq t \leq \beta,$$

1991 AMS *Mathematics Subject Classification.* Primary 30C30, Secondary 45B05.
Key words and phrases. Conformal mapping, exterior mapping function, integral equation, Kerzman-Stein kernel, Nyström method.